

Handwritten signature or scribble in blue ink.



examples.

$$\cos(x + \pi) = \cos x \cdot \underbrace{\cos \pi}_{-1} - \sin x \cdot \underbrace{\sin \pi}_0$$

$$\cos(x + \pi) = -\cos x$$

$$\cos(x + 2\pi) = \cos x$$

$$\sin(x + \pi) = \sin x \cdot \underbrace{\cos \pi}_{-1} + \cos x \cdot \underbrace{\sin \pi}_0$$

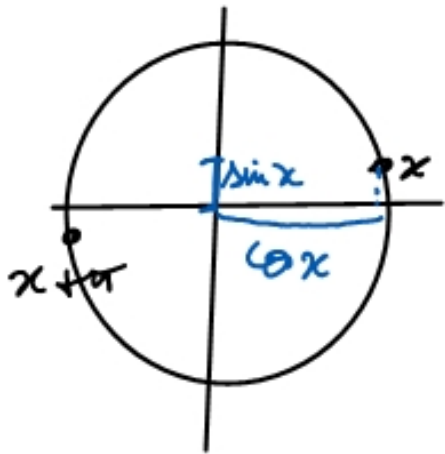
$$\sin(x + \pi) = -\sin x$$

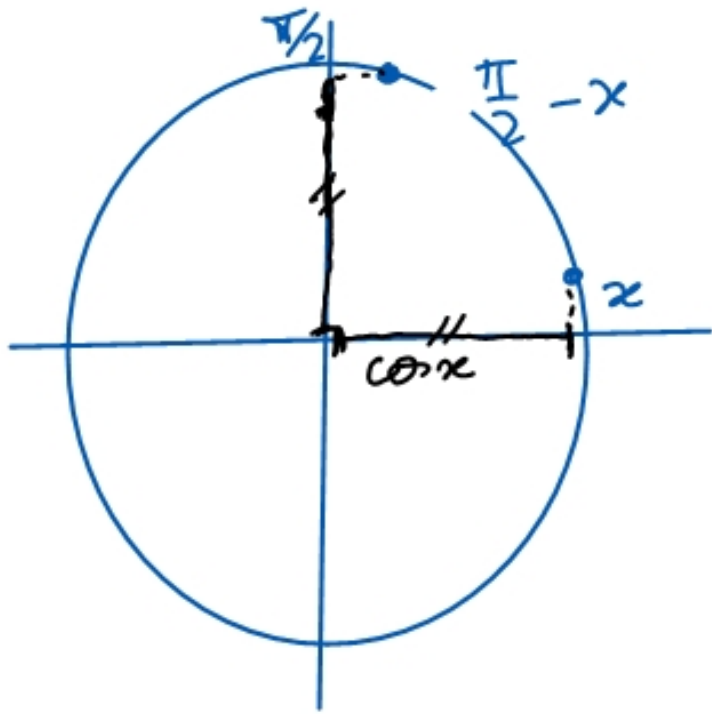
$$\cos(x + 3\pi) = -\cos x$$

$$\cos(x + 4\pi) = \cos x$$

$$\cos(x + n\pi) = (-1)^n \cos x$$

$$\sin(x + n\pi) = (-1)^n \sin x$$



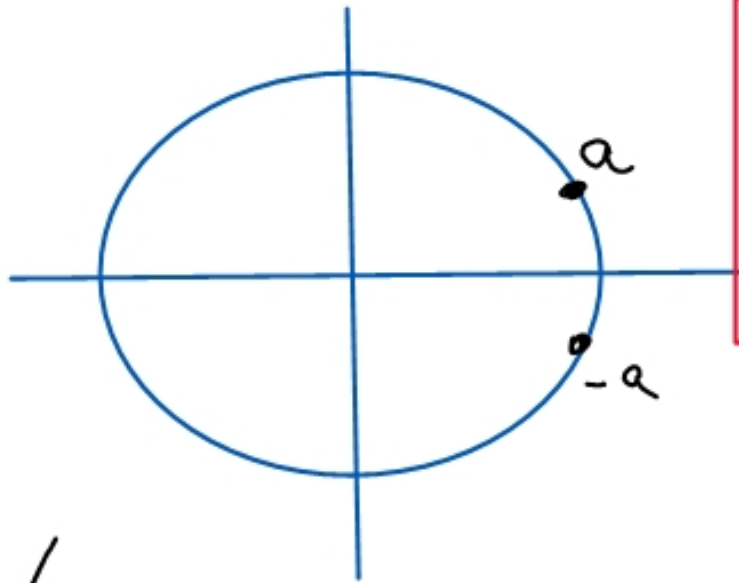


$$\sin\left(\frac{\pi}{2} - x\right) = \underbrace{\sin\left(\frac{\pi}{2}\right)}_1 \cos x - \sin x \underbrace{\cos\left(\frac{\pi}{2}\right)}_0$$

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\left(\cos\left(\frac{\pi}{2} + x\right) = -\sin x \right)$$



$$\cos a = \cos b \Leftrightarrow b = \pm a + 2k\pi$$

où $k \in \mathbb{Z}$

$$\left(\sin a = \sin b \Leftrightarrow b = a + 2k\pi \text{ ou } b = \pi - a + 2k\pi \right)$$

Pour résoudre ce genre d'équations, on se ramène

$$\text{à } \cos a = \cos b$$

ex 1. Trouver les x tels que $\cos(x+\pi) = \cos x$

$$\cos(x+\pi) = \cos x \Leftrightarrow x+\pi = \pm x + 2k\pi \text{ où } k \in \mathbb{Z}$$

