

①
$$\begin{cases} \dot{x} = y + z \\ \dot{y} = x + z \\ \dot{z} = x + y \end{cases} \Leftrightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}}_A \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

A est symétrique, donc diagonalisable

→ appelle f l'endomorphisme de \mathbb{R}^3 de matrice A dans la base canonique $B = (\vec{e}_1, \vec{e}_2, \vec{e}_3)$

$$\det(A - X I_3) = \begin{vmatrix} -X & 1 & 1 \\ 1 & -X & 1 \\ 1 & 1 & -X \end{vmatrix} = \begin{array}{l} L_1 + L_2 + L_3 \\ \begin{vmatrix} 2-X & 2-X & 2-X \\ 1 & -X & 1 \\ 1 & 1 & -X \end{vmatrix} \end{array}$$

$$= \begin{vmatrix} 2-X & 0 & 0 \\ 1 & -1-X & 0 \\ 1 & 0 & -1-X \end{vmatrix} \begin{array}{l} \text{deopt} \\ \begin{vmatrix} 2-X & -1-X & 0 \\ 0 & -1-X & \end{vmatrix} \\ \begin{array}{l} C_2 - C_1 \\ C_3 - C_1 \end{array} \end{array}$$

$$= (2-X) (-1-X)^2$$

$$= (2-X) (1+X)^2$$

2 valeurs propres 2 et -1

Espace propre pour la valeur propre 2: $E_2 = \left\{ \vec{u} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 / f(\vec{u}) = 2\vec{u} \right\}$

$$f(\vec{u}) = 2\vec{u} \Leftrightarrow \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 2 \begin{pmatrix} x \\ y \\ z \end{pmatrix} \Leftrightarrow \begin{cases} y+z = 2x \\ x+z = 2y \\ x+y = 2z \end{cases} \Leftrightarrow \begin{cases} -2x + y + z = 0 \\ x - 2y + z = 0 \\ x + y - 2z = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} -2x + y + z = 0 \\ 4+2z \\ 4+2z \end{cases} \begin{cases} -3y + 3z = 0 \\ 3y - 3z = 0 \end{cases} \Leftrightarrow \begin{cases} -2x + y + z = 0 \\ -y + z = 0 \end{cases} \quad \text{dimension 1}$$

avec $y = t$: $z = t$ et $2x = y + z = 2t \Rightarrow x = t$ $E_2 = \{(t, t, t) \mid t \in \mathbb{R}\}$.

$\vec{u} = (1, 1, 1)$ $f(\vec{u}) = 2\vec{u}$

Espace propre pour la valeur propre -1: $E_{-1} = \left\{ \vec{v} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 / f(\vec{v}) = -\vec{v} \right\}$

$$f(\vec{v}) = -\vec{v} \Leftrightarrow \begin{cases} y+z = -x \\ x+z = -y \\ x+y = -z \end{cases} \Leftrightarrow x+y+z=0 \quad \text{dim 2.}$$

on peut prendre $\vec{v} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ et $\vec{w} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$

$f(\vec{v}) = -\vec{v}$ et $f(\vec{w}) = -\vec{w}$

$\mathcal{B}' = (\vec{u}, \vec{v}, \vec{w})$ est une base de \mathbb{R}^3
car $\det_{\mathbb{R}}(\vec{u}, \vec{v}, \vec{w}) = \dots \neq 0$

$$\text{Mat}_{B'}(P) = \begin{pmatrix} P(u) \\ P(v) \\ P(w) \\ \text{d}_B \\ \text{d}_{B'} \end{pmatrix} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = A'$$

Le système dans B $\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = A \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ devient dans B' : $\begin{pmatrix} \dot{x}' \\ \dot{y}' \\ \dot{z}' \end{pmatrix} = A' \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}$

$$\begin{cases} \dot{x}' = 2x' \\ \dot{y}' = -y' \\ \dot{z}' = -z' \end{cases}$$

$$\begin{cases} x' = Ae^{2t} \\ y' = Be^{-t} \\ z' = Ce^{-t} \end{cases}$$

solutions dans B'

