

$$a^m \times a^n = a^{m+n}$$

$$(10^2 \cdot 10^3 = 10^5)$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$\frac{1}{a^n} = a^{-n}$$

$$(a^b)^c = a^{bc}$$

$$35^2 \times 20 \times 14$$

9	2268	2
	1134	2
	567	3
	189	3
	63	3
	21	3
	7	7

$$2268 = 2^2 \cdot 3^4 \cdot 7$$

⑧

$$\begin{array}{r|l} 35 & 5 \\ \hline 7 & \end{array}$$

$$35 = 5 \times 7$$

$$20 = 2^2 \cdot 5$$

$$14 = 2 \cdot 7$$

$$\begin{aligned} 35^2 \cdot 20 \cdot 14 &= (5 \cdot 7)^2 \cdot (2^2 \cdot 5) \cdot (2 \cdot 7) = 5^2 \cdot 7^2 \cdot 2^2 \cdot 5 \cdot 2 \cdot 7 \\ &= 2^3 \cdot 5^3 \cdot 7^3 = (2 \cdot 5 \cdot 7)^3 = 70^3 \end{aligned}$$

$$5 \cdot 26 \cdot 130^2$$

$$130 = 2 \cdot 5 \cdot 13$$

$$\begin{array}{r|l} 130 & 2 \\ \hline 65 & 5 \\ 13 & 13 \end{array}$$

$$\begin{aligned} 5 \cdot 26 \cdot 130^2 &= 5 \cdot 2 \cdot 13 \cdot 2^2 \cdot 5^2 \cdot 13^2 \\ &= 2^3 \cdot 5^3 \cdot 13^3 = (130)^3 \end{aligned}$$

$$\textcircled{10} \quad \frac{1}{x^{-1/2}} = x^{1/2} \quad \text{car} \quad \frac{1}{a^b} = a^{-b}$$

$$x^2 \times x^{-1/2} = x^{2-1/2} = x^{\frac{4}{2}-\frac{1}{2}} = x^{\frac{3}{2}} \quad \text{car} \quad a^b \cdot a^c = a^{b+c}$$

$$\frac{x^{-1/2}}{x} = \frac{x^{-1/2}}{x^1} = x^{-1/2-1} = x^{-3/2} \quad \text{car} \quad \frac{a^b}{a^c} = a^{b-c}$$

$$\int \frac{x^7}{\sqrt{x}} dx = \int \frac{x^7}{x^{1/2}} dx = \int x^{7-1/2} dx = \int x^{\frac{13}{2}} dx$$

$$= \frac{x}{\frac{13}{2}+1} = \frac{x}{\frac{15}{2}} = \frac{2x}{15}$$

$$\text{car} \quad \boxed{\int x^n dx = \frac{x^{n+1}}{n+1}}$$

$$\textcircled{4} \quad 8 \cdot 9 \cdot 25 = \underline{2^3 \cdot 3^2 \cdot 5^2}$$

$$\text{So } 12 = 2 \cdot 5^2 \cdot 2^2 \cdot 3 = \underline{2^3 \cdot 3 \cdot 5^2}$$

$$(6 \cdot 25)^2 \cdot 12^4 = (2 \cdot 3 \cdot 5^2)^2 (2^2 \cdot 3)^4 = 2^2 \cdot 3^2 \cdot 5^4 \cdot 2^8 \cdot 3^4 = 2^{10} \cdot 3^6 \cdot 5^4$$

$$\frac{10^2 \cdot 45}{(12 \cdot 20)^2} = \frac{(2 \cdot 5)^2 \cdot (3^2 \cdot 5)}{(2^2 \cdot 3 \cdot 2^2 \cdot 5)^2} = \frac{\cancel{2} \cdot \cancel{3} \cdot \cancel{5} \cdot 5}{2^6 \cancel{2^2} \cancel{3} \cancel{5}} = \underline{\frac{5}{2^6}} = 5 \cdot 2^{-6}$$

Formule de Pascal:

$$\binom{n}{p} + \binom{n}{p+1} = \binom{n+1}{p+1}$$

$\forall p, n$ tels
que
 $0 \leq p < n$

$$\boxed{\binom{n}{p} = \frac{n!}{p!(n-p)!} \quad \binom{n}{p+1} = \frac{n!}{(p+1)!(n-(p+1))!} = \frac{n!}{(p+1)!(n-p-1)!}}$$

$$\left. \begin{array}{l} p! = 1 \cdot 2 \cdot 3 \cdot \dots \cdot p \\ (p+1)! = 1 \cdot 2 \cdot 3 \cdot \dots \cdot p \cdot (p+1) \end{array} \right\} \Rightarrow (p+1)! = p! \cdot (p+1)$$

$$(n-p)! = 1 \cdot 2 \cdot 3 \cdot \dots \cdot (n-p-1)(n-p)$$

$$(n-p-1)! = 1 \cdot 2 \cdot 3 \cdot \dots \cdot (n-p-1)$$

$$(n-p)! = (n-p-1)! \cdot (n-p)$$

$$\binom{n}{p} + \binom{n}{p+1} = \frac{n!}{p!(n-p-1)!(n-p)} + \frac{n!}{p!(p+1)(n-p-1)!}$$

$$= \frac{n! (p+1)}{p!(n-p-1)!(n-p)(p+1)} + \frac{n! (n-p)}{p!(p+1)(n-p-1)!(n-p)}$$

$$= \frac{n!(p+1) + n!(n-p)}{(p+1)!(n-p)!} = \frac{n![\cancel{p+1} + n - \cancel{p}]}{(p+1)!(n-p)!}$$

$$= \frac{n! [n+1]}{(p+1)!(n-p)!} = \frac{(n+1)!}{(p+1)!(n-p)!} = \binom{n+1}{p+1}$$