1^{er} cas : avec + $x+\pi = x+2k\pi$ pas de renseignement sur x .

2^d cas : avec - $x+\pi = -x+2k\pi$

$$\Leftrightarrow 2x = -\pi + 2k\pi \Leftrightarrow x = -\frac{\pi}{2} + k\pi \text{ où } k \in \mathbb{Z}$$

$$J = \left\{ \dots -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \dots \right\}$$

$$J = \left\{ -\frac{\pi}{2} + k\pi \text{ où } k \in \mathbb{Z} \right\}$$

ex 2 : Trouver les x tels que $\cos(x + \frac{\pi}{2}) = -\sin x$

$$\cos(x + \frac{\pi}{2}) = -\sin x \Leftrightarrow \cos(x + \frac{\pi}{2}) = \sin(-x)$$

Rappel : $\sin x = \cos(\frac{\pi}{2} - x)$

$$\Leftrightarrow \cos(x + \frac{\pi}{2}) = \cos(\frac{\pi}{2} + x) \quad \swarrow^{-(-x)}$$

$$\Leftrightarrow x + \frac{\pi}{2} = \pm(\frac{\pi}{2} + x) + 2k\pi$$

1^{er} cas : $x + \frac{\pi}{2} = x + \frac{\pi}{2} + 2k\pi$ pas d'enseignement sur x

2^d cas : $x + \frac{\pi}{2} = -(\frac{\pi}{2} + x) + 2k\pi \Leftrightarrow 2x = -\pi + 2k\pi$

$$\Leftrightarrow x = -\frac{\pi}{2} + k\pi \text{ où } k \in \mathbb{Z}$$

$$\mathcal{S} = \left\{ -\frac{\pi}{2} + k\pi \mid k \in \mathbb{Z} \right\}$$

ex 3: $\cos(2x) = \underbrace{-\cos x}_{\parallel} \cos(x+\pi)$ On veut trouver les x

$$\cos(2x) = \cos(x+\pi) \Leftrightarrow 2x = \pm(x+\pi) + 2k\pi \text{ où } k \in \mathbb{Z}$$

