

FIP 2017

Ex 01 ①. $z_1 = \sqrt{3} + i$; $|z_1| = \sqrt{3+1} = \sqrt{2}$

$\theta = -\frac{\pi}{6}$

$$z_1 = \sqrt{2} \left(\frac{\sqrt{3}}{2} + i \frac{1}{2} \right) = \underline{\sqrt{2} e^{i\frac{\pi}{6}}} /$$

, $z_2 = -1+i$; $|z_2| = \sqrt{1+1} = \sqrt{2}$

$\theta = \frac{3\pi}{4}$

$$z_2 = \sqrt{2} \left(-\frac{1}{\sqrt{2}} + i \frac{1}{\sqrt{2}} \right) = \sqrt{2} \left(-\frac{\sqrt{2}}{2} + i \frac{\sqrt{2}}{2} \right) = \underline{\sqrt{2} e^{i\frac{3\pi}{4}}} /$$

, $z_3 = -3 = \underline{3e^{i\pi}}$

$$\cdot z_4 = z_1 \times z_2 = \sqrt{2} e^{i\left(\frac{\pi}{6} + \frac{3\pi}{4}\right)} = \sqrt{2} e^{i\frac{25\pi}{12}} = \underline{\sqrt{2} e^{i\frac{11\pi}{12}}} /$$

$$\cdot z_5 = \frac{z_1}{z_2} = \frac{\sqrt{2} e^{i\frac{\pi}{6}}}{\sqrt{2} e^{i\frac{3\pi}{4}}} = \sqrt{2} e^{i\left(\frac{\pi}{6} - \frac{3\pi}{4}\right)} = \sqrt{2} e^{i\frac{-2\pi - 9\pi}{12}} = \underline{\sqrt{2} e^{-i\frac{7\pi}{12}}} /$$

$$\cdot z_6 = 2^8 e^{i\frac{\pi}{8}} = 256 e^{i\frac{8\pi}{3}} = 256 e^{i\left(\frac{4\pi}{3} - 8\pi\right)} = \underline{256 e^{i\left(-\frac{20\pi}{3}\right)}} /$$

② $z_7 = 5e^{i\frac{\pi}{3}} = 5\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = \underline{\frac{5}{2} + i\frac{5\sqrt{3}}{2}} /$

$$z_8 = -7e^{-i\pi} = -7 \times (-1) = \underline{7} /$$

$$z_9 = 2 e^{i\frac{5\pi}{6}} = 2 \left(-\frac{\sqrt{3}}{2} + i \frac{1}{2}\right) = \underline{-\sqrt{3} + i} /$$

$$z_{10} = 8 e^{17i\frac{\pi}{2}} = 8 e^{i\left(16i\frac{\pi}{2} + i\frac{\pi}{2}\right)} = 8 e^{i\left(8\pi + \frac{\pi}{2}\right)} = 8 e^{i\frac{\pi}{2}} = \underline{8i} /$$

③ $z^2 + 3z + 5 = 0 \quad \Delta = 9 - 20 = -11$

$$z = \frac{-3 \pm i\sqrt{11}}{2}$$

$$\textcircled{1} \quad xy' + 2y = 0 \quad y = Ae^{-\frac{2}{x}} = Ae^{-2\ln x} = Ae^{\ln(x^{-2})} = Ax^{-2}$$

$\boxed{y_p = \frac{A}{x^2} \text{ bei } A \in \mathbb{R}}$

$$\textcircled{2} \quad g(x) = \frac{\ln x}{x^2}, g'(x) = \frac{\frac{1}{x}x^2 - 2x \cdot \ln x}{x^4} = \frac{x^2 - 2x^2 \ln x}{x^5} = \frac{1 - 2\ln x}{x^3}$$

$$xg'(x) + g(x) = \frac{1 - 2\ln x}{x^2} + \frac{2\ln x}{x^2} = \frac{1}{x^2}$$

daer g est une sol particulière de (E)

$$\textcircled{3} \quad \boxed{y = \frac{A + \ln x}{x^2} \text{ bei } A \in \mathbb{R}}$$

$$\textcircled{4} \quad f(1) = \frac{A + 0}{1} = A = 1 \Rightarrow \boxed{f(x) = \frac{1 + \ln x}{x^2}}$$

$$\textcircled{1} \quad \textcircled{2} \quad (\textcircled{1}) y'' + y' - 2y = 0$$

