

34) M. geo $\ln(n+1) \sim_{+\infty} \ln(n)$ car $\frac{\ln(n+1)}{\ln(n)} \xrightarrow{n \rightarrow +\infty} 1$

$$\frac{\ln(n+1)}{\ln(n)} \rightarrow \frac{\infty}{\infty} \text{ f. I}$$

$$\begin{aligned} \frac{\ln(n+1)}{\ln(n)} &= \frac{\ln\left(n\left(1+\frac{1}{n}\right)\right)}{\ln(n)} = \frac{\ln(n) + \ln\left(1+\frac{1}{n}\right)}{\ln(n)} \\ &= 1 + \frac{\ln\left(1+\frac{1}{n}\right)}{\ln(n)} \rightarrow 1 + \frac{0}{\infty} = 1 \end{aligned}$$

35) $x + \ln(x) \sim_{+\infty} x$ car $\frac{x + \ln x}{x} = 1 + \frac{\ln x}{x} \xrightarrow{x \rightarrow +\infty} 1 + 0 = 1$

do comparaison des ordres (VCC)

$$x^2 + \sin(x) \sim_{+\infty} x^2 \text{ car } \frac{x^2 + \sin(x)}{x^2} = 1 + \frac{\sin x}{x^2} \rightarrow 1 + 0 = 1$$

$\left(0 \sim_{+\infty} \frac{-1}{x^2} \leq \frac{\sin x}{x^2} \leq \frac{1}{x^2} \xrightarrow{+\infty} 0, \text{ donc } \frac{\sin x}{x^2} \xrightarrow{+\infty} 0 \right)$

$$\ln(x+2) \underset{+\infty}{\sim} \ln x, \text{ car } \frac{\ln(x+2)}{\ln x} = \frac{\ln x(1+\frac{2}{x})}{\ln x} = \frac{\ln x + \ln(1+\frac{2}{x})}{\ln x}$$

$$= 1 + \frac{\ln(1+\frac{2}{x})}{\ln x} \xrightarrow{+\infty} 1 + \frac{0}{\infty} = 1 + 0 = 1$$

$$\ln(3x^2+1) \underset{+\infty}{\sim} \ln(3x^2) \text{ par ce que précédemment}$$

$$= \ln(3) + \ln(x^2) \underset{+\infty}{\sim} \ln(x^2) = 2 \ln x,$$

$$e^{2x} + x \underset{+\infty}{\sim} e^{2x} \text{ car } \frac{e^{2x} + x}{e^{2x}} = \frac{e^{2x}}{e^{2x}} + \frac{x}{e^{2x}} = 1 + \frac{x}{e^{2x}} \xrightarrow{+\infty} 1 + 0 = 1$$

TCC

$$e^x + \ln x - 2x \underset{+\infty}{\sim} e^x \text{ car } \frac{e^x + \ln x - 2x}{e^x} = 1 + \frac{\ln x}{e^x} - \frac{2x}{e^x} \xrightarrow{+\infty} 1 + 0 - 0$$

TCC TCC

$$x^3 + 2^x \underset{+\infty}{\sim} 2^x \text{ car } \frac{x^3 + 2^x}{2^x} = 1 + \frac{x^3}{2^x} \xrightarrow{+\infty} 1 + 0$$

TCC

$$(36) \quad f(x) = \frac{(x + \ln(x))(x^2 + 4\ln(x))}{\ln(3x^2 + 1)} \underset{+\infty}{\sim} \frac{x \cdot x^2}{2 \ln x} = \frac{x^3}{2 \ln x} \rightarrow +\infty$$

\downarrow
 $+\infty (TCC)$

$$g(x) = \frac{e^{2x} + x}{e^x + \ln x - 2x} \underset{+\infty}{\sim} \frac{e^{2x}}{e^x} = e^x \xrightarrow{+\infty} +\infty$$

$$h(x) = \frac{(x^3 + 2^x) \cdot (\ln(2x) + 3^x)}{5^x - \ln(x)} \underset{+\infty}{\sim} \frac{2^x \cdot 3^x}{5^x} = \left(\frac{6}{5}\right)^x \xrightarrow{x \rightarrow +\infty} +\infty$$

$$(37) \quad x^2 + x^5 \underset{+\infty}{\sim} x^5$$

$$x^2 + x^5 \underset{0}{\sim} x^2 \quad \text{car} \quad \frac{x^2 + x^5}{x^2} = \frac{x^2(1 + x^3)}{x^2} \xrightarrow{x \rightarrow 0} 1$$

$$x^2 + x^5 \underset{2}{\sim} 4 + 32 = 36$$

$$(38) \cdot \sqrt{x+x^2} \underset{0}{\sim} \sqrt{x} \text{ car } \frac{\sqrt{x+x^2}}{\sqrt{x}} = \frac{\sqrt{x(1+x)}}{\sqrt{x}} = \frac{\sqrt{x}\sqrt{1+x}}{\sqrt{x}} \xrightarrow{x \rightarrow 0} \sqrt{1} = 1$$

$$\cdot \sqrt{-x^2+3x-2} \underset{1^+}{\sim} \sqrt{1+3-2} = \cancel{0} \text{ car } \frac{\sqrt{\quad}}{0} \rightarrow \text{NEIN}$$

