

# exercice

$$\textcircled{1} \text{ a) } \lim_{n \rightarrow +\infty} 2n + \frac{4}{n+2} = +\infty \quad \left( +\infty + \frac{4}{+\infty} = +\infty + 0 = +\infty \right)$$

$$\text{b) } \frac{3n^2 - 5n}{n-1} = \frac{n^2 (3 - 5/n)}{n (1 - 1/n)} \xrightarrow{+\infty} +\infty \times \frac{3}{1} = +\infty$$

$$\frac{n^3}{3} \xrightarrow{+\infty} +\infty \quad \text{donc } \frac{n^3}{3} + \frac{3n^2 - 5n}{n-1} \rightarrow +\infty \quad (+\infty + \infty)$$

$$\text{c) } n + (-1)^n = n \left( 1 + \frac{(-1)^n}{n} \right) = a_n$$

$$\left. \begin{array}{l} -\frac{1}{n} \leq \frac{(-1)^n}{n} \leq \frac{1}{n} \\ -\frac{1}{n} \xrightarrow{+\infty} 0 \\ \frac{1}{n} \xrightarrow{+\infty} 0 \end{array} \right) \text{ donc } \frac{(-1)^n}{n} \xrightarrow{+\infty} 0 \quad (\text{T des gendarmes})$$
$$a_n \xrightarrow{+\infty} +\infty (1+0) = +\infty$$

$$\textcircled{d} \quad x^2 + x \sin x = x^2 \left(1 + \frac{\sin x}{x}\right) = f(x)$$

$$\left| \begin{array}{l} \frac{1}{x} < \frac{\sin x}{x} < \frac{1}{x} \\ \frac{1}{x} \xrightarrow{+\infty} 0 \\ -\frac{1}{x} \xrightarrow{+\infty} 0 \end{array} \right.$$

donc  $\lim_{+\infty} \frac{\sin x}{x} = 0$  (T des gendarmes)

$$\lim_{+\infty} f(x) = +\infty \quad \left(+\infty (1+0) = +\infty \times 1 = +\infty\right)$$

$$(c) \quad \frac{3x-5}{x+1} = \frac{x(3-\frac{5}{x})}{x(1+\frac{1}{x})} \xrightarrow{+\infty} \frac{3}{1} = 3$$

$$-\frac{1}{x} \leq \frac{\sin x}{x} \leq \frac{1}{x} \quad \text{donc } \frac{\sin x}{x} \xrightarrow{+\infty} 0 \quad (\text{T des fonctions})$$

$\downarrow$   $\downarrow$   
 $+\infty$   $+\infty$   
 $0$   $0$

$$\text{donc } \frac{3x-5}{x+1} - \frac{\sin x}{x} \xrightarrow{+\infty} 3 - 0 = 3$$

$$(b) \quad \sqrt{x^2+3x} - x = \sqrt{x^2(1+\frac{3}{x})} - x$$

$$= x \sqrt{1+\frac{3}{x}} - x \xrightarrow{+\infty} +\infty - \infty \text{ F.I.}$$

$\downarrow$   
 $+\infty$   
 $0$

on peut faire un DL

$$f(x) = x \sqrt{1 + \left(\frac{3}{x}\right)} - x \quad \sqrt{1+X} = 1 + \frac{1}{2}X + o(x)$$

$$= x \left( 1 + \frac{1}{2} \frac{3}{x} + o\left(\frac{3}{x}\right) \right) - x$$

$$= \cancel{x} + \frac{3}{2} + o(3) - \cancel{x} \xrightarrow{x \rightarrow \infty} \frac{3}{2} + 0 = \frac{3}{2}$$

$$(9a) \quad \sin(2x) = 2x - \frac{(2x)^3}{3!} + o(x^3) = 2x - \frac{8}{6}x^3 + o(x^3) = \underline{2x - \frac{4}{3}x^3 + o(x^3)}$$

$$e^{-3x} = 1 - 3x + \frac{9x^2}{2} - \frac{27x^3}{6} + o(x^3) = 1 - 3x + \frac{9x^2}{2} - \frac{9x^3}{2} + o(x^3)$$

$$\sin(2x) \cdot e^{-3x} = \left( \underline{2x - \frac{4}{3}x^3 + o(x^3)} \right) \left( 1 - 3x + \frac{9x^2}{2} - \frac{9x^3}{2} + o(x^3) \right)$$

$$= 2x - 6x^2 + 9x^3 + o(x^3) - \frac{4}{3}x^3 + \dots = \underline{2x - 6x^2 + \frac{23}{3}x^3 + o(x^3)}$$

