

$$(15) \begin{cases} \boxed{x} + ay - 2z = a \\ x - 2y + az = -2 \Rightarrow L_1 - L_2 \\ ax + y - 2z = 2 \quad aL_1 - L_3 \end{cases} \left\{ \begin{array}{l} x + ay - 2z = a \\ (a+2)y - (2+a)z = a+2 \\ (a^2-1)y + (2-2a)z = a^2-2 \end{array} \right.$$

1^{er} cas : $\boxed{a = -2}$

$$\begin{cases} x - 2y - 2z = -2 \\ 0 = 0 \\ 3y + 6z = 2 \end{cases} \quad \begin{array}{l} 3 \text{ lignes et } 2 \text{ eq : } 1 \text{ paramètre} \\ \text{On pose } z = t \end{array}$$

$$\Rightarrow 3y = 2 - 6t \Rightarrow y = \frac{2}{3} - 2t$$

$$x = 2y + 2z - 2 = \frac{4}{3} - 4t + 2t - 2 = -\frac{2}{3} - 2t$$

$$\mathcal{S} = \left\{ \left(-\frac{2}{3} - 2t, \frac{2}{3} - 2t, t \right) \mid t \in \mathbb{R} \right\}$$

2^{es} cas : $\boxed{a \neq -2}$

$$\begin{cases} x + ay - 2z = a \\ \boxed{y} - z = 1 \Rightarrow \\ (a^2-1)y + (2-2a)z = a^2-2 \quad L_3 - (a^2)L_2 \end{cases} \left\{ \begin{array}{l} x + ay - 2z = a \\ y - z = 1 \\ \frac{(a^2-2a+1)z}{(a-1)^2} = -1 \end{array} \right.$$

Cas 2.1 : $\boxed{a = 1}$ L_3 devient $0 = -1$ $\mathcal{S} = \emptyset$

Cas 2.2 : $a \neq -2$ et $a \neq 1$

$$z = \frac{-1}{(a-1)^2}$$

$$y = z + 1 = \frac{(a-1)^2 - 1}{(a-1)^2} = \frac{a^2 - 2a}{(a-1)^2}$$

$$\nearrow a^3 - 2a^2 + a$$

$$x = -ay + 2z + a = \frac{-a^3 + 2a^2}{(a-1)^2} + \frac{-2}{(a-1)^2} + \frac{a(a-1)^2}{(a-1)^2}$$

$$x = \frac{a-2}{(a-1)^2}$$

$$\mathcal{S} = \left\{ \left(\frac{a-2}{(a-1)^2}, \frac{a^2-2a}{(a-1)^2}, \frac{-1}{(a-1)^2} \right) \right\}$$