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = P_{B \rightarrow B'} \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} \quad \text{ou} \quad P_{B \rightarrow B'} = \begin{pmatrix} P(u) & P(v) & P(w) \\ \text{d}_B & \text{d}_B & \text{d}_B \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & -1 & -1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & -1 & -1 \end{pmatrix} \begin{pmatrix} Ae^{2t} \\ Be^{-t} \\ Ce^{-t} \end{pmatrix} = \begin{pmatrix} Ae^{2t} + Be^{-t} \\ Ae^{2t} + Ce^{-t} \\ Ae^{2t} - Be^{-t} - Ce^{-t} \end{pmatrix}$$

$\leftarrow x(t)$
 $\leftarrow y(t)$
 $\leftarrow z(t)$

FIN

$$\textcircled{9} \det(A - X I_3) = \det \begin{pmatrix} \frac{7}{3} - X & & \\ -\frac{1}{3} & \dots & \\ \dots & \dots & \dots \end{pmatrix} = \begin{vmatrix} \frac{7-3X}{3} & -\frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{7-3X}{3} & \frac{4}{3} \\ \frac{2}{3} & -\frac{2}{3} & \frac{4-3X}{3} \end{vmatrix}$$

$$= \frac{1}{3^3} \begin{vmatrix} 7-3X & -1 & -1 \\ -1 & 7-3X & 1 \\ 2 & -2 & 4-3X \end{vmatrix} = \frac{1}{27} \begin{vmatrix} 6-3X & 6-3X & 0 \\ -1 & 7-3X & 1 \\ 2 & -2 & 4-3X \end{vmatrix} \leftarrow L_1 + L_2$$

$$= \frac{1}{27} \begin{vmatrix} 6-3X & 0 & 0 \\ -1 & 8-3X & 1 \\ 2 & -4 & 4-3X \end{vmatrix} \stackrel{\text{det}/L_1}{=} \frac{1}{27} (6-3X) \begin{vmatrix} 8-3X & 1 \\ -4 & 4-3X \end{vmatrix}$$

$C_2 - C_1$

$$= \frac{1}{27} 3(2-X) [(8-3X)(4-3X) + 4] = \frac{1}{9} (2-X) [32 - 24X - 12X + 9X^2 + 4]$$

$$= \frac{1}{9} (2-X) [9X^2 - 36X + 36] = \frac{1}{9} (2-X) 9 [X^2 - 4X + 4] = (2-X) (X-2)^2$$

$$= (2-X)(2-X)^2 = (2-X)^3$$

The seule valeur propre : 2

Espace propre pour la valeur propre 2: $E_2 = \left\{ \vec{u} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 / f(\vec{u}) = 2\vec{u} \right\}$.

$$\frac{1}{3} \begin{pmatrix} 7 & -1 & -1 \\ -1 & 7 & 1 \\ 2 & -2 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 2 \begin{pmatrix} x \\ y \\ z \end{pmatrix} \Leftrightarrow \begin{cases} 7x - y - z = 6x \\ -x + 7y + z = 6y \\ 2x - 2y + 4z = 6z \end{cases} \Leftrightarrow \begin{cases} |x| - y - z = 0 \\ -x + y + z = 0 \\ 2x - 2y - 2z = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} L_1 + L_2 \\ 2L_1 - L_2 \end{cases} \begin{cases} x - y - z = 0 \\ 0 = 0 \\ 0 = 0 \end{cases}$$

Le déterminant \neq multiple de 2, donc pas diagonalisable

② $\vec{u} = (1, 1, 0)$ et $\vec{v} = (1, -1, 2)$ conviennent

$$\textcircled{3} f(\vec{w}) = \vec{w} + 2\vec{w} \Leftrightarrow \frac{1}{3} \begin{pmatrix} 7 & -1 & -1 \\ -1 & 7 & 1 \\ 2 & -2 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + 2 \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\Leftrightarrow \begin{cases} 7x - y - z = 3 + 6x \\ -x + 7y + z = -3 + 6y \\ 2x - 2y + 4z = 6 + 6z \end{cases} \Leftrightarrow \begin{cases} x - y - z = 3 \\ -x + y + z = -3 \\ 2x - 2y - 2z = 6 \end{cases} \Leftrightarrow x - y - z = 3$$