Application de la formule de binôme à la linéarisation

Rappel: $e^{i\theta} = \cos\theta + i\sin\theta$

$$(e^{i\theta})^n = e^{in\theta} = \cos(n\theta) + i\sin(n\theta)$$

$$e^{-i\theta} = \cos\theta - i\sin\theta$$

$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

$$\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

Linéarisation de $\cos^3 x$

$$\cos^3 x = \left(\frac{e^{ix} + e^{-ix}}{2} \right)^3 = \frac{1}{8} (e^{ix} + e^{-ix})^3$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$\cos^3 x = \frac{1}{8} \left[\underbrace{e^{i3x}} + 3 \underbrace{e^{i2x} \cdot e^{-ix}} + 3 \underbrace{e^{ix} \cdot e^{-i2x}} + \underbrace{e^{-i3x}} \right]$$

$\underbrace{e^{i2x} \cdot e^{-ix}}_{e^{ix}} \quad \underbrace{e^{ix} \cdot e^{-i2x}}_{e^{-ix}}$

$$= \frac{1}{8} \left[\underbrace{e^{i3x} + e^{-i3x}} + 3 \underbrace{(e^{ix} + e^{-ix})} \right]$$

$$= \frac{1}{8} \left[2 \cos(3x) + 3 \cdot 2 \cos(x) \right] = \frac{3 \cos x}{4} + \frac{1}{4} \cos 3x$$

Linéarisation de $\sin^4 x$: $\sin^4 x = \left(\frac{e^{ix} - e^{-ix}}{2i} \right)^4$

$$\sin^4 x = \frac{1}{16} (e^{ix} - e^{-ix})^4$$

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

$$(a-b)^4 = a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4$$

$$\sin^4 x = \frac{1}{16} \left[e^{i4x} - 4 \frac{e^{i3x} e^{-ix}}{e^{i2x}} + 6 \frac{e^{i2x} e^{-i2x}}{e^0=1} - 4 \frac{e^{ix} e^{-i3x}}{e^{-i2x}} + e^{-i4x} \right]$$

$$\sin^4 x = \frac{1}{16} \left[\underbrace{e^{i4x} + e^{-i4x}}_{2\cos(4x)} - 4 \underbrace{(e^{i2x} + e^{-i2x})}_{2\cos(2x)} + 6 \right]$$

$$\sin^4 x = \frac{3}{8} - \frac{1}{2} \cos(2x) + \frac{1}{8} \cos(4x)$$

Linearisation de $f(x) = \cos^3 x \sin^2(5x)$

$$f(x) = \left(\frac{e^{ix} + e^{-ix}}{2} \right)^3 \cdot \left(\frac{e^{i5x} - e^{-i5x}}{2i} \right)^2$$

$$f(x) = -\frac{1}{32} \left[\left(e^{i3x} + 3e^{i2x}e^{-ix} + 3e^{ix}e^{-i2x} + e^{-i3x} \right) \left(e^{i10x} - 2e^0 + e^{-i10x} \right) \right]$$

$$f(x) = -\frac{1}{32} \left[\left(e^{i3x} + 3e^{i2x} + 3e^{-i2x} + e^{-i3x} \right) \left(e^{i10x} - 2 + e^{-i10x} \right) \right]$$

$$= -\frac{1}{32} \left[\underline{e^{i13x}} + 3\underline{e^{i11x}} + 3\underline{e^{i9x}} + e^{i7x} - 2e^{i3x} - 6e^{ix} \right. \\ \left. - 6e^{-ix} - 2e^{-i3x} + \underline{e^{-i7x}} + 3\underline{e^{-i9x}} + 3\underline{e^{-i11x}} + \underline{e^{-i13x}} \right]$$

$$= -\frac{1}{32} \left[\underline{2\cos(13x)} + 6\underline{\cos(11x)} + 6\underline{\cos(9x)} + 2\cos(7x) - 4\cos(3x) - 12\cos x \right]$$

$$\sin^5 x = \left(\frac{e^{ix} - e^{-ix}}{2i} \right)^5$$

$$\begin{array}{ccccccccc}
 & & & & 1 & & 2 & & 1 \\
 & & & & 1 & & 3 & & 3 & & 1 \\
 & & & & 1 & & 4 & & 6 & & 4 & & 1 \\
 & & & & 1 & & 5 & & 10 & & 10 & & 5 & & 1
 \end{array}$$

$$= \frac{1}{32i} \left(e^{i5x} - 5e^{i4x}e^{-ix} + 10e^{i3x}e^{-i2x} - 10e^{i2x}e^{-i3x} + 5e^{ix}e^{-i4x} - e^{-i5x} \right)$$

$$= \frac{1}{32i} \left(e^{i5x} - 5e^{i3x} + 10e^{ix} - 10e^{-ix} + 5e^{-i3x} - e^{-i5x} \right)$$

$$= \frac{1}{32i} \left(e^{i5x} - e^{-i5x} - 5(e^{i3x} - e^{-i3x}) + 10(e^{ix} - e^{-ix}) \right)$$

$$= \frac{1}{32i} \left(2i \sin(5x) - 10i \sin(3x) + 20i \sin(x) \right)$$

$$\sin^5 x = \frac{5}{8} \sin(x) - \frac{5}{16} \sin(3x) + \frac{1}{16} \sin(5x)$$