$$P(x) = x^2 + x - 2 = (x+2)(x-1) \quad \text{doppelte Nullst. - 2et 1}$$

$$\boxed{y_H = A e^{2x} + B e^x \text{ oder } A + B e^x}$$

$$\textcircled{5} \quad y_2 = (ax^2 + bx)e^x \quad y_2' = (2ax + b)e^x + (ax^2 + bx)e^x = (ax^2 + (b+2a)x + b)e^x$$

$$y_2'' = (2ax + b+2a)e^x + (ax^2 + (b+2a)x + b)e^x = (ax^2 + ((a+b)x + (2b+2a)))e^x$$

$$y_2'' + y_2' - 2y_2 = (ax^2 + ((a+b)x + (2b+2a)))e^x + (ax^2 + ((a+b)x + (2b+2a))x + b - 2ax^2 - 2bx)e^x \\ = (6ax + 3b + 2a)e^x = (18x - 5)e^x$$

$$\Rightarrow \begin{cases} 6a = 18 \\ 3b + 2a = -5 \end{cases} \Rightarrow \begin{cases} a = 2 \\ 3b = -5 - 4 = -9 \Rightarrow b = -3 \end{cases}$$

$$\boxed{y_2 = (2x^2 - 3x)e^x}$$

$$\textcircled{6} \quad \boxed{y = A e^{-2x} + B e^x + (2x^2 - 3x)e^x \text{ oder } A + B e^x}$$

$$\textcircled{d} \quad f(0) = A + B = 2$$

$$f'(0) = -2A e^{-2x} + B e^x + (4x - 3)e^x + (2x^2 - 3x)e^x$$

$$f'(0) = -2A + B - 3 = -1$$

$$\begin{cases} A + B = 2 \\ -2A + B = -1 \end{cases} \Rightarrow \begin{cases} A = 0 \\ B = 2 \end{cases}$$

$$\boxed{f(x) = (2x^2 - 3x + 2)e^x}$$

a) $g(x) = 2x^2 - 3x + 2 \underset{x \rightarrow \infty}{\sim} 2x^2$, donc $f(x) \underset{x \rightarrow \infty}{\sim} 2x^2 e^x \xrightarrow{x \rightarrow \infty} +\infty$

b) $f(x) \underset{x \rightarrow \infty}{\sim} 2x^2 e^x \xrightarrow{x \rightarrow -\infty} 0$ (T des croissances comparées)

c) $f(x) = (4x-3)e^x + (2x^2-3x+2)e^x = (2x^2+x-1)e^x$

$$f'(x) = 0 \Leftrightarrow 2x^2+x-1=0 \quad \Delta = 1+8=9$$

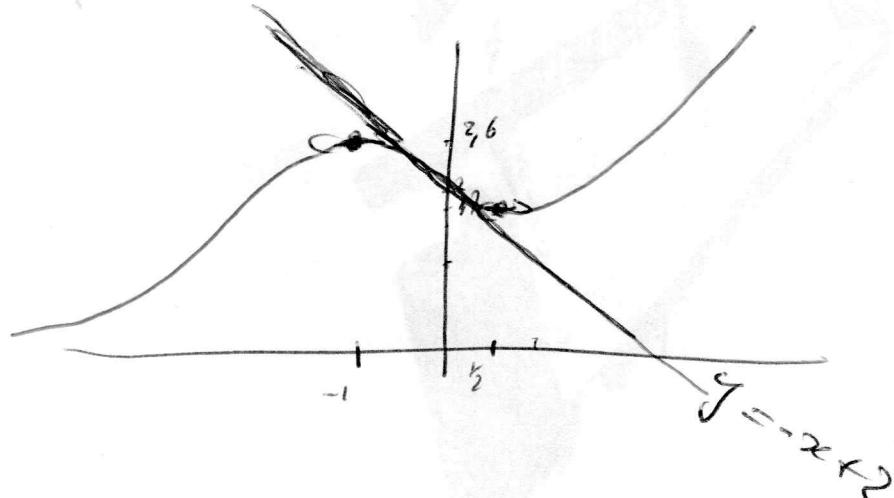
$$x = \frac{-1 \pm 3}{2} = \frac{1}{2} \text{ ou } -1$$

x	$-\infty$	-1	$\frac{1}{2}$	$+\infty$
$f''(x)$	+	0	-	+
$f(x)$	↑	3,6	↓	↑

$$f(-1) = (-1+3+2)e^{-1} = 4e^{-1} \approx 1,4$$

$$f\left(\frac{1}{2}\right) = \left(\frac{1}{2}-\frac{3}{2}+2\right)e^{\frac{1}{2}} = \left(\frac{1}{2}-\frac{3}{2}+2\right)e^{\frac{1}{2}} = (-1+2)e^{\frac{1}{2}} = e^{\frac{1}{2}} \approx 1,4$$

