

T D Combinatoire

$$\textcircled{1} \quad \forall n \geq 1, 1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Initialisation: Pour $n=1$: $1^2 = 1$ et $\frac{1 \times 2 \times 3}{6} = 1$ Ok

Hérédité: Je prends un $n \geq 1$. Je suppose que $1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$

On ajoute $(n+1)^2$: $1^2 + 2^2 + \dots + n^2 + (n+1)^2 = \frac{n(n+1)(2n+1)}{6} + (n+1)^2$

$$= \frac{n(n+1)(2n+1) + 6(n+1)^2}{6} = \frac{(n+1)[n(2n+1) + 6(n+1)]}{6}$$

$$= \frac{(n+1)[2n^2 + 7n + 6]}{6}$$

Calculons $(n+2)(2n+3)$:

$$(n+2)(2n+3) = 2n^2 + 3n + 4n + 6 = 2n^2 + 7n + 6.$$

Ah bon, alors:

$$1^2 + 2^2 + \dots + (n+1)^2 = \frac{(n+1)(n+2)(2n+3)}{6} \quad \text{Ok}$$

Conclusion: $\forall n \in \mathbb{N}, 1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$

D'après le principe de récurrence.