

$$\textcircled{1} \begin{cases} x + 2y + z = 4 \\ 2x - y - z = 0 \\ x - 2y + 2z = 1 \end{cases} \Leftrightarrow \begin{array}{l} 2L_1 - L_2 \\ L_1 - L_3 \end{array} \left\{ \begin{array}{l} x + 2y + z = 4 \\ 5y + 3z = 8 \\ 4y - z = 3 \end{array} \right.$$

$$\Leftrightarrow \begin{array}{l} L_2 + 3L_3 \\ L_1 - L_3 \end{array} \left\{ \begin{array}{l} x + 2y + z = 4 \\ 5y + 3z = 8 \\ 17y = 17 \end{array} \right. \Rightarrow y = 1 \Rightarrow z = 1 \Rightarrow x = 1$$

$$\mathcal{S} = \{(1, 1, 1)\}$$

$$\begin{cases} x + 2y + z = 2 \\ x - y - 2z = -1 \\ 2x + y - z = 3 \end{cases} \Leftrightarrow \begin{array}{l} L_1 - L_2 \\ 2L_1 - L_3 \end{array} \left\{ \begin{array}{l} x + 2y + z = 2 \\ 3y + 3z = 3 \\ 3y + 3z = 1 \end{array} \right. \Rightarrow z = 1$$

pas de solutions

$$\underline{\mathcal{S} = \emptyset}$$

$$\textcircled{3} \begin{cases} x + 2y + z = 2 \\ x - y - 2z = -1 \\ 2x + y - z = 1 \end{cases} \Leftrightarrow \begin{matrix} l_1 - l_2 \\ 2l_1 - l_3 \end{matrix} \begin{cases} x + 2y + z = 2 \\ 3y + 3z = 3 \\ 3y + 3z = 3 \end{cases} \Leftrightarrow \begin{cases} x + 2y + z = 2 \\ y + z = 1 \end{cases}$$

3 inconnues - 2 eq = dimension 1. 1 paramètre

On choisit $y = t \Rightarrow l_2: z = 1 - t \Rightarrow l_1: x = 2 - 2t - (1 - t)$
 $x = 1 - t$

$$S = \left\{ (1-t; t; 1-t) \text{ où } t \in \mathbb{R} \right\}$$

c'est la droite de vecteur directeur $\vec{d} \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$ passant par $A \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

$$(5) \begin{cases} x + 2y + 2z = 4 \\ 2x - y - z = 0 \\ x + 2y - 2z = 1 \\ x + y + z = 1 \end{cases} \Leftrightarrow$$

$$\begin{array}{l} 2l_1 - l_2 \\ l_1 - l_3 \\ l_1 - l_4 \end{array}$$

$$\begin{cases} x + 2y + 2z = 4 \\ 5y + 5z = 8 \\ 4z = 3 \\ y + z = 3 \end{cases}$$

$$\Leftrightarrow \begin{array}{l} \frac{1}{2} - 5l_4 \\ \frac{1}{4} - 5l_4 \end{array} \begin{cases} x + 2y + 2z = 4 \\ 0 = -7 \\ 4z = 3 \\ y + z = 3 \end{cases}$$

$$\underline{S = \emptyset}$$

$$\textcircled{5} \begin{cases} x + y + 2z = 4 \\ 2x - y - z = 0 \\ x + 2y - 2z = 1 \\ x + y + z = 3 \end{cases} \xrightarrow{\substack{L_1 - L_2 \\ L_1 - L_3 \\ L_1 - L_4}} \begin{cases} x + y + 2z = 4 \\ 3y + 5z = 8 \\ -y + 4z = 3 \\ z = 1 \end{cases}$$

$$\xrightarrow{L_2 + 3L_4} \begin{cases} x + y + 2z = 4 \\ 3y + 5z = 8 \\ 17z = 17 \\ z = 1 \end{cases} \xrightarrow{L_1 - L_4} \begin{cases} x + y + 2z = 4 \\ 3y + 5z = 8 \\ z = 1 \\ z = 1 \end{cases} \Rightarrow 3y = 3 \Rightarrow y = 1 \xrightarrow{x=1}$$

