

ATS 2015. FOURIER

① $\cos(u(-t)) = \cos(-ut) = \cos(ut) = \cos(t)$ fonction paire

Etude sur $[0, \pi]$ + sym / oy (periode 2π)

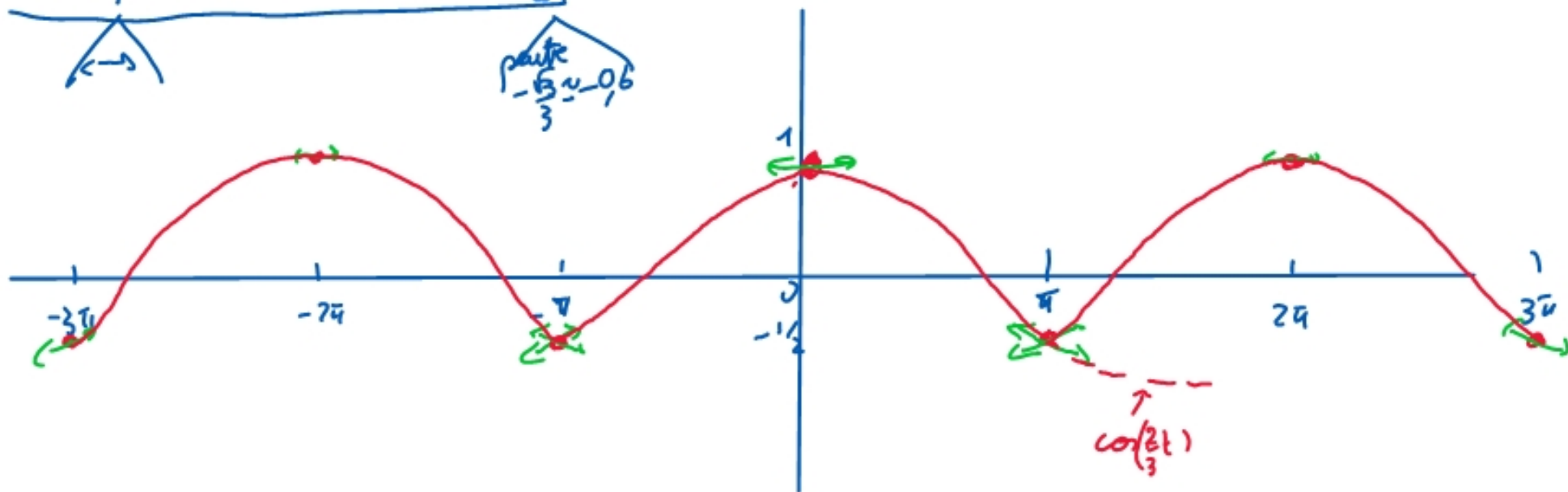
$f(t) = \cos\left(\frac{2}{3}t\right)$ $f'(t) = -\frac{2}{3}\sin\left(\frac{2}{3}t\right)$ s'annule pd $\frac{2}{3}t = k\pi$

$t = \frac{3k\pi}{2}$ ici $t=0$

$f'(t) = -\frac{2}{3}\sin\left(\frac{2\pi}{3}\right) = -\frac{2\sqrt{3}}{3}$

t	0	π
f'(t)	0	$-\frac{\sqrt{3}}{3}$
f(t)	1	$-\frac{1}{2}$

\leftarrow (pointe) $-\frac{\sqrt{3}}{3} \approx -0,6$

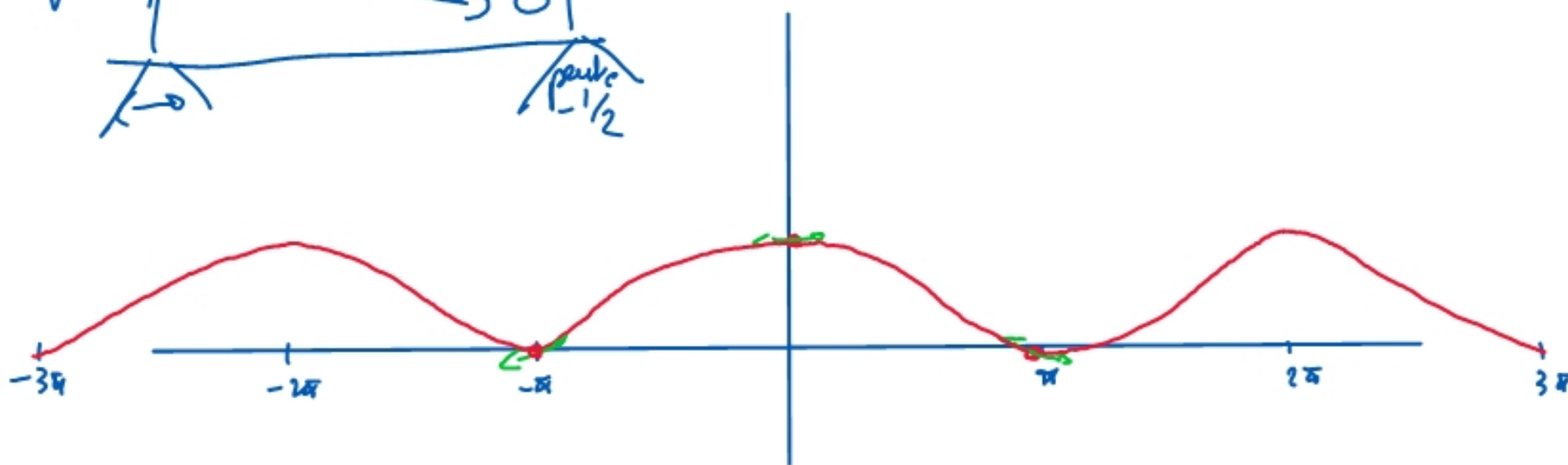
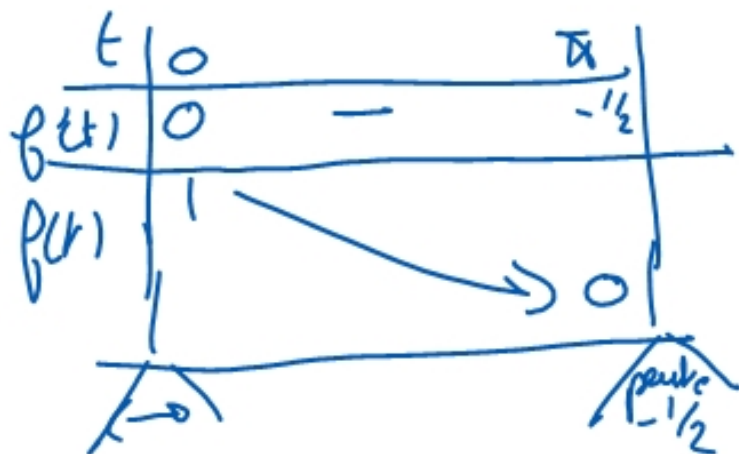