On peut prendre $\vec{w} = (1, -1, -1)$

$\mathcal{B}' = (\vec{u}, \vec{v}, \vec{w})$ est une base de \mathbb{R}^3

$$\text{Mat}_{\mathcal{B}}(f) = \begin{pmatrix} f(\vec{u}) \\ f(\vec{v}) \\ f(\vec{w}) \end{pmatrix}_{\mathcal{B}'} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix} = A'$$

$\leftarrow f(\vec{w}) = \vec{v} + 2\vec{w}$

④ le système pourri $\begin{cases} 3 \frac{dx}{dt} = 7x - y - z \\ \vdots \end{cases}$ devient dans \mathcal{B}' $\begin{cases} \frac{dz'}{dt} = 2z' & \text{①} \\ \frac{dy'}{dt} = 2y' & \text{②} \\ \frac{dx'}{dt} = y' + 2z' & \text{③} \end{cases}$

① $x'(t) = Ae^{2t}$ ② $y' = Be^{2t}$

③ $\frac{dz'}{dt} - 2z' = Be^{2t}$

Hom: $z'_H = Ce^{2t}$

Sol partic: $z'_2 = dte^{2t}$

$\dot{z}'_2 - 2z'_2 = \frac{de^{2t} + 2dte^{2t}}{z'_2} - 2dte^{2t} = Be^{2t} \Rightarrow d = B$

$z'_2 = Bte^{2t}$

Sol de l'eq complète: $z' = Ce^{2t} + Bte^{2t}$

$$\textcircled{1} \begin{cases} \dot{x} = y+z \\ \dot{y} = x+z \\ \dot{z} = x+y \end{cases} \Leftrightarrow \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}}_A \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

J'appelle $B = (\vec{i}, \vec{j}, \vec{k})$ la base canonique de \mathbb{R}^3

J'appelle f l'endomorphisme de \mathbb{R}^3 tel que $\text{Mat}_B(f) = A$

$$\det(A - X I_3) = \begin{vmatrix} -X & 1 & 1 \\ 1 & -X & 1 \\ 1 & 1 & -X \end{vmatrix} = \begin{vmatrix} 2-X & 1 & 1 \\ 1 & -X & 1 \\ 1 & 1 & -X \end{vmatrix}$$

$$= \begin{vmatrix} 2-X & 0 & 0 \\ 1 & -1-X & 0 \\ 1 & 0 & -1-X \end{vmatrix} = (2-X) \begin{vmatrix} -1-X & 0 \\ 0 & -1-X \end{vmatrix} = (2-X)(-1-X)^2$$

$$= (2-X)(1+X)^2$$

deux valeurs propres : 2 et -1

espace propre pour la valeur propre 2: $E_2 = \left\{ \vec{u} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 / f(\vec{u}) = 2\vec{u} \right\}$.

$$f(\vec{u}) = 2\vec{u} \Leftrightarrow \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 2 \begin{pmatrix} x \\ y \\ z \end{pmatrix} \Leftrightarrow \begin{cases} y + z = 2x \\ x + z = 2y \\ x + y = 2z \end{cases} \Leftrightarrow \begin{cases} -2x + y + z = 0 \\ x - 2y + z = 0 \\ x + y - 2z = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} L_1 + 2L_2 \\ L_1 + 2L_3 \end{cases} \begin{cases} -2x + y + z = 0 \\ -3y + 3z = 0 \Leftrightarrow 3y - 3z = 0 \end{cases} \Leftrightarrow \begin{cases} -2x + y + z = 0 \\ -y + z = 0 \end{cases}$$

3 inconnues
2 eq
donc 1.

$\vec{u} = (1, 1, 1)$ convient

$f(\vec{u}) = 2\vec{u}$

espace propre pour -1: $E_{-1} = \left\{ \vec{v} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 / f(\vec{v}) = -\vec{v} \right\}$

$$\begin{cases} y + z = -x \\ x + z = -y \\ x + y = -z \end{cases} \Leftrightarrow \begin{cases} x + y + z = 0 \\ x + y + z = 0 \\ x + y + z = 0 \end{cases} \Leftrightarrow x + y + z = 0$$

$\vec{v} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$ et $\vec{w} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

$f(\vec{v}) = -\vec{v}$ $f(\vec{w}) = -\vec{w}$

$B' = (\vec{u}, \vec{v}, \vec{w})$ est une base de \mathbb{R}^3 car $\det_B(\vec{u}, \vec{v}, \vec{w}) = \begin{vmatrix} 1 & 1 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & -1 \end{vmatrix} \stackrel{\text{Sarrus}}{=} 1 \cdot (-1 \cdot -1) - 1 \cdot 1 \cdot (-1) = 1 - (-1 - 1) = 3 \neq 0$

