

$$\textcircled{1} \cdot U_n = \frac{n^2 + 1}{-3n^4 + n + 1} \sim \frac{n^2}{-3n^4} = -\frac{1}{3n^2}$$

$$\sum_{n=1}^{+\infty} \frac{1}{n^2} \text{ CV (Riemann; } 2 > 1) \Rightarrow \sum_{n=1}^{+\infty} -\frac{1}{3n^2} \text{ CV} \Rightarrow \underline{\sum_{n=1}^{+\infty} u_n \text{ CV}} \quad (\text{T des } \mathcal{N})$$

$$\cdot U_n = \frac{2^n + n^3}{3^n + n} \quad 2^n + n^3 \underset{+\infty}{\sim} 2^n \text{ car } \frac{2^n + n^3}{2^n} = 1 + \frac{n^3}{2^n} \rightarrow 1 + 0$$

$\Downarrow$  FI L.T.C.C.

$$U_n \sim \frac{2^n}{3^n} = \left(\frac{2}{3}\right)^n \cdot \sum_{n=0}^{+\infty} \left(\frac{2}{3}\right)^n \text{ CV (série géométrique de raison } \frac{2}{3} \in ]-1, 1[ )$$

$$\left( \sum_{n=0}^{+\infty} \left(\frac{2}{3}\right)^n = \frac{1}{1 - \frac{2}{3}} = \frac{3}{3-2} = 3 \right)$$

$$\Rightarrow \underline{\sum_{n=1}^{+\infty} u_n \text{ CV}} \quad (\text{T des } \mathcal{N})$$

$$(2) \sin\left(\frac{1}{n^2}\right) = a_n \underset{n \rightarrow +\infty}{\sim} \frac{1}{n^2}. \quad \sum_1^{+\infty} \frac{1}{n^2} \text{ CV (Riemann; } 2 > 1) \Rightarrow \underline{\sum_1^{+\infty} a_n \text{ CV}} (\forall \text{ da } \mathcal{N})$$

$$b_n = \sqrt{n} \sin\left(\frac{1}{n}\right) \underset{n \rightarrow +\infty}{\sim} \sqrt{n} \cdot \frac{1}{n} = \frac{1}{\sqrt{n}} = \frac{1}{n^{0.5}} \quad \sum_1^{+\infty} \frac{1}{n^{0.5}} \text{ DIV (Riemann)} \Rightarrow \underline{\sum_1^{+\infty} b_n \text{ DIV}} (\forall \text{ da } \mathcal{N})$$

$$c_n = \frac{\sin(\sqrt{n})}{\sqrt{n}} \underset{n \rightarrow +\infty}{\sim} \frac{1}{\sqrt{n}} = \frac{1}{n^{0.5}} = \frac{1}{n^{1/2}} = \frac{1}{n^{3/2}}$$

$$\sin(x) = x - \frac{x^3}{6} \dots \underset{x \rightarrow 0}{\sim} x$$

$$\sin\left(\frac{1}{n}\right) = \frac{1}{n} + o\left(\frac{1}{n}\right) \underset{n \rightarrow +\infty}{\sim} \frac{1}{n}$$

$$\sum_1^{+\infty} \frac{1}{n^{3/2}} \text{ CV (Riemann)} \Rightarrow \sum_1^{+\infty} c_n \text{ CV} (\forall \text{ da } \mathcal{N})$$

$$(3) u_n = e^{\frac{1}{n^2}} - 1 = 1 + \frac{1}{n^2} + o\left(\frac{1}{n^2}\right) - 1 = \frac{1}{n^2} + o\left(\frac{1}{n^2}\right) \underset{n \rightarrow +\infty}{\sim} \frac{1}{n^2}$$

$$\sum_1^{+\infty} \frac{1}{n^2} \text{ CV (Riemann; } 2 > 1) \Rightarrow \sum_1^{+\infty} u_n \text{ CV} (\forall \text{ da } \mathcal{N})$$

