



①  $\frac{n+1}{n+2} \sim_{n \rightarrow \infty} \frac{n}{n} = 1$ .  $\sum \frac{n+1}{n^2} x^n$  et  $\sum \underbrace{1}_{u_n} x^n$  out in rayon

$$\left| \frac{u_{n+1}}{u_n} \right| = \frac{|x|^{n+1}}{|x|^n} = |x| \xrightarrow{n \rightarrow \infty} |x|$$

Si  $|x| < 1$  alors  $\sum u_n$  CV  $\left. \begin{array}{l} \text{Règle de D'Alembert} \\ \text{Si } |x| > 1 \text{ alors } \sum u_n \text{ D.V.} \end{array} \right\} \text{ donc } \underline{R=1}$

②  $\sum \left( \frac{2^n + n}{n^3} \right) x^n$   $a_n = \frac{2^n + n}{n^3} \sim_{n \rightarrow \infty} \frac{2^n}{n^3} = b_n$

$\sum a_n x^n$  et  $\sum b_n x^n$  out in rayon  $u_n = \frac{2^n}{n^3} x^n$

$$\left| \frac{u_{n+1}}{u_n} \right| = \left| \frac{2^{n+1} x^{n+1} n^3}{(n+1)^3 2^n x^n} \right| = \frac{2(n)^3}{(n+1)^3} |x| \sim_{n \rightarrow \infty} 2|x| \rightarrow 2|x|$$

Si  $2|x| < 1$  alors  $\sum u_n$  CV  $\left. \begin{array}{l} \text{D'Alembert} \\ \text{Si } 2|x| > 1 \text{ alors } \sum u_n \text{ D.V.} \end{array} \right\}$

Si  $|x| < \frac{1}{2}$  alors  $\sum u_n$  CV  
Si  $|x| > \frac{1}{2}$  ——— D.V

$R = \frac{1}{2}$

$$c) \sum \underbrace{\left( e - \left(1 + \frac{1}{n}\right)^n \right)}_{a_n} z^n$$

$$a_n = e - \left(1 + \frac{1}{n}\right)^n = e - e^{n \ln\left(1 + \frac{1}{n}\right)} = e - e^{n \left[ \frac{1}{n} - \frac{1}{2n^2} + o\left(\frac{1}{n^2}\right) \right]}$$

$$= e - e^{1 - \frac{1}{2n} + o\left(\frac{1}{n}\right)} = e - e^1 \cdot e^{-\frac{1}{2n} + o\left(\frac{1}{n}\right)}$$

$$= e - e \left[ 1 - \frac{1}{2n} + o\left(\frac{1}{n}\right) \right] = e - e + \frac{e}{2n} + o\left(\frac{1}{n}\right)$$

$$a_n \underset{n \rightarrow \infty}{\sim} \frac{e}{2n} = b_n. \quad \sum_{n=0}^{\infty} b_n z^n = \sum \left( \frac{e}{2n} z^n \right) \leftarrow U_n$$

$$\left| \frac{U_{n+1}}{U_n} \right| = \frac{\frac{e}{2(n+1)} |z|^{n+1}}{\frac{e}{2n} |z|^n} = |z| \frac{n}{n+1} \xrightarrow{n \rightarrow \infty} |z|$$

Si  $|z| < 1$  alors  $\sum b_n z^n \text{ CV}$  ) D'Abel  
 Si  $|z| > 1$  ————— D'V )  $R = 1$

2a.  $S(x) = \sum_{n=1}^{\infty} \frac{x^n}{n(n+1)}$       $\frac{1}{n(n+1)} \sim \frac{1}{n^2}$

$\left| \frac{x^{n+1}}{(n+1)^2} \frac{n^2}{x^n} \right| = \left( \frac{n}{n+1} \right)^2 |x| \rightarrow |x|$       $R=1$

$\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$       $S(x) = \sum_{n=1}^{\infty} \frac{x^n}{n} - \sum_{n=1}^{\infty} \frac{x^n}{n+1}$

$-\ln(1-x)$       $?$

$\sum_{n=1}^{\infty} \frac{x^n}{n+1} = \sum_{n=2}^{\infty} \frac{x^{n-1}}{n}$

$= \frac{1}{x} \sum_{n=2}^{\infty} \frac{x^n}{n} = \frac{1}{x} \left[ \sum_{n=1}^{\infty} \frac{x^n}{n} - x \right] = \frac{1}{x} (-\ln(1-x) - x)$

$S(x) = -\ln(1-x) - \left( \frac{1}{x} (-\ln(1-x) - x) \right) = -\ln(1-x) + \frac{\ln(1-x)}{x} + 1$