↑
JAMAIS

Chgt $x = x - 1$ $x = 1 + x$

$$\begin{aligned} \sqrt{-x^2+3x-2} &= \sqrt{-(x+1)^2+3(x+1)-2} = \sqrt{-x^2-2x-1+3x+3-2} \\ &= \sqrt{x-x^2} \underset{x \rightarrow 0}{\sim} \sqrt{x} = \sqrt{x-1} \end{aligned}$$

$$\boxed{\sqrt{-x^2+3x-2} \underset{1}{\sim} \sqrt{x-1}}$$

$$\textcircled{39} \quad \tan(3x) = 3x + o(x) \underset{0}{\sim} 3x$$

$$\sin(2x) = 2x + o(x) \underset{0}{\sim} 2x$$

$$\Rightarrow \frac{\tan(3x)}{\sin(2x)} \underset{0}{\sim} \frac{3x}{2x} = \frac{3}{2} \xrightarrow{x \rightarrow 0} \frac{3}{2}$$

ou bien, comme la semaine dernière :

$$\frac{\tan(3x)}{\sin(2x)} = \frac{3x + o(x)}{2x + o(x)} = \frac{\cancel{x}(3 + o(1))}{\cancel{x}(2 + o(1))} \rightarrow \frac{3}{2}$$

$$(29) \quad f(x) = \frac{\ln(1+x)}{1-x} = \frac{x - \frac{x^2}{2} + \frac{x^3}{3} + o(x^3)}{1-x}$$

$$x - \frac{x^2}{2} + \frac{x^3}{3} + o(x^3)$$

$$-(x - x^2)$$

$$\frac{x^2}{2} + \frac{x^3}{3} + o(x^3)$$

$$-(\frac{x^2}{2} - \frac{x^3}{2})$$

$$+ \frac{5}{6}x^3 + o(x^3)$$

$$1-x$$

$$x + \frac{x^2}{2} + \frac{5}{6}x^3$$

$$x^2 \cdot \frac{x^2}{2} = \frac{x^4}{2}$$

$$\frac{1}{3} + \frac{1}{2} = \frac{2+3}{6} = \frac{5}{6}$$

$$f(x) = x + \frac{x^2}{2} + \frac{5}{6}x^3 + o(x^3)$$

$$g(x) = 2 - \sqrt{4-4x-x^2} = 2 - \sqrt{4(1-x-\frac{x^2}{4})} = 2 - 2\sqrt{1 + \underbrace{(-x-\frac{x^2}{4})}_X}$$

$$\text{DL de } \sqrt{1+X} : \quad \sqrt{1+X} = 1 + \frac{1}{2}X - \frac{1}{8}X^2 + \frac{1}{16}X^3 + o(X^3),$$

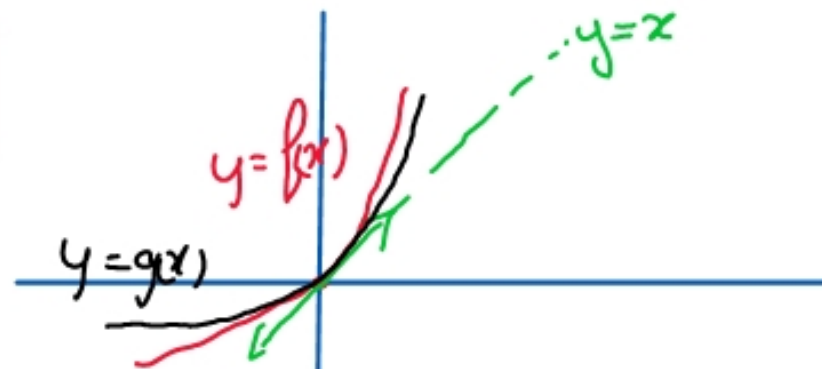
$$\begin{aligned}
g(x) &= 2 - 2 \left(1 + \frac{1}{2} \left(-x - \frac{x^2}{4} \right) - \frac{1}{8} \left(-x - \frac{x^2}{4} \right)^2 + \frac{1}{16} \left(-x - \frac{x^2}{4} \right)^3 + \mathcal{O}(x^3) \right), \\
&= 2 - 2 \left(1 - \frac{1}{2}x - \frac{x^2}{8} - \frac{1}{8} \left(x^2 + 2 \cdot x \cdot \frac{x^2}{4} + \text{possible} \right) + \frac{1}{16} \left(-x^3 + \text{possible} \right) \right) \\
&= 2 - 2 \left(1 - \frac{1}{2}x - \frac{x^2}{8} - \frac{1}{8}x^2 - \frac{1}{16}x^3 - \frac{1}{16}x^3 + \mathcal{O}(x^3) \right) \\
&= 2 - 2 \left(1 - \frac{1}{2}x - \frac{1}{4}x^2 - \frac{1}{8}x^3 + \mathcal{O}(x^3) \right) \\
&= 2 - 2 + x + \frac{1}{2}x^2 + \frac{1}{4}x^3 + \mathcal{O}(x^3)
\end{aligned}$$

