

ATS 2018 (AMARCHE ANTHONY)

① déjà fait au 2018 SLAGIMAN ANTHONY

$$\textcircled{2} \frac{x}{e^x - 1} = \frac{x}{x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \mathcal{O}(x^5)} = \frac{1}{1 + \frac{x}{2} + \frac{x^2}{6} + \frac{x^3}{24} + \mathcal{O}(x^3)}$$

On effectue alors la division Euclidienne.

$$\begin{array}{r} 1 \\ - (1 + \frac{x}{2} + \frac{x^2}{8} + \frac{x^3}{24} + \mathcal{O}(x^3)) \\ \hline -\frac{x}{2} - \frac{x^2}{8} - \frac{x^3}{24} + \mathcal{O}(x^3) \\ - (-\frac{x}{2} - \frac{x^2}{4} - \frac{x^3}{12} + \mathcal{O}(x^3)) \\ \hline +\frac{x^2}{12} + \frac{1}{24}x^3 + \mathcal{O}(x^3) \\ +\frac{x^2}{12} + \frac{x^3}{24} + \mathcal{O}(x^3) \\ \hline 0 + \mathcal{O}(x^3) \\ - \mathcal{O}(x^3) \\ \hline 0 \end{array} \quad \left| \begin{array}{r} 1 + \frac{x}{2} + \frac{x^2}{6} + \frac{x^3}{24} + \mathcal{O}(x^3) \\ \hline 1 - \frac{x}{2} + \frac{x^2}{12} + \mathcal{O}(x^3) \end{array} \right.$$

Ainsi,  $\frac{x}{e^x - 1} = 1 - \frac{x}{2} + \frac{x^2}{12} + \mathcal{O}(x^3)$