$S(x) = \frac{-x \ln(1-x) + \ln(1-x) + x}{x}$

$$c) S(x) = \sum_1^{\infty} n x^n = ?$$

$$\sum_0^{\infty} x^n = \frac{1}{1-x}. \text{ On dérive : } \sum_{0 \leq n} n x^{n-1} = \frac{1}{(1-x)^2}$$

On multiplie par x :

$$\sum_1^{\infty} n x^n = \frac{x}{(1-x)^2}$$

$$d) S_1(x) = \sum_0^{\infty} n \frac{x^n}{n!}$$

$$\sum_0^{\infty} \frac{x^n}{n!} = e^x. \text{ On dérive : } \sum_1^{\infty} n \frac{x^{n-1}}{n!} = e^x$$

On multiplie par x

$$\sum_0^{\infty} n \frac{x^n}{n!} = x e^x$$

$$S(x) = \sum_0^{\infty} n^2 \frac{x^n}{n!}$$

On dérive :

$$\sum_0^{\infty} n^2 \frac{x^{n-1}}{n!} = (x+1)e^x$$

On multiplie par x :

$$\sum_0^{\infty} n^2 \frac{x^n}{n!} = (x^2 + x)e^x$$

$$\begin{aligned}
 \textcircled{e} \int_{x=1}^{\infty} \frac{(n^2+n+3)x^n}{n!} &= \sum_1^{\infty} \frac{n^2}{n!} x^n + \sum_1^{\infty} \frac{n}{n!} x^n + 3 \sum_1^{\infty} \frac{x^n}{n!} \\
 &= (x^2+x)e^x + xe^x + 3(e^x - 1) \\
 &= (x^2+2x+3)e^x - 3
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{3(a)} \quad e^t &= \sum_0^{+\infty} \frac{t^n}{n!} = 1 + t + \frac{t^2}{2!} + \dots \quad \text{+1 ER} \\
 e^t - 1 &= \sum_1^{+\infty} \frac{t^n}{n!} \Rightarrow \frac{e^t - 1}{t} = \sum_1^{+\infty} \frac{t^{n-1}}{n!}
 \end{aligned}$$

Respon  $\infty$

On intègre:  $\int_0^x \frac{e^t - 1}{t} dt = \sum_1^{+\infty} \left[ \frac{t^n}{n \cdot n!} \right]_0^x = \sum_1^{+\infty} \frac{x^n}{n \cdot n!}$

$$\textcircled{3(b)} \quad g(x) = \int_0^x \frac{d^2 t}{F} dt = \int_0^x \sum_0^{+\infty} \frac{(-1)^n t^{2n}}{(2n+1)! t} dt = \int_0^x \sum_0^{+\infty} \frac{(-1)^n t^{2n-1}}{(2n+1)!} dt = \sum_0^{+\infty} \frac{(-1)^n}{(2n+1)!} \left[ \frac{t^{2n}}{2n+1} \right]_0^x$$

$$g(x) = \sum_0^{+\infty} \frac{(-1)^n x^{2n}}{(2n+1)(2n+1)!}$$

(1a)  $S(x) = \sum \frac{n+1}{n+2} x^n$        $\frac{n+1}{n+2} \sim 1$

$S(x)$  a le în rașon pmo  $\sum_0^{+\infty} x^n$       Rayon 1

(1b)  $S(x) = \sum \frac{2^{n+1}}{n^3} x^n$        $\frac{2^{n+1}}{n^3} \sim \frac{2^n}{n^3}$

$S(x)$  a le în rașon pmo  $\sum_{n=0}^{+\infty} \left( \frac{2^n}{n^3} x^n \right)$       (r des ~)

$\left| \frac{u_{n+1}}{u_n} \right| = \frac{2^{n+1} |x|^{n+1}}{(n+1)^3} \cdot \frac{n^3}{2^n |x|^n} = 2|x| \left( \frac{n}{n+1} \right)^3 \xrightarrow{n \rightarrow +\infty} 2|x|$

Si  $2|x| < 1$ , alas  $\sum u_n$  CV  
 Si  $2|x| > 1$  —————> DV ) D'Alambert

Si  $|x| < \frac{1}{2}$  alas  $\sum u_n$  CV

Si  $|x| > \frac{1}{2}$  alas  $\sum u_n$  Div

Dac  $R = \frac{1}{2}$

$$c) \sum \underbrace{\left( e - \left(1 + \frac{1}{n}\right)^n \right)}_{a_n} z^n$$

$$a_n = e - \left(1 + \frac{1}{n}\right)^n = e - e^{n \ln\left(1 + \frac{1}{n}\right)} = e - e^{n \left[ \frac{1}{n} - \frac{1}{2n^2} + o\left(\frac{1}{n}\right) \right]}$$

$$= e - e^{1 - \frac{1}{2n} + o\left(\frac{1}{n}\right)} = e - e^1 \cdot e^{\left(-\frac{1}{2n} + o\left(\frac{1}{n}\right)\right)} \quad \text{DL}$$