$$\mathcal{S} = \{ (1, 1, 1) \}$$

$$\textcircled{6} \begin{cases} x+y+z=2 \\ 2x+2y+2z=4 \\ 3x+3y+3z=6 \end{cases} \Leftrightarrow \begin{matrix} 2L_1 - L_2 \\ 3L_1 - L_3 \end{matrix} \begin{cases} x+y+z=2 \\ 0=0 \\ 0=0 \end{cases}$$

$$\Leftrightarrow x+y+z=2 \quad \begin{matrix} 3 \text{ lignes} - 1 \text{ eq} = \text{deux lignes} \\ 2 \text{ paramètres} \end{matrix}$$

$$x=t \text{ et } y=t' : z=2-t-t'$$

$$\underline{S = \{ (t, t', 2-t-t') \text{ où } t, t' \in \mathbb{R} \}}$$

C'est le plan de vecteurs directeurs $\vec{u} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ $\vec{v} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$

passant par $A \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}$

$$\begin{aligned}
 (14) \quad & \left. \begin{aligned} ax + (a-2)y + (a-3)z &= a-1 \\ 2x + (a-1)y + (a-2)z &= 3 \\ 2x & - \boxed{3} = 1 \end{aligned} \right\}
 \end{aligned}$$

x, y, z sont les inconnues
 a est un paramètre.

$$\begin{aligned}
 (\Rightarrow) \quad & \left. \begin{aligned} L_1 + (a-3)L_3 & \left\{ \begin{aligned} (a+2a-6)x + (a-2)y & = a-1+a-3 \\ (2+a-4)x + (a-1)y & = 3+a-2 \\ 2x & - z = 1 \end{aligned} \right. \end{aligned} \right\}
 \end{aligned}$$

$$(\Rightarrow) \quad \left\{ \begin{aligned} (3a-6)x + (a-2)y & = 2a-4 \\ (2a-2)x + (a-1)y & = a+1 \\ 2x & - z = 1 \end{aligned} \right.$$

$$(\Rightarrow) \quad \left\{ \begin{aligned} (a-2)3x + (a-2)y & = (a-2)2 \\ (a-1)2x + (a-1)y & = a+1 \\ 2x & - z = 1 \end{aligned} \right.$$

1^{er} cas $a=2$

le système devient $\begin{cases} 0=0 \\ 2x+y=3 \\ 2x-3z=1 \end{cases} \Leftrightarrow \begin{cases} 2x+y=3 \\ 2x-3z=1 \end{cases}$

3 inconnues - 2 eq = dim 1 1 paramètre $x=t$

$\Rightarrow L_1: y=3-2t$ $L_2: z=t-1$

$S = \{ (t; 3-2t; t-1) \text{ où } t \in \mathbb{R} \}$

2^e cas: $a=1$

le système devient $\begin{cases} L_1 \\ L_2 \\ L_3 \end{cases} 0=2$ $S = \emptyset$

3^e cas $a \neq 1$ et $a \neq 2$

le système devient $\begin{cases} 3x + \boxed{y} = 2 \\ 2x + y = \frac{a+1}{a-1} \\ 2x - 3z = 1 \end{cases}$

$$\begin{cases} 3x + y & = 2 \\ 2x + y & = \frac{a+1}{a-1} \\ 2x - z & = 1 \end{cases} \Leftrightarrow L_1 - L_2 \begin{cases} 3x + y & = 2 \\ x & = 2 - \frac{a+1}{a-1} = \frac{2a-2-(a+1)}{a-1} = -\frac{a-3}{a-1} \\ 2x - z & = 1 \end{cases}$$

