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(26) $a_n = (-1)^n \sqrt{n} \sin\left(\frac{1}{n}\right) = (-1)^n \sqrt{n} \left(\frac{1}{n} + o\left(\frac{1}{n}\right)\right) = \frac{(-1)^n}{\sqrt{n}} + o\left(\frac{1}{\sqrt{n}}\right)$. On ne peut pas conclure
donc DL à l'ordre 3.

$$a_n = (-1)^n \sqrt{n} \left(\frac{1}{n} - \frac{1}{6n^3} + o\left(\frac{1}{n^3}\right) \right) = \underbrace{\frac{(-1)^n}{\sqrt{n}}}_{u_n} - \underbrace{\frac{(-1)^n}{6n^{5/2}} + o\left(\frac{1}{n^{5/2}}\right)}_{v_n}$$

$\sum u_n$ CV (CSA)

$$|v_n| \sim \frac{1}{6n^{5/2}} \quad \sum \frac{1}{n^{5/2}} \text{ CV (Riemann)} \Rightarrow \sum \frac{1}{6n^{5/2}} \text{ CV} \Rightarrow \sum |v_n| \text{ CV} \quad (\forall \epsilon > 0)$$

$$\Rightarrow \sum v_n \text{ CVA} \Rightarrow \underline{\sum u_n} \text{ CV}$$

$$\Rightarrow \sum a_n \text{ CV} \quad (\text{CV} + \text{CV} = \text{CV})$$

$$(26) \quad a_n = (-1)^n \sin\left(\frac{1}{n^{1/3}}\right) = (-1)^n \left[\frac{1}{n^{1/3}} - \frac{1}{6n} + o\left(\frac{1}{n}\right) \right] = \underbrace{(-1)^n \frac{1}{n^{1/3}}}_{u_n} - \underbrace{\frac{(-1)^n}{6n} + o\left(\frac{1}{n}\right)}_{v_n}$$

On ne peut pas conclure pour $\sum u_n$, donc DL plus gd :

$$a_n = (-1)^n \left[\frac{1}{n^{1/3}} - \frac{1}{6n} + \frac{1}{5! n^{5/3}} + o\left(\frac{1}{n^{5/3}}\right) \right] = \underbrace{\frac{(-1)^n}{n^{1/3}}}_{u_n} - \underbrace{\frac{(-1)^n}{6n}}_{v_n} + \underbrace{\frac{(-1)^n}{5! n^{5/3}} + o\left(\frac{1}{n^{5/3}}\right)}_{w_n}$$

• $\sum u_n$ et $\sum v_n$ CV (CSA)

• $|\omega_n| \sim \frac{1}{5! n^{5/3}} \quad \sum \frac{1}{n^{5/3}} \text{ CV (Riemann)} \Rightarrow \sum \frac{1}{5! n^{5/3}} \text{ CV}$

$\Rightarrow \sum |\omega_n| \text{ CV (rd, N)} \Rightarrow \sum \omega_n \text{ CV (CVA} \Rightarrow \text{CV)}$

$\sum a_n \text{ CV (CV} + \text{CV} = \text{CV)}$

$$(27) \quad u_n = \frac{(-1)^n}{n^\alpha (-1)^n} = \frac{(-1)^n}{n^\alpha \left[1 + \frac{(-1)^n}{n^\alpha}\right]} \quad \text{DL de } \frac{1}{1+x} = 1 - x + o(x)$$

$$u_n = \frac{(-1)^n}{n^\alpha} \left[1 - \frac{(-1)^n}{n^\alpha} + o\left(\frac{1}{n^\alpha}\right)\right] = \frac{(-1)^n}{n^\alpha} - \frac{(-1)^{2n}}{n^{2\alpha}} + o\left(\frac{1}{n^{2\alpha}}\right) = \underbrace{\frac{(-1)^n}{n^\alpha}}_{a_n} - \underbrace{\frac{1}{n^{2\alpha}} + o\left(\frac{1}{n^{2\alpha}}\right)}_{b_n}$$