(4)  $u_n = \tan(\pi \sqrt{n^2+1})$   ~~$\sim \tan(\pi n) = 0$~~   $\sim \tan(\pi n) = 0$  on ne peut pas composer l' $\sim$  de  $\sqrt{n^2+1}$  avec  $\tan$ .

$$u_n = \tan(\pi n \sqrt{1 + \frac{1}{n^2}})$$

$$\sqrt{1+x} = 1 + \frac{1}{2}x + o(x)$$

$$= \tan(\pi n (1 + \frac{1}{2n^2} + o(\frac{1}{n^2}))) = \tan(\pi n + \frac{\pi}{2n} + o(\frac{1}{n}))$$

$$= \tan(\frac{\pi}{2n} + o(\frac{1}{n}))$$
 car la fonction  $\tan$  est  $\pi$ -périodique

$$u_n = \frac{\pi}{2n} + o(\frac{1}{n}) + o(\frac{1}{n}) = \frac{\pi}{2n} + o(\frac{1}{n}) \sim \frac{\pi}{2n}$$

$$\sum_{n=1}^{+\infty} \frac{1}{n} \text{ DIV (Riemann)} \Rightarrow \sum_{n=1}^{+\infty} \frac{\pi}{2n} \text{ DIV} \Rightarrow \sum_{n=1}^{+\infty} o(\frac{1}{n}) \text{ DIV } (\forall \text{ des } \mathcal{N})$$

(19)  $a_n = \frac{n^3}{2^n + n} \sim \frac{n^3}{2^n} = b_n$   $(\frac{2^{n+1}}{2^n} = 1 + \frac{1}{2^n} \xrightarrow{+\infty} 1 + 0 \leftarrow \forall \text{cc})$

$$\left| \frac{b_{n+1}}{b_n} \right| = \frac{(n+1)^3}{2^{n+1}} \cdot \frac{2^n}{n^3} = \frac{(n+1)^3}{2n^3} \sim \frac{n^3}{2n^3} = \frac{1}{2} \quad \left| \frac{b_{n+1}}{b_n} \right| \rightarrow \frac{1}{2} < 1$$

$$\sum b_n \text{ CV (D'Alembert)} \Rightarrow \sum a_n \text{ CV } (\forall \text{ des } \mathcal{N})$$

$$(20) \quad a_n = e^{-n} \quad \frac{a_{n+1}}{a_n} = \frac{e^{-(n+1)}}{e^{-n}} = \frac{e^{-n-1}}{e^{-n}} = \frac{e^{-n} \cdot e^{-1}}{e^{-n}} = e^{-1} \rightarrow e^{-1} < 1$$

donc  $\sum a_n$  CV

$$b_n = \left(\frac{1}{n}\right)^n \quad \left| \frac{b_{n+1}}{b_n} \right| = \frac{\left(\frac{1}{n+1}\right)^{n+1}}{\left(\frac{1}{n}\right)^n} = \frac{n^n}{(n+1)^{n+1}} = \frac{n^n}{(n+1)^n (n+1)} = \left(\frac{n}{n+1}\right)^n \cdot \frac{1}{n+1}$$

$$\frac{n}{n+1} < 1 \Rightarrow \left(\frac{n}{n+1}\right)^n < 1 \Rightarrow 0 \leq \left| \frac{b_{n+1}}{b_n} \right| < 1 \cdot \frac{1}{n+1} \rightarrow 0, \text{ car } \lim_{n \rightarrow \infty} \left| \frac{b_{n+1}}{b_n} \right| = 0$$

(V de Jordan)

$\Rightarrow$   $\sum b_n$  CV (D'Alembert)

$$(21) \quad u_n = \frac{1}{n!} \quad \left| \frac{u_{n+1}}{u_n} \right| = \frac{n!}{(n+1)!} = \frac{1}{n+1} \rightarrow 0. \quad \sum u_n \text{ CV (D'Alembert)}$$

$$\textcircled{1} u_n = \frac{n^2 + 1}{-3n^4 + n^4} \underset{+\infty}{\sim} \frac{n^2}{-3n^4} = -\frac{1}{3n^2}$$