$$g(x) = x + \frac{1}{2}x^2 + \frac{1}{4}x^3 + \mathcal{O}(x^3)$$

$$f(x) = x + \frac{1}{2}x^2 + \frac{5}{6}x^3 + \mathcal{O}(x^3)$$

f et g sont au dessus de la tg car $\frac{1}{2}x^2 \geq 0$

$$f(x) - g(x) = \left(\frac{5}{6} - \frac{1}{4} \right) x^3 + \mathcal{O}(x^3) = \frac{7}{12} x^3 + \mathcal{O}(x^3) \begin{cases} > 0 \text{ qd } x > 0 \\ < 0 \text{ qd } x < 0 \end{cases} \begin{matrix} f > g \\ f < g \end{matrix}$$



$$\textcircled{31} \quad f(x) = \sqrt{\frac{x^3}{x-1}} = \sqrt{\frac{x^3}{x(1-\frac{1}{x})}} = \sqrt{\frac{x^2}{1-\frac{1}{x}}} = x \sqrt{\frac{1}{1-\frac{1}{x}}} \quad \begin{array}{l} \nearrow \text{en } +\infty \\ \text{ } \end{array}$$

$$\text{DL: } \frac{1}{1-x} = 1 + x + x^2 + o(x^2)$$

en $-\infty$ $\sqrt{x^2}$ devient $-x$

$$f(x) = x \sqrt{1 + \frac{1}{x} + \frac{1}{x^2} + o\left(\frac{1}{x^2}\right)} \quad \rightarrow 0$$

DL de $\sqrt{1+x}$

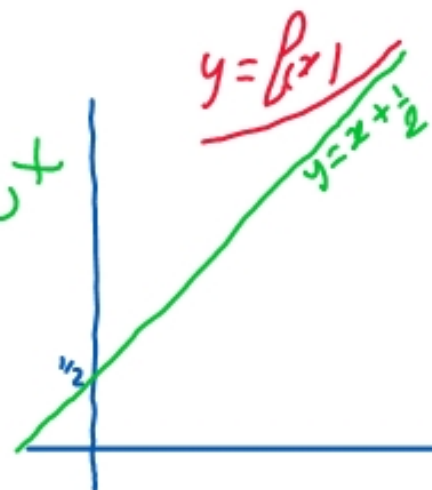
$$= x \left(1 + \frac{1}{2} \left(\frac{1}{x} + \frac{1}{x^2} + o\left(\frac{1}{x^2}\right) \right) - \frac{1}{8} \left(\frac{1}{x} + \frac{1}{x^2} + o\left(\frac{1}{x^2}\right) \right)^2 \right)$$

$$= x \left(1 + \frac{1}{2x} + \frac{1}{2x^2} - \frac{1}{8x^2} + o\left(\frac{1}{x}\right) \right) \quad \frac{1}{2} - \frac{1}{8} = \frac{3}{8}$$

$$f(x) = x + \frac{1}{2} + \frac{3}{8x} + o\left(\frac{1}{x}\right)$$

$$\text{en } +\infty \quad f(x) - \left(x + \frac{1}{2}\right) = \frac{3}{8x} + o\left(\frac{1}{x}\right) \rightarrow 0^+$$

asymptote $y = x + \frac{1}{2}$ Courbe au dessus car $\frac{3}{8x} > 0$



$$(34) \quad \frac{\ln(n+1)}{\ln(n)} \rightarrow \frac{0}{0} \quad \cdot \quad \text{F.T.}$$