• $\sum a_n$ (CSA)

• $b_n \sim \frac{1}{n^{2\alpha}}$ $\sum_{n=1}^{+\infty} \frac{1}{n^{2\alpha}}$ $\left\{ \begin{array}{l} \text{CV qd } 2\alpha > 1 \text{ car } \alpha > \frac{1}{2} \\ \text{DIV qd } \alpha \leq \frac{1}{2} \end{array} \right.$

$\Rightarrow \sum b_n$ $\left\{ \begin{array}{l} \text{CV qd } \alpha > \frac{1}{2} \\ \text{DIV qd } \alpha \leq \frac{1}{2} \end{array} \right.$ (T des \mathcal{N})

$\Rightarrow \sum u_n$ $\left\{ \begin{array}{l} \text{CV qd } \alpha > \frac{1}{2} \\ \text{DIV qd } \alpha \leq \frac{1}{2} \end{array} \right.$ (CV + CV = CV
CV + DIV = DIV)

$$(36) \quad a_n = \frac{1}{n(n+1)} \underset{n \rightarrow \infty}{\sim} \frac{1}{n^2} \quad \sum_1^{\infty} \frac{1}{n^2} \text{ CV (Riemann)} \Rightarrow \underline{\sum a_n \text{ CV (TDSU)}}$$

$$\frac{1}{n(n+1)} \stackrel{E(m)+}{=} \frac{a}{n} + \frac{b}{n+1}$$

$$E(m)=0 \text{ car } \deg(\text{num}) < \deg(\text{den})$$

$$a = \frac{1}{n+1} \Big|_{n=0} = 1$$

$$b = \frac{1}{n} \Big|_{n=-1} = -1$$

$$\frac{1}{n} - \frac{1}{n+1} = \frac{n+1 - n}{n(n+1)} = \frac{1}{n(n+1)}$$

$$\sum_{n=1}^N \frac{1}{n(n+1)} = \sum_{n=1}^N \left(\frac{1}{n} - \frac{1}{n+1} \right) = \sum_{n=1}^N \frac{1}{n} - \sum_{n=1}^N \frac{1}{n+1} = \sum_{n=1}^N \frac{1}{n} - \sum_{n=2}^{N+1} \frac{1}{n}$$

décalage d'indice

$$\sum_1^N \frac{1}{n} = \frac{1}{1} + \sum_2^N \frac{1}{n} \quad \text{et} \quad \sum_{n=2}^{N+1} \frac{1}{n} = \sum_2^N \frac{1}{n} + \frac{1}{N+1}$$

$$\sum_1^N \frac{1}{n} - \sum_2^{N+1} \frac{1}{n} = 1 + \cancel{\sum_2^N \frac{1}{n}} - \left(\cancel{\sum_2^N \frac{1}{n}} + \frac{1}{N+1} \right) = 1 - \frac{1}{N+1} \xrightarrow{N \rightarrow \infty} 1$$

donc

$$\boxed{\sum_1^{+\infty} a_n = 1}$$

$$(28) u_n = \ln \left(1 + \frac{(-1)^n}{\sqrt{n}} \right) = \frac{(-1)^n}{\sqrt{n}} - \frac{(-1)^{2n}}{2n} + \mathcal{O}\left(\frac{1}{n}\right) = \underbrace{\frac{(-1)^n}{\sqrt{n}}}_{a_n} - \underbrace{\frac{1}{2n}}_{b_n} + \mathcal{O}\left(\frac{1}{n}\right)$$

$$\underline{\sum a_n} \text{ (CSA)} \cdot b_n \sim \frac{1}{2n} \cdot \sum_n \perp \text{DIV (Riemann)} \Rightarrow \sum_n \perp \text{DIV}$$

$$\Rightarrow \underline{\sum b_n} \text{ DIV (TdsN)} \Rightarrow \underline{\sum u_n} \text{ DIV (CV + DIV = DN)}$$

$$(5) u_n = n+2 - \sqrt{n^2+4n+1} = n+2 - n \sqrt{1 + \frac{4}{n} + \frac{1}{n^2}}$$