1^{er} cas: $2x = x + \pi + 2k\pi \Leftrightarrow \boxed{x = \pi + 2k\pi}$ ($\dots -\pi, \pi, 3\pi, 5\pi \dots$)

2^{er} cas: $2x = -x - \pi + 2k\pi \Leftrightarrow 3x = -\pi + 2k\pi$

$$\Leftrightarrow \boxed{x = -\frac{\pi}{3} + \frac{2k\pi}{3}}$$

$$\left(\dots -\frac{\pi}{3}; \frac{\pi}{3} \dots \right)$$

$$\begin{array}{cccccc} \underline{-\pi} & ; & -\frac{\pi}{3} & ; & \frac{\pi}{3} & ; & \underline{\pi} & ; & \frac{5\pi}{3} & ; & \frac{7\pi}{3} & ; & \underline{3\pi} \\ & & \uparrow & & \uparrow & & \uparrow & & \uparrow & & \uparrow & & \uparrow \\ & & k=0 & & k=1 & & k=2 & & k=3 & & k=4 & & k=5 \end{array}$$

Le deuxième paquet contient le premier, donc

$$J = \left\{ -\frac{\pi}{3} + \frac{2k\pi}{3} \text{ où } k \in \mathbb{Z} \right\}$$

ex 4: Quel est l'ensemble de définition de la fonction ?
 $x \mapsto f(x) = \ln(\cos(2x) - \sin x)$?

\mathbb{R} faut que $\cos(2x) - \sin x > 0$

On résout $\cos(2x) - \sin x = 0$, puis on fait un tableau de signe

$$\cos(2x) = \sin x \iff \cos(2x) = \cos\left(\frac{\pi}{2} - x\right)$$

$$\Leftrightarrow \cos x = \pm \left(\frac{\pi}{2} - x\right) + 2k\pi \quad \text{ou } k \in \mathbb{Z}$$

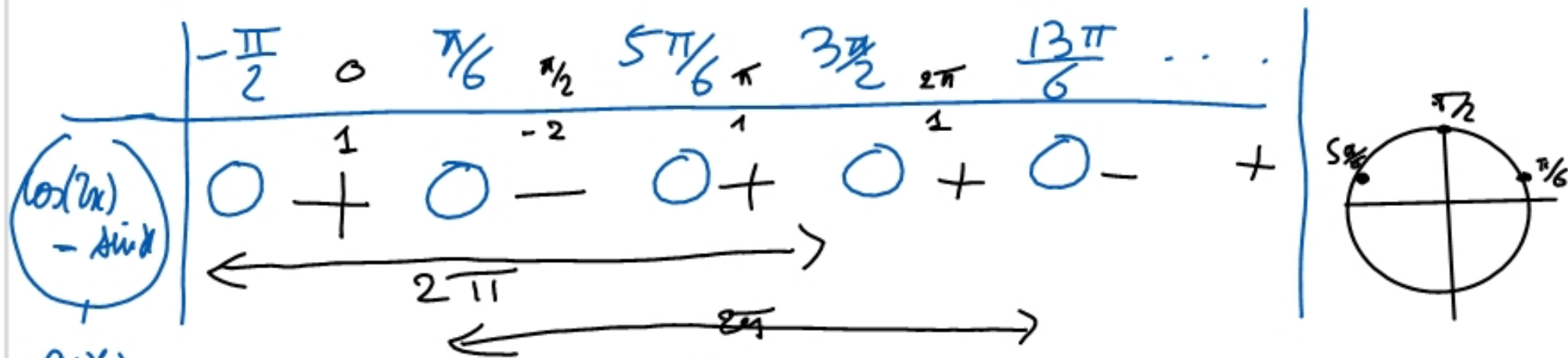
$$1^{\text{e}} \text{ cas: } \cos x = \frac{\pi}{2} - x + 2k\pi \Leftrightarrow 3x = \frac{\pi}{2} + 2k\pi \Leftrightarrow x = \frac{\pi}{6} + \frac{2k\pi}{3}$$

