



⑧ DL à l'ordre 3 en  $x_0 = 4$  de  $\sqrt{x}$

On fait le chgt:  $X = x - 4$  car qd  $x \rightarrow 4$ ,  $X \rightarrow 0$

$$\sqrt{x} = \sqrt{4+X} = \sqrt{4\left(1+\frac{X}{4}\right)} = 2\sqrt{1+\frac{X}{4}}$$

$\rightarrow 0$  on peut faire le DL

$$\sqrt{x} = 2\left(1 + \frac{1}{2}\frac{X}{4} - \frac{1}{8}\left(\frac{X}{4}\right)^2 + \frac{1}{16}\left(\frac{X}{4}\right)^3 + \mathcal{O}(X^3)\right)$$

$$= 2\left(1 + \frac{X}{8} - \frac{X^2}{128} + \frac{X^3}{512} + \mathcal{O}(X^3)\right)$$

$$= 2 + \frac{X}{4} - \frac{X^2}{64} + \frac{X^3}{512} + \mathcal{O}(X^3)$$

$$\sqrt{x} = 2 + \frac{x-4}{4} - \frac{(x-4)^2}{64} + \frac{(x-4)^3}{512} + \mathcal{O}((x-4)^3)$$

tgte:  $y = 2 + \frac{x-4}{4} \Leftrightarrow y = 2 + \frac{x}{4} - 1$

$y = 1 + \frac{x}{4}$

position: coube en dessous de la tgte car  $-\frac{(x-4)^2}{64} < 0$

(8b) ordre 2 au voisinage de  $\frac{\pi}{3}$  de  $e^{\cos x} = f(x)$



chgt  $X = x - \frac{\pi}{3} \Rightarrow x = X + \frac{\pi}{3}$

$$f(x) = e^{\cos(X + \frac{\pi}{3})} = e^{\cos X \cos(\frac{\pi}{3}) - \sin X \sin(\frac{\pi}{3})} = e^{\frac{1}{2} \cos X - \frac{\sqrt{3}}{2} \sin X}$$

ne tend pas vers 0, pas de DL

$$f(x) = e^{\frac{1}{2}(1 - \frac{X^2}{2} + o(X^2)) - \frac{\sqrt{3}}{2}(X + o(X^2))} = e^{\frac{1}{2} - \frac{\sqrt{3}}{2}X - \frac{X^2}{4} + o(X^2)}$$

ne tend pas vers 0! pas de DL

$$e^{a+b} = e^a \cdot e^b$$

$f(x) = e^{\frac{1}{2}} \cdot e^{-\frac{\sqrt{3}}{2}X - \frac{X^2}{4} + o(X^2)}$  tend vers 0, on peut utiliser la DL de  $e^x$

$$= e^{\frac{1}{2}} \left[ 1 + \left(-\frac{\sqrt{3}}{2}X - \frac{X^2}{4} + o(X^2)\right) + \frac{1}{2} \left(-\frac{\sqrt{3}}{2}X - \frac{X^2}{4} + o(X^2)\right)^2 + o(X^2) \right]$$

$$= e^{\frac{1}{2}} \left[ 1 - \frac{\sqrt{3}}{2}X - \frac{X^2}{4} + o(X^2) + \frac{3}{8}X^2 + \dots \right] = e^{\frac{1}{2}} - \frac{\sqrt{3}}{2}e^{\frac{1}{2}}X + e^{\frac{1}{2}}\frac{X^2}{8} + o(X^2)$$

*pour être sûr*

$$f(x) = e^{\frac{1}{2}} - \frac{\sqrt{3}}{2}e^{\frac{1}{2}}\left(x - \frac{\pi}{3}\right) + \frac{e^{\frac{1}{2}}}{8}\left(x - \frac{\pi}{3}\right)^2 + o\left(x - \frac{\pi}{3}\right)^2$$

tpc  $y = e^{\frac{1}{2}} - \frac{\sqrt{3}}{2}e^{\frac{1}{2}}\left(x - \frac{\pi}{3}\right)$

comme au dessus car  $\frac{e^{\frac{1}{2}}}{8}\left(x - \frac{\pi}{3}\right)^2 > 0$