$$\sqrt{1+X} = (1+X)^{1/2} = 1 + \frac{1}{2}X + \frac{\frac{1}{2}(-\frac{1}{2})}{2}X^2 + \mathcal{O}(X^2) = 1 + \frac{1}{2}X - \frac{1}{8}X^2 + \mathcal{O}(X^2)$$

$$u_n = n+2 - n \left[1 + \frac{1}{2} \left(\frac{4}{n} + \frac{1}{n^2} \right) - \frac{1}{8} \left(\frac{4}{n} + \frac{1}{n^2} \right)^2 + \mathcal{O}\left(\frac{1}{n^2}\right) \right]$$

$$= n+2 - n \left[1 + \frac{2}{n} + \frac{1}{2n^2} - \frac{1}{8} \left(\frac{16}{n^2} + \dots \right) + \mathcal{O}\left(\frac{1}{n^2}\right) \right] = n+2 - n \left[1 + \frac{2}{n} - \frac{3}{2n^2} + \mathcal{O}\left(\frac{1}{n^2}\right) \right]$$

$$= n+2 - n - 2 + \frac{3}{2n} + \mathcal{O}\left(\frac{1}{n}\right) = \frac{3}{2n} + \mathcal{O}\left(\frac{1}{n}\right) \sim \frac{3}{2n}$$

$$\sum_n \perp \text{DIV (Riemann)} \Rightarrow \sum \frac{3}{2n} \text{ DIV} \Rightarrow \underline{\sum a_n} \text{ DIV (TdsN)}$$

(26) $a_n = (-1)^n \sqrt{n} \sin\left(\frac{1}{n}\right) = (-1)^n \sqrt{n} \left(\frac{1}{n} + o\left(\frac{1}{n}\right)\right) = \frac{(-1)^n}{\sqrt{n}} + o\left(\frac{1}{\sqrt{n}}\right)$. On ne peut pas conclure
 donc DL à l'ordre 3 ↑ DL

$$a_n = (-1)^n \sqrt{n} \left(\frac{1}{n} - \frac{1}{6n^3} + o\left(\frac{1}{n^3}\right)\right) = \underbrace{\frac{(-1)^n}{\sqrt{n}}}_{u_n} - \underbrace{\frac{(-1)^n}{6n^{5/2}}}_{v_n} + o\left(\frac{1}{n^{3/2}}\right)$$

$\sum u_n$ CV (CSA)

$$|u_n| \sim \frac{1}{6n^{5/2}} \quad \sum_{n=1}^{+\infty} \frac{1}{n^{5/2}} \text{ CV (Riemann)} \Rightarrow \sum \frac{1}{6n^{5/2}} \text{ CV} \Rightarrow \sum |u_n| \text{ CV} \quad (\forall d, \cup)$$

$$\Rightarrow \sum v_n \text{ CV} \quad (\text{CVA} \Rightarrow \text{CV})$$

$$\Rightarrow \sum a_n \text{ CV} \quad (\text{CV} + \text{CV} = \text{CV})$$

$$(26) \quad a_n = (-1)^n \sin\left(\frac{1}{n\sqrt{3}}\right) = (-1)^n \left[\frac{1}{n\sqrt{3}} - \frac{1}{6n} + o\left(\frac{1}{n}\right) \right] = \underbrace{\frac{(-1)^n}{n\sqrt{3}}}_{u_n} - \underbrace{\frac{(-1)^n}{6n} + o\left(\frac{1}{n}\right)}_{v_n}$$

On ne peut pas conclure pour $\sum u_n$, donc DL plus gd

$$a_n = (-1)^n \left[\frac{1}{n\sqrt{3}} - \frac{1}{6n} + \frac{1}{5!n^5\sqrt{3}} + o\left(\frac{1}{n^5\sqrt{3}}\right) \right] = \underbrace{\frac{(-1)^n}{n\sqrt{3}}}_{u_n} - \underbrace{\frac{(-1)^n}{6n}}_{v_n} + \underbrace{\frac{(-1)^n}{5!n^5\sqrt{3}} + o\left(\frac{1}{n^5\sqrt{3}}\right)}_{w_n}$$