$$\sum_{n=1}^{+\infty} \frac{1}{n^2} \text{ CV (Riemann)} \Rightarrow \sum_{n=1}^{+\infty} -\frac{1}{3n^2} \text{ CV} \Rightarrow \sum_{n=1}^{+\infty} u_n \text{ CV (T des } \sim)$$

$$v_n = \frac{2^n + n^3}{3^n + n} \underset{+\infty}{\sim} \frac{2^n}{3^n} = \left(\frac{2}{3}\right)^n \left( \text{car } \frac{2^n + n^3}{2^n} = 1 + \frac{n^3}{2^n} \rightarrow 1 + 0 \leftarrow \text{TCC} \right)$$

$$\sum_{n=0}^{+\infty} \left(\frac{2}{3}\right)^n \text{ CV (Série Géométrique de raison } \frac{2}{3} \in ]-1, 1[ \text{ )}$$

$$\left( \sum_{n=0}^{+\infty} \left(\frac{2}{3}\right)^n = \frac{1}{1 - \frac{2}{3}} = \frac{3}{3-2} = 3 \right)$$

$$\Rightarrow \underline{\sum_{n=0}^{+\infty} u_n \text{ CV (T des } \sim)}$$



$$\textcircled{2} \quad a_n = \sin\left(\frac{1}{n^2}\right) = \frac{1}{n^2} + \mathcal{O}\left(\frac{1}{n^2}\right) \underset{+\infty}{\sim} \frac{1}{n^2} \quad \sum_1^{\infty} \frac{1}{n^2} \text{CV (Riemann)} \Rightarrow \underline{\sum_1^{\infty} \frac{1}{n^2} \text{CV (TdsV)}}$$

$$b_n = \sqrt{n} \sin\left(\frac{1}{n}\right) \underset{+\infty}{\sim} \sqrt{n} \frac{1}{n} = \frac{1}{\sqrt{n}} \quad \sum_1^{+\infty} \frac{1}{\sqrt{n}} \text{Div (Riemann } \frac{1}{2} < 1) \Rightarrow \underline{\sum_1^{+\infty} \frac{1}{\sqrt{n}} \text{Div (TdsV)}}$$

$$c_n = \frac{\sin\left(\frac{1}{n}\right)}{\sqrt{n}} \underset{+\infty}{\sim} \frac{\frac{1}{n}}{\sqrt{n}} = \frac{1}{n\sqrt{n}} = \frac{1}{n^{1+1/2}} = \frac{1}{n^{3/2}} = \frac{1}{n^{3/2}}$$

$$\sum_1^{+\infty} \frac{1}{n^{3/2}} \text{CV (Riemann)} \Rightarrow \sum_1^{+\infty} c_n \text{CV (TdsV)}$$

$$\textcircled{3} \quad u_n = e^{\frac{1}{n^2}} - 1 \quad e^x = 1 + x + \mathcal{O}(x^2)$$

$$u_n = 1 + \frac{1}{n^2} + \mathcal{O}\left(\frac{1}{n^2}\right) - 1 = \frac{1}{n^2} + \mathcal{O}\left(\frac{1}{n^2}\right) \underset{+\infty}{\sim} \frac{1}{n^2}$$

$$\sum_1^{+\infty} \frac{1}{n^2} \text{CV (Riemann)} \Rightarrow \underline{\sum_1^{+\infty} u_n \text{CV (TdsV)}}$$

$$(4) u_n = \tan(\pi \sqrt{n^2+1}) = \tan\left(\pi n \sqrt{1+\frac{1}{n^2}}\right)$$

$$\sqrt{1+X} = (1+X)^{1/2} = 1 + \frac{1}{2}X + o(X)$$

$$u_n = \tan\left(\pi n \left(1 + \frac{1}{2n^2} + o\left(\frac{1}{n^2}\right)\right)\right) = \tan\left(\pi n + \frac{\pi}{2n} + o\left(\frac{1}{n}\right)\right)$$

$$= \tan\left(\frac{\pi}{2n} + o\left(\frac{1}{n}\right)\right) \quad \text{car } \tan \text{ est } \pi\text{-périodique}$$