$$\dots; \frac{\pi}{6}; \frac{5\pi}{6}; \frac{9\pi}{6} = \frac{3\pi}{2}; \frac{13\pi}{6}; \dots$$

$$2^{\text{e}} \text{ cas: } \cos x = -\frac{\pi}{2} + x + 2k\pi \Leftrightarrow x = -\frac{\pi}{2} + 2k\pi$$

$$\dots; -\frac{\pi}{2}; \frac{3\pi}{2}; \dots$$

$$\mathcal{S} = \left\{ \frac{\pi}{6} + \frac{2k\pi}{3} \mid k \in \mathbb{Z} \right\}$$



$g(x)$

$$g(0) = \cos(0) - \sin(0) = 1$$

$$g\left(\frac{\pi}{2}\right) = \cos(\pi) - \sin\left(\frac{\pi}{2}\right) = -1 - 1 = -2$$

$$g(\pi) = \cos(2\pi) - \sin(\pi) = 1 - 0 = 1$$

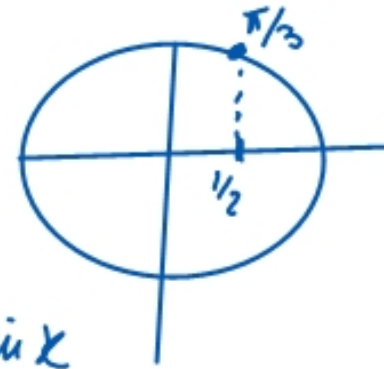
$$g(x+2\pi) = \cos(2x+4\pi) - \sin(x+2\pi) = \cos(2x) - \sin x = g(x)$$

g est 2π -periodique

les x pour les quels $g(x) > 0$ sont $]-\frac{\pi}{2}, \frac{\pi}{6}[;]\frac{5\pi}{6}, \frac{3\pi}{2}[;$
 \dots

ex: Mettre $\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x$ sous la forme $\sin(x - y)$

$\sin(?)$ $\cos(?)$



$$\begin{aligned}\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x &= \sin\left(\frac{\pi}{3}\right)\cos x - \cos\left(\frac{\pi}{3}\right)\sin x \\ &= \sin\left(\frac{\pi}{3} - x\right) = -\sin\left(x - \frac{\pi}{3}\right)\end{aligned}$$

$A \sin(\omega x - \varphi)$

amplitude pulsation déphasage

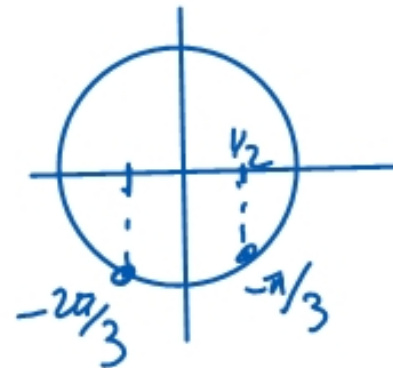
ex Mettre $\sqrt{3} \cos x - \sin x$ sous la forme $A \sin(x - y)$

$$A \sin(x - y) = A \sin x \cos y - A \cos x \sin y$$

$$\Rightarrow \begin{cases} A \cos y = -1 \\ -A \sin y = \sqrt{3} \end{cases} \Rightarrow \begin{cases} A^2 \cos^2 y = 1 \\ A^2 \sin^2 y = 3 \end{cases} \Rightarrow A^2 = 4 \Rightarrow A = \pm 2$$

choix $A = 2$:

$$\begin{cases} \cos y = -\frac{1}{2} \\ \sin y = -\frac{\sqrt{3}}{2} \end{cases}$$



$$y = -\frac{2\pi}{3}$$

$$\sqrt{3} \cos x - \sin x = 2 \sin\left(x + \frac{2\pi}{3}\right)$$

Amplitude 2 : Pulsation $\omega = 1$ déphasage: $-\frac{2\pi}{3}$