• $\sum u_n$ et $\sum v_n$ CV (CSA)

• $|\omega_n| \sim \frac{1}{5!n^5\sqrt{3}}$ $\sum \frac{1}{n^5\sqrt{3}}$ (Riemann) $\Rightarrow \sum \frac{1}{5!n^5\sqrt{3}}$ CV

$\Rightarrow \sum |\omega_n|$ CV (+dobV) $\Rightarrow \sum \omega_n$ CV (CVA \Rightarrow CV)

$\sum a_n$ CV (CV + CV = CV)

$$(36) \quad a_n = \frac{1}{n(n+1)} \underset{+\infty}{\sim} \frac{1}{n^2} \quad \sum_1^{\infty} \frac{1}{n^2} \text{ CV (Riemann)} \Rightarrow \underline{\sum a_n \text{ CV (TdrU)}}$$

$$\frac{1}{n(n+1)} = \frac{a}{n} + \frac{b}{n+1}$$

$$a = \frac{1}{n+1} \Big|_{n=0} = 1 \quad b = \frac{1}{n} \Big|_{n=-1} = -1 \quad \text{vérif: } \frac{1}{n} - \frac{1}{n+1} = \frac{n+1-n}{n(n+1)} = \frac{1}{n(n+1)}$$

$$\sum_{n=1}^N \frac{1}{n(n+1)} = \sum_{n=1}^N \left(\frac{1}{n} - \frac{1}{n+1} \right) = \sum_{n=1}^N \frac{1}{n} - \sum_{n=1}^N \frac{1}{n+1} = \sum_{n=1}^N \frac{1}{n} - \underbrace{\sum_{n=2}^{N+1} \frac{1}{n}}_{\text{décalage d'indice}}$$

$$\sum_1^N \frac{1}{n} = 1 + \sum_2^N \frac{1}{n} \quad \text{et} \quad \sum_{n=2}^{N+1} \frac{1}{n} = \sum_2^N \frac{1}{n} + \frac{1}{N+1}$$

$$\sum_1^N \frac{1}{n} - \sum_2^{N+1} \frac{1}{n} = \left(1 + \cancel{\sum_2^N \frac{1}{n}} \right) - \left(\cancel{\sum_2^N \frac{1}{n}} + \frac{1}{N+1} \right) = 1 - \frac{1}{N+1} \xrightarrow{N \rightarrow +\infty} 1$$

donc

$$\boxed{\sum_1^{+\infty} a_n = 1}$$

$$(28) u_n = \ln \left(1 + \frac{(-1)^n}{\sqrt{n}} \right) = \frac{(-1)^n}{\sqrt{n}} - \frac{(-1)^{2n}}{2n} + O\left(\frac{1}{n}\right) = \underbrace{\frac{(-1)^n}{\sqrt{n}}}_{a_n} - \underbrace{\frac{1}{2n}}_{b_n} + O\left(\frac{1}{n}\right)$$

$$\begin{aligned} \sum a_n \text{ CV (CSA)} \quad b_n \sim \frac{1}{2n} \quad \sum \frac{1}{n} \text{ DIV (Riemann)} &\Rightarrow \sum \frac{1}{2n} \text{ DIV} \\ &\Rightarrow \sum b_n \text{ DIV (T de V)} \Rightarrow \sum u_n \text{ DIV (CV + DIV = DIV)} \end{aligned}$$

$$(5) u_n = n+2 - \sqrt{n^2 + 4n + 1} = n+2 - n \sqrt{1 + \frac{4}{n} + \frac{1}{n^2}} \rightarrow 0$$

$$\sqrt{1+x} = (1+x)^{\frac{1}{2}} = 1 + \frac{1}{2}x + \frac{\frac{1}{2}(-\frac{1}{2})}{2}x^2 + O(x^2) = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + O(x^2)$$

$$u_n = n+2 - n \left[\right]$$