Use sub method: $L_2: x = \frac{a-3}{a-1}$

$$L_1: y = 2 - 3x = 2 - \frac{3a-9}{a-1} = \frac{2a-2-(3a-9)}{a-1} = \frac{-a+7}{a-1}$$

$$L_3: z = 2x - 1 = \frac{2a-6}{a-1} - 1 = \frac{2a-6-(a-1)}{a-1} = \frac{a-5}{a-1}$$

$$S = \left\{ \left(\frac{a-3}{a-1}, \frac{-a+7}{a-1}, \frac{a-5}{a-1} \right) \right\}$$

$$(15) \begin{cases} x + ay - 2z = a \\ x - 2y + az = -2 \\ ax + y - 2z = 2 \end{cases} \Leftrightarrow \begin{cases} L_2 \leftarrow L_2 - L_1 \\ L_3 \leftarrow aL_1 - L_3 \end{cases} \begin{cases} x + ay - 2z = a \\ (-2-a)y + (a+2)z = -2-a \\ (a^2-1)y + (2-a)z = a^2-2 \end{cases}$$

1^{er} cas: $a = -2$

le système devient

$$\begin{cases} x - 2y - 2z = -2 \\ \text{---} = 0 \\ 3y + 6z = 2 \end{cases}$$

3 lignes - 2 eq = 1 paramètre. $z = t \Rightarrow L_3: y = \frac{2}{3} - 2t$

$$\Rightarrow L_1: x = 2y + 2z - 2 = \frac{4}{3} - 4t + 2t - 2 = -2t - \frac{2}{3}$$

$$\underline{S = \left\{ \left(-2t - \frac{2}{3}; \frac{2}{3} - 2t; t \right) \text{ où } t \in \mathbb{R} \right\}}$$

2^{es} cas: $a \neq -2$ le système devient:

$$\textcircled{1} \begin{cases} x + 2y + z = 4 \\ 2x - y - z = 0 \\ x - 2y + z = 1 \end{cases} \Leftrightarrow \begin{matrix} 2L_1 - L_2 \\ L_1 - L_3 \end{matrix} \left\{ \begin{array}{l} x + 2y + z = 4 \\ 5y + 3z = 8 \\ 4y - z = 3 \end{array} \right.$$

$$\Leftrightarrow L_2 + 3L_3 \left\{ \begin{array}{l} x + 2y + z = 4 \\ 17y = 17 \\ 4y - z = 3 \end{array} \right. \Leftrightarrow \begin{cases} x + 2y + z = 4 \\ y = 1 \\ 4y - z = 3 \end{cases}$$

$$L_3 \Rightarrow z = 4y - 3 = 1$$

$$L_1: x = 4 - 2y - z = 1$$

$$\mathcal{J} = \{ (1, 1, 1) \}$$

$$\textcircled{2} \quad \left\{ \begin{array}{l} \boxed{x} + 2y + 3z = 2 \\ x - y - 2z = -1 \\ 2x + y - 3z = 3 \end{array} \right. \Leftrightarrow \begin{array}{l} L_1 - L_2 \\ 2L_1 - L_3 \end{array} \left\{ \begin{array}{l} x + 2y + 3z = 2 \\ \boxed{3y} + 3z = 3 \\ 3y + 3z = 1 \end{array} \right.$$

$$\Leftrightarrow \left\{ \begin{array}{l} x + 2y + 3z = 2 \\ 3y + 3z = 3 \\ 0 = -2 \end{array} \right.$$

$L_3 - L_1$
no de soluta. $S = \emptyset$

$$\textcircled{3} \left\{ \begin{array}{l} \boxed{x} + 2y + z = 2 \\ x - y - 2z = -1 \\ 2x + y - z = 1 \end{array} \right. \Leftrightarrow \left. \begin{array}{l} L_2 \leftarrow L_1 - L_2 \\ L_3 \leftarrow 2L_1 - L_3 \end{array} \right\} \begin{array}{l} x + 2y + z = 2 \\ 3y + 3z = 3 \\ 3y + 3z = 3 \end{array}$$