$\text{Mat}_{B'}(f) = \begin{pmatrix} f(\vec{u}) \\ f(\vec{v}) \\ f(\vec{w}) \end{pmatrix}_{\text{de } B'}$ $\begin{pmatrix} f(\vec{u}) \\ f(\vec{v}) \\ f(\vec{w}) \end{pmatrix}_{\text{de } B'}$ $\begin{pmatrix} f(\vec{u}) \\ f(\vec{v}) \\ f(\vec{w}) \end{pmatrix}_{\text{de } B'}$ $= \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = A'$

le système dans \mathcal{B} est $\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = A \begin{pmatrix} x \\ y \\ z \end{pmatrix}$. Il devient dans \mathcal{B}' : $\begin{pmatrix} \dot{x}' \\ \dot{y}' \\ \dot{z}' \end{pmatrix} = A' \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}$

$$\begin{pmatrix} \dot{x}' \\ \dot{y}' \\ \dot{z}' \end{pmatrix} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} \Rightarrow \begin{cases} \dot{x}' = 2x' \\ \dot{y}' = -y' \\ \dot{z}' = -z' \end{cases}$$

$$\begin{cases} x'(t) = Ae^{2t} \\ y'(t) = Be^{-t} \\ z'(t) = Ce^{-t} \end{cases}$$

Solutions dans \mathcal{B}'

Formules de changement de base: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \underset{\begin{pmatrix} Ae^{2t} \\ Be^{-t} \\ Ce^{-t} \end{pmatrix}}{P_{\mathcal{B} \rightarrow \mathcal{B}'}} \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}$ où $P_{\mathcal{B} \rightarrow \mathcal{B}'} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{pmatrix}$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} Ae^{2t} + Be^{-t} + Ce^{-t} \\ Ae^{2t} - Be^{-t} \\ Ae^{2t} - Ce^{-t} \end{pmatrix} \begin{matrix} \leftarrow x(t) \\ \leftarrow y(t) \\ \leftarrow z(t) \end{matrix}$$

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~~$$\frac{1}{3} \begin{vmatrix} 7-x & -1 & -1 \\ -1 & 7-x & 1 \\ 2 & -2 & 4-x \end{vmatrix}$$~~

$$\begin{vmatrix} \frac{7-x}{3} & -\frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{7-x}{3} & \frac{1}{3} \\ \frac{2}{3} & -\frac{2}{3} & \frac{4-x}{3} \end{vmatrix} = \begin{vmatrix} \frac{7-3x}{3} \\ -\frac{1}{3} \end{vmatrix}$$

$$= \frac{1}{3^3} \begin{vmatrix} 7-3x & -1 & -1 \\ -1 & 7-3x & 1 \\ 2 & -2 & 4-3x \end{vmatrix}$$

$$= \frac{1}{3^3} \begin{vmatrix} 6-3x & -1 & -1 \\ 6-3x & 7-3x & 1 \\ 0 & -2 & 4-3x \end{vmatrix} = \frac{1}{3^3} \begin{matrix} L_2 - L_1 \\ L_3 - L_1 \end{matrix} \begin{vmatrix} 6-3x & -1 & -1 \\ 0 & 8-3x & 2 \\ 0 & -2 & 4-3x \end{vmatrix}$$

$C_1 + C_2 \quad C_2 \quad C_3$

$$\stackrel{\text{det of } C_1}{=} \frac{1}{27} (6-3x) \begin{vmatrix} 8-3x & 2 \\ -2 & 4-3x \end{vmatrix} = \frac{1}{27} \cdot 3(2-x) [(8-3x)(4-3x) + 4]$$

$$= \frac{1}{9} (2-x) [9x^2 - 24x - 12x + 32 + 4] = \frac{1}{9} (2-x) [9x^2 - 36x + 36]$$

$$= \frac{1}{9} (2-x) 9(x^2 - 4x + 4) = (2-x)(x-2)^2 = \underline{(2-x)^3}$$

calcul de $E_2 = \{ \vec{u} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 / f(\vec{u}) = 2\vec{u} \}$