$$u = \frac{1}{2} \quad f(t) = \cos\left(\frac{1}{2}t\right)$$

Am $[-\pi, \pi]$

paire 2π -périodique

$$f'(t) = -\frac{1}{2} \sin\left(\frac{1}{2}t\right) \text{ s'annule en } 0, 2\pi, \dots$$



3) a)

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} \cos\left(\frac{1}{2}t\right) dt = \frac{1}{\pi} \int_0^{\pi} \cos\left(\frac{1}{2}t\right) dt = \frac{1}{\pi} \left[\frac{\sin\left(\frac{1}{2}t\right)}{\frac{1}{2}} \right]_0^{\pi}$$

$$a_0 = \frac{\sin\left(\frac{1}{2}\pi\right)}{\frac{1}{2}\pi}$$

$$a_n = \frac{2}{2\pi} \int_{-\pi}^{\pi} = \frac{1}{\pi} \int_0^{\pi} \cos(ut) \cos(nt) dt$$

$$= \frac{2}{\pi} \int_0^{\pi} \left(\cos((u+n)t) + \cos((u-n)t) \right) dt$$

$$= \frac{1}{\pi} \left[\frac{\sin((u+n)t)}{u+n} + \frac{\sin((u-n)t)}{u-n} \right]_0^{\pi}$$

$$= \frac{1}{\pi} \left(\frac{\sin((u+n)\pi)}{u+n} + \frac{\sin((u-n)\pi)}{u-n} \right)$$

$$= \frac{1}{\pi} \left(\frac{(-1)^n \sin(u\pi)}{u+n} + \frac{(-1)^n \sin(u\pi)}{u-n} \right) = \frac{(-1)^n \sin(u\pi)}{\pi} \left(\frac{1}{u+n} + \frac{1}{u-n} \right)$$

$$= \frac{(-1)^n \sin(u\pi)}{\pi} \frac{u-n + u+n}{u^2 - n^2}$$

$$a_n = \frac{(-1)^n 2u \sin(u\pi)}{\pi (u^2 - n^2)}, \quad \forall n \geq 1$$

$$\sin(a \pm n\pi) = \sin a \cos(n\pi) \pm \cos a \sin n\pi$$

$\begin{matrix} (-1)^n \\ \downarrow \\ \cos \end{matrix}$

(3b) $b_n = 0$ car f est paire

$$S_f(x) = \frac{\sin(u\pi)}{u\pi} + \sum_{n=1}^{+\infty} \frac{(-1)^n u \sin(u\pi)}{\pi(u^2 - n^2)} \cos(n\pi x) \quad \forall x \in \mathbb{R}$$

(4a) f est de classe C' par morceaux, donc S_f CV vers $\frac{f(x^-) + f(x^+)}{2}$
(T de DIRICHLET)

Ici, f est continue, donc $\frac{f(x^-) + f(x^+)}{2} = f(x)$,

$$S_f(x) = f(x), \quad \forall x \in \mathbb{R}$$

$$(4b) S_f(x) = \frac{\sin(u\pi)}{u\pi} + \sum_{n=1}^{+\infty} \frac{(-1)^n u \sin(u\pi)}{\pi(u^2 - n^2)} \cos(n\pi x)$$

DIRICHLET
 \downarrow
 $(-1)^{x-1} = 1$

$$\underline{S_f(x)} = \frac{\sin(u\pi)}{u\pi} + \sum_{n=1}^{+\infty} \frac{u \sin(u\pi)}{\pi(u^2 - n^2)} = \underline{f(x) = \cos(u\pi)}$$