↳ On peut faire le DL

$$= \frac{\pi}{2n} + o\left(\frac{1}{n}\right) + o\left(\frac{1}{n}\right) = \frac{\pi}{2n} + o\left(\frac{1}{n}\right) \underset{n \rightarrow \infty}{\sim} \frac{\pi}{2n}$$

$$\sum_{n=1}^{+\infty} \frac{1}{n} \text{DIV (Riemann)} \Rightarrow \sum_{n=1}^{+\infty} \frac{\pi}{2n} \text{DIV} \Rightarrow \underline{\sum_{n=1}^{+\infty} u_n \text{DIV}} \quad (\forall \text{ ds } \mathcal{N})$$

$$(19) a_n = \frac{n^3}{2^{n+1}} \underset{n \rightarrow \infty}{\sim} \frac{n^3}{2^n} = b_n \quad \left| \frac{b_{n+1}}{b_n} \right| = \frac{\frac{(n+1)^3}{2^{n+1}}}{\frac{n^3}{2^n}} = \frac{(n+1)^3 \cdot 2^n}{2^{n+1} \cdot n^3} = \frac{1}{2} \left(\frac{n+1}{n}\right)^3 \xrightarrow{n \rightarrow \infty} \frac{1}{2} < 1$$

donc  $\sum b_n \text{cv}$  (D'Alembert)  $\Rightarrow \underline{\sum a_n \text{cv}}$  ( $\forall \text{ ds } \mathcal{N}$ )

(20) •  $a_n = e^{-n}$       $\left| \frac{a_{n+1}}{a_n} \right| = \frac{e^{-(n+1)}}{e^{-n}} = \frac{e^{-n-1}}{e^{-n}} = e^{-1} \xrightarrow{n \rightarrow +\infty} e^{-1} < 1$ . Donc  $\sum a_n$  CV  
(D'Alembert)

•  $b_n = \left(\frac{1}{n}\right)^n$       $\left| \frac{b_{n+1}}{b_n} \right| = \frac{\left(\frac{1}{n+1}\right)^{n+1}}{\left(\frac{1}{n}\right)^n} = \frac{n^n}{(n+1)^{n+1}} = \frac{n^n}{(n+1)^n} \cdot \frac{1}{n+1} = \left(\frac{n}{n+1}\right)^n \cdot \frac{1}{n+1} \xrightarrow{n \rightarrow +\infty} 0 < 1$

donc  $\sum b_n$  CV (D'Alembert)

(21) •  $u_n = \frac{1}{n!}$       $\left| \frac{u_{n+1}}{u_n} \right| = \frac{n!}{(n+1)!} = \frac{\cancel{2 \dots n}}{2 \cdot 3 \dots n \cdot (n+1)} = \frac{1}{n+1} \xrightarrow{n \rightarrow +\infty} 0 < 1$

donc  $\sum u_n$  CV (D'Alembert)

•  $v_n = \frac{x^n}{n!}$  ( $x \in \mathbb{R}^*$ )      $\left| \frac{v_{n+1}}{v_n} \right| = \frac{|x|^{n+1}}{(n+1)!} \cdot \frac{n!}{|x|^n} = \frac{|x|}{n+1} \xrightarrow{n \rightarrow +\infty} 0 < 1$

donc  $\sum_{n=1}^{+\infty} v_n$  CV (D'Alembert)



$$(II) \quad u_n = \frac{1}{\frac{1}{2} + \frac{2}{3} + \dots + \frac{n-1}{n}}$$

$$\frac{1}{2} \leq \frac{1}{2} + \frac{2}{3} + \dots + \frac{n-1}{n} \leq 1 + 1 + \dots + 1 \leq n$$

$$\Rightarrow \frac{1}{n} \leq u_n \leq \frac{1}{\frac{1}{2}} = 2$$

$$\sum_1^{+\infty} \frac{1}{n} \text{ D}iV \text{ (Riemann)} \Rightarrow \underline{\sum_1^{+\infty} u_n} \text{ D}iV \text{ (T de comparaison)}$$