$$\begin{aligned} \frac{\ln(n+1)}{\ln(n)} &= \frac{\ln\left(n\left(1+\frac{1}{n}\right)\right)}{\ln(n)} = \frac{\ln(n) + \ln\left(1+\frac{1}{n}\right)}{\ln(n)} \\ &= 1 + \frac{\ln\left(1+\frac{1}{n}\right)}{\ln(n)} \xrightarrow{n \rightarrow +\infty} 1 + \frac{0}{\infty} = 1 + 0 = 1 \end{aligned}$$

$$\frac{\ln(n+1)}{\ln(n)} \xrightarrow{+\infty} 1, \text{ donc } \ln(n+1) \sim \ln(n)$$

$$(35) \quad x + \ln(x) \underset{+\infty}{\sim} x \text{ car } \frac{x + \ln(x)}{x} = 1 + \frac{\ln(x)}{x} \xrightarrow{+\infty} 1 + 0 = 1$$

$$x^2 + \sin x \underset{+\infty}{\sim} x^2 \text{ car } \frac{x^2 + \sin x}{x^2} = 1 + \frac{\sin x}{x^2} \rightarrow 1 + 0 = 1$$

$$\left(\begin{array}{c} -\frac{1}{x^2} \\ \vdots \\ 0 \end{array} \leq \frac{\sin x}{x^2} \leq \begin{array}{c} \frac{1}{x^2} \\ \vdots \\ 0 \end{array} \right), \text{ donc } \frac{\sin x}{x^2} \xrightarrow{+\infty} 0 \text{ (T des gendarmes) }$$

O (T des coïncidences comparables)
TCC

$$\ln(x+2) \underset{+\infty}{\sim} \ln(x) \text{ car } \frac{\ln(x+2)}{\ln(x)} = \frac{\ln(x(1+\frac{2}{x}))}{\ln(x)} = \frac{\ln(x) + \ln(1+\frac{2}{x})}{\ln(x)}$$

$$= 1 + \frac{\ln(1+\frac{2}{x})}{\ln(x)} \xrightarrow{+\infty} 1 + \frac{0}{\infty} = 1 + 0 = 1$$

$$\ln(3x^2+1) \underset{+\infty}{\sim} \ln(3x^2) \quad (\uparrow \text{puisque que au dessus})$$

$$\ln(3x^2) = \ln(3) + \ln(x^2) \underset{+\infty}{\sim} \ln(x^2) = 2\ln(x)$$

$$e^{2x} + x \underset{+\infty}{\sim} e^{2x} \text{ car } \frac{e^{2x} + x}{e^{2x}} = 1 + \frac{x}{e^{2x}} \xrightarrow{+\infty} 1 + 0 = 1$$

$\xrightarrow{+\infty} 0 \text{ (TCC)}$

$$e^x + \ln x - 2x \underset{+\infty}{\sim} e^x \text{ car } \frac{e^x + \ln x - 2x}{e^x} = 1 + \frac{\ln x}{e^x} - 2\frac{x}{e^x} \xrightarrow{+\infty} 1 + 0 - 0 = 1$$

$\downarrow \quad \downarrow$
 $o(\infty) \quad o(\infty)$

$$x^3 + 2^x \underset{+\infty}{\sim} 2^x \text{ car } \frac{x^3 + 2^x}{2^x} = 1 + \frac{x^3}{2^x} \xrightarrow{+\infty} 1 + 0 = 1$$

$\xrightarrow{+\infty} 0 \text{ (TCC)}$

$$\textcircled{36} \quad f(x) = \frac{(x + \ln(x)) (x^2 + \sin(x))}{\ln(3x^2 + 1)} \underset{+\infty}{\sim} \frac{x \cdot x^2}{2 \ln(x)} = \frac{x^3}{2 \ln(x)} \xrightarrow{+\infty} +\infty \quad (\text{TCC})$$

$$g(x) = \frac{e^{2x} + x}{e^x + \ln(x) - 2x} \underset{+\infty}{\sim} \frac{e^{2x}}{e^x} = e^{2x-x} = e^x \xrightarrow{+\infty} +\infty$$

$$h(x) = \frac{(x^3 + 2^x)(\ln(2x) + 3^x)}{5^x - \ln(x)} \underset{+\infty}{\sim} \frac{2^x \cdot 3^x}{5^x} = \left(\frac{6}{5}\right)^x \xrightarrow{+\infty} +\infty$$

$$\textcircled{37} \quad x^2 + x^5 \underset{+\infty}{\sim} x^5 \quad \text{car } \frac{x^2 + x^5}{x^5} = 1 + \frac{x^2}{x^5} = 1 + \frac{1}{x^3} \xrightarrow{+\infty} 1 + 0 = 1$$