(10) DL ordre 4 en 0 de  $\frac{\sqrt{1+\sin x}}{\cos x}$

ordre 4 :  $1 + \sin x = 1 + \boxed{x - \frac{x^3}{6} + \mathcal{O}(x^5)}$

$$\sqrt{1+\sin x} = 1 + \frac{1}{2} \boxed{\phantom{x}} - \frac{1}{8} \boxed{\phantom{x^2}} + \frac{1}{16} \boxed{\phantom{x^3}} - \frac{1}{32} \boxed{\phantom{x^4}} + \text{possible}$$

$$\sqrt{1+\sin x} = 1 + \frac{1}{2} \left( x - \frac{x^3}{6} + \mathcal{O}(x^5) \right) - \frac{1}{8} \left( x^2 - \frac{x^4}{3} + p \right) + \frac{1}{16} (x^3) - \frac{1}{32} x^4$$

$$= 1 + \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{96}x^3 + \frac{1}{96}x^4 + \mathcal{O}(x^4)$$

$$- \frac{1}{12} + \frac{1}{16} = \frac{-4+3}{48} = -\frac{1}{48}$$

$$\frac{1}{24} - \frac{1}{32} = \frac{4-3}{96} = \frac{1}{96}$$

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{24} + \mathcal{O}(x^4)$$

Division

$$\begin{array}{r}
 1 + \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{48}x^3 + \frac{1}{96}x^4 + o(x^4) \\
 - \left( 1 - \frac{x^2}{2} + \frac{x^4}{24} + o(x^4) \right) \\
 \hline
 \frac{1}{2}x + \frac{3}{8}x^2 - \frac{1}{48}x^3 - \frac{1}{32}x^4 + o(x^4) \\
 - \left( \frac{1}{2}x - \frac{x^3}{4} + o(x^4) \right) \\
 \hline
 \frac{3}{8}x^2 + \frac{11}{48}x^3 - \frac{1}{32}x^4 + o(x^4) \\
 - \left( \frac{3}{8}x^2 - \frac{3}{16}x^4 + o(x^4) \right) \\
 \hline
 \frac{11}{48}x^3 + \frac{5}{32}x^4 + o(x^4) \\
 - \left( \frac{11}{48}x^3 + o(x^4) \right) \\
 \hline
 \frac{5}{32}x^4 + o(x^4)
 \end{array}$$

$$\begin{array}{r}
 1 - \frac{x^2}{2} + \frac{x^4}{24} + o(x^4) \\
 \hline
 1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{11}{48}x^3 + \frac{5}{32}x^4 + o(x^4)
 \end{array}$$

$$-\frac{1}{8} + \frac{1}{2} = \frac{-1+4}{8} = \frac{3}{8}$$

$$\frac{1}{96} - \frac{1}{24} = \frac{1-4}{96} = \frac{-3}{96} = \frac{-1}{32}$$

$$-\frac{1}{48} + \frac{1}{4} = \frac{-1+12}{48} = \frac{11}{48}$$

$$-\frac{1}{32} + \frac{3}{16} = \frac{-1+6}{32} = \frac{5}{32}$$

hp:  $y = 1 + \frac{1}{2}x$

Compte au dessus, car  $\frac{3}{8}x^2 > 0$

$$(12) \lim_{x \rightarrow e} \frac{\ln(\ln(x))}{x^2 - 3ex + 2e^2} = \frac{0}{0} \quad \text{F. I.}$$

chgt  $x = e + X \Rightarrow x = e + X \quad \text{qd } x \rightarrow e, X \rightarrow 0$

$$f(x) = \frac{\ln(\ln(e+X))}{(e+X)^2 - 3e(e+X) + 2e^2} = \frac{\ln(\ln(e(1+\frac{X}{e})))}{e^2 + 2eX + X^2 - 3e^2 - 3eX + 2e^2}$$

$$= \frac{\ln(\ln(e) + \ln(1+\frac{X}{e}))}{-eX + X^2} = \frac{\ln(1 + \frac{X}{e} + o(X))}{-eX + X^2}$$

$$= \frac{\frac{X}{e} + o(X)}{-eX + X^2} = \frac{X(\frac{1}{e} + o(1))}{X(-e + X)} \rightarrow \frac{\frac{1}{e}}{-e} = -\frac{1}{e^2}$$

8a) DL à l'ordre 3 au voisinage de 4 de  $\sqrt{x}$

On se ramène en 0 grâce au chgt:  $X = x - 4$  (qd  $x \rightarrow 4$ ,  $X \rightarrow 0$ )  
 $\Rightarrow x = X + 4$