$$f(\vec{u}) = 2\vec{u} \Leftrightarrow \frac{1}{3} \begin{pmatrix} 7 & -1 & -1 \\ -1 & 7 & 1 \\ 2 & -2 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 2 \begin{pmatrix} x \\ y \\ z \end{pmatrix} \Leftrightarrow \begin{cases} 7x - y - z = 6x \\ -x + 7y + z = 6y \\ 2x - 2y + 4z = 6z \end{cases} \Leftrightarrow \begin{cases} x - y - z = 0 \\ -x + y + z = 0 \\ 2x - 2y - 2z = 0 \end{cases}$$

$$\Leftrightarrow x - y - z = 0 \quad \dim(E_2) = 2 \neq \text{multiplicité de } 2$$

donc A n'est pas diagonalisable

$$\textcircled{2} f(\vec{u}) = \frac{1}{3} \begin{pmatrix} 7 & -1 & -1 \\ -1 & 7 & 1 \\ 2 & -2 & 4 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 6 \\ 6 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} = 2\vec{u} \quad \text{donc est avec } \vec{v}$$

\textcircled{3} On cherche ^{un} $\vec{w} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ tel que $f(\vec{w}) = \vec{v} + 2\vec{w}$

$$\frac{1}{3} \begin{pmatrix} 7 & -1 & -1 \\ -1 & 7 & 1 \\ 2 & -2 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + 2 \begin{pmatrix} x \\ y \\ z \end{pmatrix} \Leftrightarrow \begin{cases} 7x - y - z = 3 + 6x \\ -x + 7y + z = -3 + 6y \\ 2x - 2y + 4z = 6 + 6z \end{cases} \Leftrightarrow \begin{cases} x - y - z = 3 \\ x + y + z = -3 \\ 2x - 2y - 2z = 6 \end{cases}$$

$$\Leftrightarrow x - y - z = 3$$

$$\underline{\vec{w}} = (4, 1, 0) \text{ convient}$$

\textcircled{4} $\mathcal{B}' = (\vec{u}, \vec{v}, \vec{w})$ est une base de \mathbb{R}^3 car $\det(\vec{u}, \vec{v}, \vec{w}) = \dots \neq 0$

$$\text{Mat}_{B'}(f) = \begin{pmatrix} \text{Coord} \\ f(\vec{u}), f(\vec{v}), f(\vec{w}) \\ \downarrow B' \downarrow B' \downarrow B' \end{pmatrix} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix} = A'$$

$$f(\vec{u}) = 2\vec{u} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}_{B'}$$

$$f(\vec{w}) = \vec{v} + 2\vec{w}$$

Le syst d'équations des B' :

$$\begin{pmatrix} \dot{x}' \\ \dot{y}' \\ \dot{z}' \end{pmatrix} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}$$

$$x' = A e^{2t}$$

$$z' = C e^{2t}$$

$$\begin{cases} \dot{x}' = 2x' \\ \dot{y}' = 2y' + z' \\ \dot{z}' = 2z' \end{cases}$$

$$\underline{\dot{y}' - 2y' = C e^{2t}}$$

Hom $y'_H = B e^{2t}$

Sol part $y'_p = \alpha t e^{2t}$

$$\dot{y}'_p = (\alpha + 2\alpha t) e^{2t}$$

$$y'_p - 2y'_p = \alpha e^{2t} + 2\alpha t e^{2t} - 2\alpha t e^{2t} = C e^{2t} \Rightarrow \alpha = C \quad y'_p = C t e^{2t}$$

Sols de l'eq complète

$$\underline{y' = B e^{2t} + C t e^{2t}}$$

chgt de base: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \underset{\mathbb{B} \rightarrow \mathbb{B}'}{P} \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}$ et $\underset{\mathbb{B} \rightarrow \mathbb{B}'}{P} = \begin{pmatrix} 1 & 1 & 4 \\ 0 & -1 & 1 \\ 0 & 2 & 0 \end{pmatrix}$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 1 & 4 \\ 1 & -1 & 1 \\ 0 & 2 & 0 \end{pmatrix} \begin{pmatrix} Ae^{2t} \\ Be^{2t} + tce^{2t} \\ Ce^{2t} \end{pmatrix}$$

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