$$\Leftrightarrow \begin{cases} x + 2y + z = 2 \\ y + z = 1 \end{cases}$$

3 inconnues - 2 eq effectives = dimension 1. 1 paramètre

avec $y = t \Rightarrow z = 1 - t \Rightarrow L_1: x = 2 - 2t - (1 - t) + \cancel{t}$

$$\Rightarrow x = -t + 1$$

$$\mathcal{L} = \{ (-t + 1; t; 1 - t) \text{ où } t \in \mathbb{R} \}$$

C'est la droite de vecteur directeur $\vec{d} \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$ passant par $A \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$$\textcircled{4} \begin{cases} \boxed{x} + 2y + 2z = 4 \\ 2x - y - z = 0 \\ x + 2y - 2z = 1 \\ x + y + z = 1 \end{cases}$$

 (\Leftrightarrow)

$$\left. \begin{array}{l} 2L_1 - L_2 \\ L_1 - L_3 \\ L_1 - L_4 \end{array} \right\} \begin{cases} x + 2y + 2z = 4 \\ 5y + 5z = 8 \\ 4z = 3 \\ \boxed{y} + z = 3 \end{cases}$$

$$\Leftrightarrow L_2 - 5L_4 \left\{ \begin{array}{l} x + 2y + 2z = 4 \\ 0 = -7 \\ 4z = 3 \\ y + z = 3 \end{array} \right.$$

$$\underline{S = \emptyset}$$

$$\begin{array}{l}
 \textcircled{5} \quad \left\{ \begin{array}{l} \boxed{x} + y + 2z = 4 \\ 2x - y - z = 0 \\ x + 2y - 2z = 1 \\ x + y + z = 3 \end{array} \right. \Leftrightarrow \begin{array}{l} 2L_1 - L_2 \\ L_1 - L_3 \\ L_1 - L_4 \end{array} \left\{ \begin{array}{l} x + y + 2z = 4 \\ 3y + 5z = 8 \\ \boxed{-y} + 4z = 3 \\ z = 1 \end{array} \right.
 \end{array}$$

$$\Leftrightarrow L_2 + 3L_3 \left\{ \begin{array}{l} x + y + 2z = 4 \\ \boxed{17z = 17} \\ -y + 4z = 3 \\ \boxed{z = 1} \end{array} \right. \quad \text{c'est bon m}$$

$$\Leftrightarrow \left\{ \begin{array}{l} x + y + 2z = 4 \\ -y + 4z = 3 \\ z = 1 \end{array} \right. \Rightarrow y = 4z - 3 = 1 \Rightarrow x = 4 - y - z = 1$$

$$\underline{S = \{ (1, 1, 1) \}}$$

$$\textcircled{6} \quad \left\{ \begin{array}{l} \boxed{x} + y + z = 2 \\ 2x + 2y + 2z = 4 \\ 3x + 3y + 3z = 6 \end{array} \right. \Leftrightarrow \left. \begin{array}{l} L_2 - 2L_1 \\ L_3 - 3L_1 \end{array} \right\} \begin{array}{l} x + y + z = 2 \\ 0 = 0 \\ 0 = 0 \end{array}$$

$$\Leftrightarrow x + y + z = 2$$

3 inconnues - 1 eq = dimension 2. 2 paramètres.

$$x = t \text{ et } y = t' \quad z = 2 - t - t'$$

$$\mathcal{S} = \left\{ (t; t'; 2 - t - t') \text{ où } (t, t') \in \mathbb{R}^2 \right\}$$

C'est le plan de vecteurs directeurs $\vec{u} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ $\vec{v} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$ passant par $A \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}$

$$\textcircled{14} \begin{cases} ax + (a-2)y + (a-3)z = a-1 \\ 2x + (a-1)y + (a-2)z = 3 \\ 2x \quad \quad \quad - \boxed{3} = 1 \end{cases}$$

$$\begin{aligned} & \begin{cases} L_1 \leftarrow L_1 + (a-3)L_3 & (3a-6)x + (a-2)y & = 2a-4 \\ \Rightarrow L_2 \leftarrow L_2 + (a-2)L_3 & (2a-2)x + (a-1)y & = a+1 \\ & 2x \quad \quad \quad - 3 & = 1 \end{cases} \end{aligned}$$