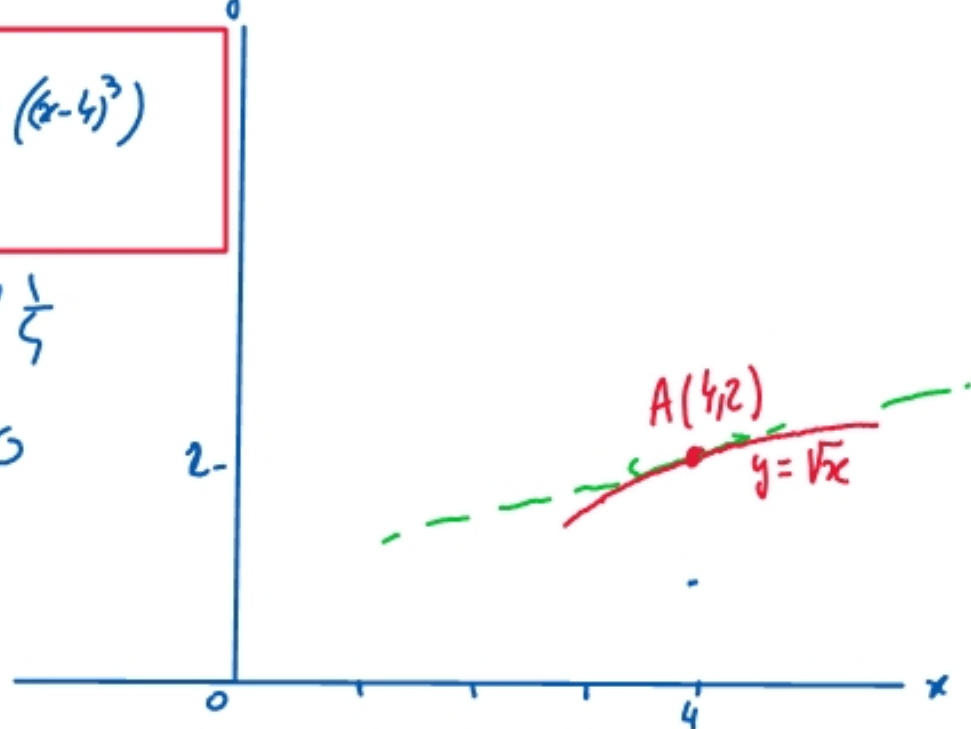
$$\sqrt{x} = \sqrt{4+X} = 2 \sqrt{1 + \frac{X}{4}} = 2 \left( 1 + \frac{1}{2} \left( \frac{X}{4} \right) - \frac{1}{8} \left( \frac{X}{4} \right)^2 + \frac{1}{16} \left( \frac{X}{4} \right)^3 + \mathcal{O}(X^3) \right)$$

$$\sqrt{x} = 2 \left( 1 + \frac{X}{8} - \frac{X^2}{128} + \frac{X^3}{2 \cdot 512} + \mathcal{O}(X^3) \right) = 2 + \frac{X}{4} - \frac{X^2}{64} + \frac{X^3}{512} + \mathcal{O}(X^3)$$

$$\sqrt{x} = 2 + \frac{x-4}{4} - \frac{(x-4)^2}{64} + \frac{(x-4)^3}{512} + \mathcal{O}((x-4)^3)$$

l'oe:  $y = 2 + \frac{x-4}{4} \Leftrightarrow y = \frac{x}{4} + 1$  pente  $\frac{1}{4}$

coube en dessous de la l'oe car  $-\frac{(x-4)^2}{64} \leq 0$



(8b) DL à l'ordre 2 de  $e^{\cos x}$  au voisinage de  $\frac{\pi}{3}$

chgt  $X = x - \frac{\pi}{3} \Rightarrow x = X + \frac{\pi}{3}$

ne tient pas vers 0, pas de DL



$$\cos\left(X + \frac{\pi}{3}\right) = \cos X \cos \frac{\pi}{3} - \sin X \sin \frac{\pi}{3} = \frac{1}{2} \cos X - \frac{\sqrt{3}}{2} \sin X$$

$\downarrow$  DL                       $\downarrow$  DL

$$= \frac{1}{2} \left(1 - \frac{X^2}{2} + o(X^2)\right) - \frac{\sqrt{3}}{2} (X + o(X^2))$$

$$\cos(x) = \frac{1}{2} - \frac{\sqrt{3}}{2} X - \frac{X^2}{4} + o(X^2)$$

$$e^{\cos x} = e^{\left(\frac{1}{2} - \frac{\sqrt{3}}{2} X - \frac{X^2}{4} + o(X^2)\right)} \rightarrow \frac{1}{2} : \text{ne tient pas vers } 0 \cdot \text{pas de DL}$$

$$= e^{\frac{1}{2}} \cdot e^{\left(-\frac{\sqrt{3}}{2} X - \frac{X^2}{4} + o(X^2)\right)} \rightarrow 0 \cdot \text{on peut faire la DL}$$

$$= e^{\frac{1}{2}} \cdot \left[1 + \left(-\frac{\sqrt{3}}{2} X - \frac{X^2}{4} + o(X^2)\right) + \frac{1}{2} \left(-\frac{\sqrt{3}}{2} X - \frac{X^2}{4} + o(X^2)\right)^2 + o(X^2)\right]$$

$$= e^{\frac{1}{2}} \left[1 - \frac{\sqrt{3}}{2} X - \frac{X^2}{4} + o(X^2) + \frac{3}{8} X^2\right] = e^{\frac{1}{2}} \left[1 - \frac{\sqrt{3}}{2} X + \frac{1}{8} X^2 + o(X^2)\right]$$

$$\underline{e^{\cos x} = e^{\frac{1}{2}} - \frac{\sqrt{3}}{2} e^{\frac{1}{2}} \left(x - \frac{\pi}{3}\right) + \frac{1}{8} e^{\frac{1}{2}} \left(x - \frac{\pi}{3}\right)^2 + o\left(\left(x - \frac{\pi}{3}\right)^2\right)}$$