d) $f'(0) = -1$ et $f(0) = 2$ telle: $y = -x + 2$



e) $A = \int_0^1 f(x) dx = \int_0^1 (2x^2 - 3x + 2)e^x dx$ et pp $\begin{cases} u = 2x^2 - 3x + 2 & u' = 4x - 3 \\ v = e^x & v' = e^x \end{cases}$

$$A = \left[(2x^2 - 3x + 2)e^x \right]_0^1 - \int_0^1 (4x-3)e^x dx$$

$$\begin{aligned} &= (e^1 - 2) - \left(\left[(4x-3)e^x \right]_0^1 - \int_0^1 4e^x dx \right) && \begin{cases} u = 4x-3 & u' = 4 \\ v = e^x & v' = e^x \end{cases} \\ &= e - 2 - (e^1 + 3 - [4e^x]_0^1) && = e - 2 - e - 3 + 4e^1 - 4 = 4e - 9 \end{aligned}$$

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6x05

$$\textcircled{1} \quad z^2 = -12 \Leftrightarrow z = \pm \sqrt{12} i \Leftrightarrow \boxed{z = \pm 2\sqrt{3} i} \quad \begin{array}{l} \operatorname{Re}(z) = 0 \\ |\operatorname{Im}(z)| = \pm 2\sqrt{3} \end{array}$$

$$z_1 = 2\sqrt{3}i = 2\sqrt{3} e^{i\frac{\pi}{2}} \quad \text{modulus } 2\sqrt{3}; \text{ argument } \frac{\pi}{2}$$

$$z_2 = -2\sqrt{3}i = 2\sqrt{3} e^{-i\frac{\pi}{2}}$$

$$\textcircled{2} \quad z = 1+i\sqrt{3} \quad |z| = \sqrt{1+3} = 2$$

$$z = 2\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = 2e^{i\frac{\pi}{3}} \quad \text{modulus } 2; \text{ argument } \frac{\pi}{3}$$

$$z^2 = (1+i\sqrt{3})^2. \quad \text{les 2 racines carres de } 2e^{i\frac{\pi}{3}} \text{ sont } \frac{\pm \sqrt{2} e^{i\frac{\pi}{6}}}{2}$$

car les 2 racines carres de Re^2 sont $\pm \sqrt{2} e^{i\frac{\pi}{2}}$

$$\textcircled{3} \quad z^4 + 2z^2 + 4 = 0 \quad \text{on pose } T = z^2$$

$$T^2 + 2T + 4 = 0 \quad \Delta = 4 - 16 = -12 = (2\sqrt{3}i)^2$$

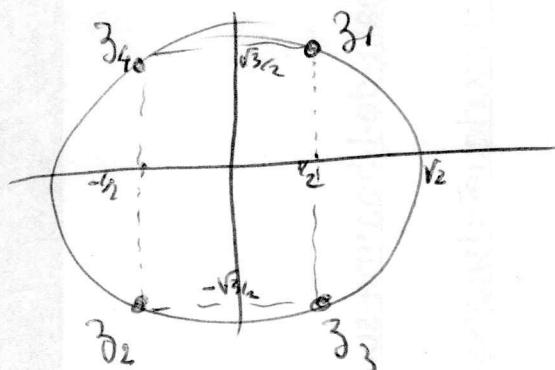
$$T = \frac{-2 \pm 2\sqrt{3}i}{2} = -1 \pm \sqrt{3}i = 2 e^{\pm i\frac{\pi}{3}}$$

$$z = \sqrt{2} e^{\pm i\frac{\pi}{3}}, \text{ donc quatre solutions :}$$

$$z_1 = \sqrt{2} e^{i\frac{\pi}{3}} \quad z_2 = -\sqrt{2} e^{i\frac{\pi}{3}} = \sqrt{2} e^{i(\frac{\pi}{3} + \pi)} = \sqrt{2} e^{-i\frac{2\pi}{3}}$$

$$z_3 = \sqrt{2} e^{-i\frac{\pi}{3}} \quad z_4 = -\sqrt{2} e^{-i\frac{\pi}{3}} = \sqrt{2} e^{i(\frac{\pi}{3} + \pi)} = \sqrt{2} e^{i\frac{2\pi}{3}}$$

Elles sont toutes les 4 sur le cercle de rayon $\sqrt{2}$:



ils forment un rectangle
dont les longueurs des côtés sont
 $2 \times \frac{1}{2} = 1$ et $2 \times \frac{\sqrt{3}}{2} = \sqrt{3}$