$$x^2 + x^5 \underset{0}{\sim} x^2 \quad \text{car } \frac{x^2 + x^5}{x^2} = 1 + x^3 \xrightarrow{x \rightarrow 0} 1$$

$$x^2 + x^5 \underset{2}{\sim} 4 + 32 = 36$$

$$(38) \quad \sqrt{x+x^2} \underset{0^+}{\sim} \sqrt{x} \quad \text{car} \quad \frac{\sqrt{x+x^2}}{\sqrt{x}} = \sqrt{\frac{x+x^2}{x}} = \sqrt{1+x} \xrightarrow{x \rightarrow 0} \sqrt{1} = 1$$

$$\sqrt{-x^2+3x-2} \underset{1^+}{\sim} ? \quad \text{car ?}$$

$$\text{chgt } X = x-1 \Rightarrow x = 1+X$$

$$\begin{aligned} \sqrt{-x^2+3x-2} &= \sqrt{-(1+X)^2+3(1+X)-2} \\ &= \sqrt{-(1+2X+X^2)+3+3X-2} = \sqrt{X-X^2} \underset{0^+}{\sim} \sqrt{X} = \sqrt{x-1} \end{aligned}$$

donc $\sqrt{-x^2+3x-2} \underset{1^+}{\sim} \sqrt{x-1}$

$$(39) \quad \lim_0 \frac{\tan(3x)}{\sin(2x)} = \frac{0}{0} \quad \text{F.T.}$$

$$\frac{\tan(3x)}{\sin(2x)} = \frac{3x + o(x)}{2x + o(x)} = \frac{x(3+o(1))}{x(2+o(1))} \xrightarrow{x \rightarrow 0} \frac{3}{2}$$

$$\left. \begin{array}{l} \\ \end{array} \right\} \underset{0}{\sim} \frac{3x}{2x} = \frac{3}{2} \rightarrow \frac{3}{2}$$

$$(29) \quad f(x) = \frac{\ln(1+x)}{1-x} = \frac{x - \frac{x^2}{2} + \frac{x^3}{3} + o(x^3)}{1-x}$$

$$\begin{array}{r} x - \frac{x^2}{2} + \frac{x^3}{3} + o(x^3) \quad | \quad \frac{1-x}{x + \frac{x^2}{2} + \frac{5}{6}x^3} \\ - (x - x^2) \\ \hline \frac{x^2}{2} + \frac{x^3}{3} + o(x^3) \\ - (\frac{x^2}{2} - \frac{x^3}{2}) \\ \hline \frac{5}{6}x^3 + o(x^3) \end{array}$$

$$\frac{1}{3} + \frac{1}{2} = \frac{2+3}{6} = \frac{5}{6}$$

$$f(x) = x + \frac{x^2}{2} + \frac{5}{6}x^3 + o(x^3)$$

$$g(x) = 2 - \sqrt{4-4x-x^2}$$

$$= 2 - \sqrt{4(1-x-\frac{x^2}{4})} = 2 - 2\sqrt{1 + (-x-\frac{x^2}{4})} \quad DL$$

$$\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 + o(x^3)$$

$o(x^2)$

$$g(x) = 2 - 2\sqrt{1 + (-x - \frac{x^2}{4})}$$

$$= 2 - 2 \left(1 + \frac{1}{2}(-x - \frac{x^2}{4}) - \frac{1}{8}(-x - \frac{x^2}{4})^2 + \frac{1}{16}(-x - \frac{x^2}{4})^3 + \mathcal{O}(x^3) \right)$$

$$= 2 - 2 \left(1 - \frac{1}{2}x - \frac{x^2}{8} - \frac{x^2}{8} - \frac{x^3}{16} - \frac{x^3}{16} + \mathcal{O}(x^3) \right)$$

$$= 2 - 2 + x + \frac{x^2}{2} + \frac{x^3}{4} + \mathcal{O}(x^3)$$

$$g(x) = x + \frac{x^2}{2} + \frac{x^3}{4} + \mathcal{O}(x^3)$$

$$\frac{5}{6} - \frac{1}{4} = \frac{10-3}{12}$$

$$f(x) = x + \frac{x^2}{2} + \frac{5}{6}x^3 + \mathcal{O}(x^3)$$

⇓

$$f(x) - g(x) = \frac{1}{12}x^3 + \mathcal{O}(x^3)$$

> 0 qd x > 0

fauc dessus de g

< 0 qd x < 0

fauc dessous de g