$\begin{matrix} a-1+a-3 \\ \downarrow \\ \hline \end{matrix}$

1^{er} cas: $a=2$

le système devient

$$\begin{cases} 0 = 0 \\ 2x + y = 3 \\ 2x - z = 1 \end{cases}$$

3 inconnues - 2 eq
1 paramètre

avec $x=t$: $z = 2x - 1 = 2t - 1$ et $y = 3 - 2x = 3 - 2t$

$S = \{(t; 3-2t; 2t-1) \text{ ou } t \in \mathbb{R}\}$

$$2^{\text{e}} \text{ cas : } \underline{a=1} \quad L_2 \left. \vphantom{\begin{matrix} 2 \\ 2 \\ 2 \end{matrix}} \right\} 0 = 2 \quad \underline{S = \emptyset}$$

3^e cas : $a \neq 2$ et $a \neq 1$
 Le système devient

$$\left. \begin{array}{l} L_1/a-2 \\ L_2/a-1 \end{array} \right\} \begin{array}{l} 3x + y = 2 \\ 2x + y = \frac{a+1}{a-1} \\ 2x - 3y = 1 \end{array}$$

$$\Leftrightarrow L_1 - L_2 \left\{ \begin{array}{l} 3x + y = 2 \\ x \\ 2x - 3y = 1 \end{array} \right. \begin{array}{l} = 2 \\ = 2 - \frac{a+1}{a-1} = \frac{2a-2-(a+1)}{a-1} = \frac{a-3}{a-1} \\ = 1 \end{array}$$

$$L_2: \boxed{x = \frac{a-3}{a-1}}$$

$$L_1: y = 2 - 3x = 2 - \frac{a-3}{a-1} = \frac{2a-2-(a-3)}{a-1} \quad \boxed{y = \frac{a+1}{a-1}}$$

$$L_3: z = 2x - 1 = \frac{2(a-3) - (a-1)}{a-1} \quad \boxed{z = \frac{a-5}{a-1}}$$

$$\underline{S = \left\{ \left(\frac{a-3}{a-1}, \frac{a+1}{a-1}, \frac{a-5}{a-1} \right) \right\}}$$

$$\textcircled{15} \begin{cases} x + ay - 2z = a \\ x - 2y + az = -2 \\ ax + y - 2z = 2 \end{cases} \Leftrightarrow \begin{matrix} L_1 - L_2 \\ aL_1 - L_3 \end{matrix} \begin{cases} x + ay - 2z = a \\ (a+2)y - (2+a)z = a+2 \\ (a^2-1)y + (-2a)z = a^2-2 \end{cases}$$

1^{er} cas : $a = -2$

le système devient

$$\begin{cases} x - 2y - 2z = -2 \\ 0 = 0 \\ 3y + 6z = 2 \end{cases} \quad \begin{matrix} 3 \text{ lignes} - 2 \text{ eq} \\ \\ 1 \text{ param} \end{matrix}$$

on prend $z = t \rightarrow y = \frac{2-6t}{3} = \frac{2}{3} - 2t = y$

$$x = -2 + 2y + 2z = -2 + \frac{4}{3} - 4t + 2t = -2t - \frac{2}{3} = x$$

$$\mathcal{S} = \left\{ \left(-2t - \frac{2}{3}; \frac{2}{3} - 2t; t \right) \text{ où } t \in \mathbb{R} \right\}$$

2^e cas : $a = 1$

$$L_3 : 0 = -1$$

pas de sol

$\frac{3}{2} \cos$
quand $a \neq -2$ et $a \neq 1$

le système devient

$$\left. \begin{array}{l} L_2/a+2 \\ L_1/a-1 \end{array} \right\}$$

$$x + ay - 2z = a$$

$$y - \boxed{z} = 1$$

$$(a+1)y - 2z = \frac{a^2-2}{a-1}$$

