



$$1. \begin{cases} |z_1 z_1'| = 1 \Rightarrow z_1' = \frac{1}{z_1} \quad (z_1 \neq 0) \\ |z_1 z_1'| = \sqrt{3} |z_1| \end{cases} \Rightarrow \begin{cases} z_1 + \frac{1}{z_1} = \sqrt{3} i z_1 \Rightarrow z_1^2 + \frac{1}{z_1} = \sqrt{3} i z_1^2 \Rightarrow z_1^2 (1 - \sqrt{3} i) = -\frac{1}{z_1} \\ z_1^2 = -\frac{1}{z_1} + \frac{\sqrt{3} i}{z_1} = e^{i \frac{2\pi}{3}} \end{cases} \Rightarrow \boxed{z_1 = \pm e^{i \frac{2\pi}{3}} \text{ et } z_1' = \pm \frac{1}{z_1} e^{-i \frac{2\pi}{3}}}$$

$$2. F(x) = \frac{\int_0^x e^{\sqrt{t}} dt}{x} \quad F'(x) = \frac{x e^{\sqrt{x}} - \int_0^x e^{\sqrt{t}} dt}{x^2} = \frac{n(x)}{x^2} \quad (\text{La dérivée de } \int_0^x e^{\sqrt{t}} dt \text{ vaut } e^{\sqrt{x}})$$

$$F' \text{ est du signe de } n(x). \quad n'(x) = e^{\sqrt{x}} + \frac{\sqrt{x}}{2} e^{\sqrt{x}} - e^{\sqrt{x}} \geq 0$$

x	0	+\infty
n'		+
n	0	↗

n(0) = 0 et n est croissante, donc n(x) > 0, \forall x \in \mathbb{R}^+

x	0	+\infty
F'		+
F		↗

F est croissante

3. On trouve det $M_a = a^2(a-1)^2$. Le det s'annule qd $a=0$ et $a=1$

M_a est inversible lorsque $\boxed{a \neq 0 \text{ et } a \neq 1}$

$$4. x^4 - 1 = (x^2)^2 - 1^2 = (x^2+1)(x^2-1) = (x^2+1)(x+1)(x-1)$$

$$F(x) = \frac{2x^2}{x^4-1} = \frac{a}{x+1} + \frac{b}{x-1} + \frac{cx+d}{x^2+1} = \frac{2x^2}{(x^2+1)(x+1)(x-1)}$$

$$a = \frac{2x^2}{(x^2+1)(x-1)} \Big|_{x=-1} = \frac{2}{-4} = -\frac{1}{2} \quad b = \frac{2x^2}{(x^2+1)(x+1)} \Big|_{x=1} = \frac{1}{2}$$

Comme F est paire: $c=0$

$$\text{Valeur en } 0: 0 = -\frac{1}{2} - \frac{1}{2} + d \quad d=1$$

$$\int_{-1/2}^{1/2} \frac{2x^2}{x^4-1} dx = 2 \int_0^{1/2} \frac{2x^2}{x^4-1} dx = 2 \int_0^{1/2} \left(\frac{1/2}{x-1} - \frac{1/2}{x+1} + \frac{1}{x^2+1} \right) dx$$

$$= 2 \left[\frac{1}{2} \ln|x-1| - \frac{1}{2} \ln|x+1| + \arctan x \right]_0^{1/2}$$

$$= 2 \left[\frac{1}{2} \ln\left(\frac{1}{2}\right) - \frac{1}{2} \ln\left(\frac{3}{2}\right) + \arctan \frac{1}{2} \right]$$

$$= \ln \frac{1}{2} - \ln \frac{3}{2} + 2 \arctan \frac{1}{2}$$

$$= -\ln 2 - \ln 3 + \ln 2 + 2 \arctan \frac{1}{2}$$

$$= \underline{\underline{2 \arctan \frac{1}{2} - \ln 3}}$$