

$$\boxed{114} \left\{ \begin{array}{l} ax + (a-2)y + (a-3)z = a-1 \quad 2l_1 - a l_3 \\ ax + (a-1)y + (a-2)z = 3 \quad \Leftrightarrow l_2 - l_3 \end{array} \right. \left. \begin{array}{l} (2a-4)y + (3a-6)z = \overset{a-2}{\cancel{2a-2}} \\ (a-1)y + (a-1)z = 2 \\ 2x \quad \quad -z = 1 \end{array} \right.$$

1^{er} cas : $\boxed{a=1}$ $l_2: 0=2$ impossible $|\mathcal{F}| = \emptyset$

2^{er} cas : $\boxed{a=2}$ $\left\{ \begin{array}{l} 0=0 \\ y+z=2 \\ 2x-z=1 \end{array} \right.$ 3 équations et 2 eq : dimension 1
On pose $x=t$

$\Rightarrow l_3: z = 2t-1 \Rightarrow l_2: y = 2 - (2t-1) = 3-2t$

$|\mathcal{F}| = \left\{ (t, 3-2t, 2t-1) \mid t \in \mathbb{R} \right\}$

3^{er} cas : $\boxed{a \neq 1 \text{ et } a \neq 2}$ $\left\{ \begin{array}{l} 2y + 3z = 1 \quad \leftarrow \frac{l_1}{a-2} \\ \boxed{y} + z = \frac{2}{a-1} \quad \leftarrow \frac{l_2}{a-1} \\ 2x - z = 1 \end{array} \right.$

$\Leftrightarrow l_1 - 2l_2 \left\{ \begin{array}{l} z = 1 - \frac{4}{a-1} = \frac{a-1-4}{a-1} = \frac{a-5}{a-1} \\ y + z = \frac{2}{a-1} \\ 2x - z = 1 \end{array} \right.$

$\Rightarrow l_2: y = \frac{2}{a-1} - \frac{a-5}{a-1} = \frac{7-a}{a-1}$

$l_3: 2x = z+1 = \frac{a-5}{a-1} + \frac{a-1}{a-1} = \frac{2a-6}{a-1} \Rightarrow x = \frac{a-3}{a-1}$

$|\mathcal{F}| = \left\{ \left(\frac{a-3}{a-1}, \frac{7-a}{a-1}, \frac{a-5}{a-1} \right) \right\}$