10) DL à l'ordre 4 en 0 de  $\frac{\sqrt{1+\sin x}}{\cos x}$

$$\sin x = x - \frac{x^3}{6} + \mathcal{O}(x^5)$$

$$1 + \sin x = 1 + \left( x - \frac{x^3}{6} + \mathcal{O}(x^5) \right) \quad X$$

$$\sqrt{1+X} = 1 + \frac{1}{2}X - \frac{1}{8}X^2 + \frac{1}{16}X^3 - \frac{1}{32}X^4 + \mathcal{O}(X^5)$$

$$\sqrt{1+\sin x} = 1 + \frac{1}{2} \left[ x - \frac{x^3}{6} + \mathcal{O}(x^5) \right] - \frac{1}{8} \left[ \quad \right]^2 + \frac{1}{16} \left[ \quad \right]^3 - \frac{1}{32} \left[ \quad \right]^4 + \mathcal{O}(x^5)$$

$$= 1 + \frac{1}{2}x - \frac{x^3}{12} + \mathcal{O}(x^5) - \frac{1}{8} \left( x^2 - \frac{2x^4}{3} + \dots \right) + \frac{1}{16} \left( x^3 + \dots \right) - \frac{1}{32} \left( x^4 + \dots \right)$$

$$= 1 + \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{48}x^3 + \frac{1}{96}x^4 + \mathcal{O}(x^5)$$

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{24} + \mathcal{O}(x^6)$$

$$\frac{1}{16} - \frac{1}{12} = \frac{3-4}{48} = -\frac{1}{48}$$

$$\frac{1}{24} - \frac{1}{32} = \frac{4-3}{96} = \frac{1}{96}$$



$$1 + \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{48}x^3 + \frac{1}{96}x^4 + \mathcal{O}(x^4)$$

$$- \left( 1 - \frac{x^2}{2} + \frac{x^4}{24} + \mathcal{O}(x^4) \right)$$

$$\frac{1}{2}x + \frac{3}{8}x^2 - \frac{1}{48}x^3 - \frac{x^4}{32} + \mathcal{O}(x^4)$$

$$- \left( \frac{1}{2}x - \frac{1}{4}x^3 + \mathcal{O}(x^4) \right)$$

$$\frac{3}{8}x^2 + \frac{11}{48}x^3 - \frac{x^4}{32} + \mathcal{O}(x^4)$$

$$- \left( \frac{3}{8}x^2 - \frac{3}{16}x^4 + \mathcal{O}(x^4) \right)$$

$$\frac{11}{48}x^3 + \frac{5}{32}x^4 + \mathcal{O}(x^4)$$

$$- \left( \frac{11}{48}x^3 + \mathcal{O}(x^4) \right)$$

$$\frac{5}{32}x^4 + \mathcal{O}(x^4)$$

$$1 - \frac{x^2}{2} + \frac{x^4}{24} + \mathcal{O}(x^4)$$

$$1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{11}{48}x^3 + \frac{5}{32}x^4 + \mathcal{O}(x^4)$$

$$-\frac{1}{8} + \frac{1}{2} = \frac{-1+4}{8} = \frac{3}{8}$$

$$\frac{1}{96} - \frac{1}{24} = \frac{1-4}{96} = \frac{-3}{96}$$

$$= \frac{-1}{32}$$

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$$(12) \lim_{x \rightarrow e} \frac{\ln(\ln(x))}{x^2 - 3ex + 2e^2} = \frac{0}{0} \quad \text{F. I}$$

chgt  $X = x - e \Rightarrow x = e + X$  qd  $x \rightarrow e, X \rightarrow 0$

$$f(x) = \frac{\ln(\ln(e+x))}{(e+x)^2 - 3e(e+x) + 2e^2} = \frac{\ln(\ln(e(1+\frac{x}{e})))}{\cancel{e^2} + 2eX + X^2 - 3\cancel{e^2} - 3eX + 2\cancel{e^2}}$$

$$= \frac{\ln(\ln(e) + \overbrace{\ln(1+\frac{x}{e})}^{DL})}{-eX + X^2} = \frac{\ln(1 + \frac{x}{e} + o(x))}{-eX + X^2}$$

$$= \frac{\frac{x}{e} + o(x)}{-eX + X^2} = \frac{\cancel{x} (\frac{1}{e} + o(1))}{\cancel{x} (-e + X)} \xrightarrow{x \rightarrow 0} \frac{\frac{1}{e}}{-e} = -\frac{1}{e^2}$$