$$= e - e \left[ 1 - \frac{1}{2n} + o\left(\frac{1}{n}\right) \right] = e - e + \frac{e}{2n} + o\left(\frac{1}{n}\right)$$

$$\sum a_n z^n \text{ et } \sum \underbrace{\left( \frac{e}{2n} z^n \right)}_{u_n} \text{ ont } \hat{m} \text{ rayon} \quad \sim \frac{e}{2n}$$

$$\left| \frac{u_{n+1}}{u_n} \right| = \frac{\frac{e}{2(n+1)} |z|^{n+1}}{\frac{e}{2n} |z|^n} = |z| \frac{n}{n+1} \xrightarrow{n \rightarrow \infty} |z|$$

Si  $|z| < 1$  alors  $\sum$  CV )  
 Si  $|z| > 1$  ————— Div )

$$\underline{R=1}$$

(2a)  $S(x) = \sum_{n=1}^{\infty} \frac{x^n}{n(n+1)}$  a le m rayon pue  $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$  car  $\frac{1}{n(n+1)} \sim \frac{1}{n^2}$

$$\left| \frac{x^{n+1}}{(n+1)^2} \frac{n^2}{x^n} \right| = \left( \frac{n}{n+1} \right)^2 |x| \rightarrow |x| \quad \text{D'Alembert} \Rightarrow \underline{R=1}$$

$$\frac{1}{n(n+1)} = \frac{1}{n} + \frac{-1}{n+1}$$

$$S(x) = \sum_{n=1}^{\infty} \frac{x^n}{n} - \sum_{n=1}^{\infty} \frac{x^n}{n+1}$$

$$\sum_{n=1}^{\infty} \frac{x^n}{n+1} = \sum_{n=2}^{\infty} \frac{x^{n-1}}{n} - \ln(1-x)$$

$$= \frac{1}{x} \sum_{n=2}^{\infty} \frac{x^n}{n} = \frac{1}{x} \left[ \sum_{n=1}^{\infty} \frac{x^n}{n} - x \right] = \frac{1}{x} (-\ln(1-x) - x)$$

$$S(x) = -\ln(1-x) - \left( \frac{1}{x} (-\ln(1-x) - x) \right) = -\ln(1-x) + \frac{\ln(1-x) + x}{x}$$

$$S(x) = \frac{-x \ln(1-x) + \ln(1-x) + x}{x}$$



$$c) S(x) = \sum_1^{\infty} n x^n$$

$$\sum_0^{\infty} x^n = \frac{1}{1-x} \quad \text{On dérive : } \sum_0^{\infty} n x^{n-1} = \frac{1}{(1-x)^2}$$

On multiplie par  $x$  :

$$\sum_1^{\infty} n x^n = \frac{x}{(1-x)^2}$$

$$\bullet S(x) = \sum_0^{\infty} n \frac{x^n}{n!}$$

$$\sum_0^{\infty} \frac{x^n}{n!} = e^x \quad \text{On dérive } \sum_1^{\infty} \frac{n x^{n-1}}{n!} = e^x$$

On multiplie par  $x$

$$\sum_0^{\infty} \frac{n x^n}{n!} = x e^x$$

①

On dérive :

$$\sum_0^{\infty} \frac{n^2 x^{n-1}}{n!} = (x+1)e^x$$

On multiplie par  $x$

$$\sum_0^{\infty} \frac{n^2 x^n}{n!} = (x^2 + x) e^x$$

$$\begin{aligned}
 \textcircled{e) } \sum_{n=1}^{\infty} \frac{(n^2+n+3)x^n}{n!} &= \sum_{n=1}^{\infty} \frac{n^2}{n!} x^n + \sum_{n=1}^{\infty} \frac{n}{n!} x^n + 3 \sum_{n=1}^{\infty} \frac{x^n}{n!} \\
 &= (x^2+x)e^x + xe^x + 3(e^x-1) \\
 &= \underline{(x^2+2x+3)e^x - 3}
 \end{aligned}$$

$$\textcircled{3a) } e^t = \sum_0^{+\infty} \frac{t^n}{n!} = 1 + t + \frac{t^2}{2!} + \dots \quad \forall t \in \mathbb{R}$$

$$e^t - 1 = \sum_1^{+\infty} \frac{t^n}{n!} \Rightarrow \frac{e^t - 1}{t} = \sum_1^{+\infty} \frac{t^{n-1}}{n!}$$

$$\text{On int\egre: } \int_0^x \frac{e^t - 1}{t} dt = \sum_1^{+\infty} \left[ \frac{t^{n-1}}{n \cdot n!} \right]_0^x = \underline{\sum_1^{+\infty} \frac{x^n}{n \cdot n!}}$$