partie principale:  $2x$

$$(2b) \ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + O(x^4)$$

$$\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} + O(x^4)$$

$$\begin{array}{r} x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + O(x^4) \\ - (x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + O(x^4)) \\ \hline -x^2 - \frac{x^4}{2} + O(x^4) \end{array}$$

$$-(-x^2 - \frac{x^3}{2} - \frac{x^4}{3} + O(x^4))$$

$$\frac{x^3}{2} - \frac{x^4}{6} + O(x^4)$$

$$-(\frac{x^3}{2} + \frac{x^4}{4} + O(x^4))$$

$$-\frac{5x^4}{12} + O(x^4)$$

$$-(-\frac{5x^4}{12} + O(x^4))$$

$$O(x^4)$$

$$-x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} + O(x^4)$$

$$-1 + x - \frac{x^2}{2} + \frac{5x^3}{12} + O(x^3)$$

$$-\frac{1}{2} + \frac{1}{3} = \frac{-3+2}{6} = -\frac{1}{6}$$

$$-\frac{1}{6} - \frac{1}{4} = \frac{-2-3}{12} = -\frac{5}{12}$$

$$\frac{\ln(1+x)}{\ln(1-x)} = \frac{x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \mathcal{O}(x^4)}{-x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} + \mathcal{O}(x^4)}$$

$$= \frac{1 - \frac{x}{2} + \frac{x^2}{3} - \frac{x^3}{4} + \mathcal{O}(x^3)}{-1 - \frac{x}{2} - \frac{x^2}{3} - \frac{x^3}{4} + \mathcal{O}(x^3)} = - \frac{1 - \frac{x}{2} + \frac{x^2}{3} - \frac{x^3}{4} + \mathcal{O}(x^3)}{1 + \frac{x}{2} + \frac{x^2}{3} + \frac{x^3}{4} + \mathcal{O}(x^3)}$$

$\times$

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + \mathcal{O}(x^3)$$

$$\frac{\ln(1+x)}{\ln(1-x)} = - \left(1 - \frac{x}{2} + \frac{x^2}{3} - \frac{x^3}{4} + \mathcal{O}(x^3)\right) \cdot \frac{1}{1+x}$$

$$= - \left(1 - \frac{x}{2} + \frac{x^2}{3} - \frac{x^3}{4} + \mathcal{O}(x^3)\right) \left(1 - \frac{x}{2} + \frac{x^2}{3} + \frac{x^3}{4} + \mathcal{O}(x^3)\right) + \left( \frac{\quad}{\quad} \right)^2 - \left( \frac{\quad}{\quad} \right)^3 + \text{pole}$$

$$(4a) \quad \frac{\cos(x) - 1}{x^2} = f(x)$$

$$\mathcal{D}_{\frac{1}{2}}(0): \quad \cos(x) = 1 - \frac{x^2}{2} + \mathcal{O}(x^2)$$

$$f(x) = \frac{1 - \frac{x^2}{2} + \mathcal{O}(x^2) - 1}{x^2} = \frac{x^2(-\frac{1}{2} + \mathcal{O}(1))}{x^2} \rightarrow -\frac{1}{2} + \mathcal{O} = -\frac{1}{2}$$

$$(4b) \quad \frac{\cos(x) - 1}{x \ln(1+x)} = \frac{1 - \frac{x^2}{2} + \mathcal{O}(x^2) - 1}{x(x + \mathcal{O}(x))} \leftarrow \text{order 2}$$

$$= \frac{x^2(-\frac{1}{2} + \mathcal{O}(1))}{x^2(1 + \mathcal{O}(1))} \xrightarrow{x \rightarrow 0} \frac{-\frac{1}{2} + 0}{1 + 0} = -\frac{1}{2}$$

$$(4c) \quad \frac{x - \sin x}{x(1 - \cos x)} = \frac{x - (x - \frac{x^3}{6} + \mathcal{O}(x^3))}{x(1 - (1 - \frac{x^2}{2} + \mathcal{O}(x^2)))} \leftarrow \text{order 3}$$

$$= \frac{\frac{x^3}{6} + \mathcal{O}(x^3)}{x^2(\frac{1}{2} + \mathcal{O}(1))} = \frac{x^3(\frac{1}{6} + \mathcal{O}(1))}{x^3(\frac{1}{2} + \mathcal{O}(1))} \xrightarrow{0} \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{2}{6} = \frac{1}{3}$$

$$(6a) \quad \frac{\ln(\cos \beta x)}{\ln(\cos \alpha x)} = \frac{\ln\left(1 - \frac{9x^2}{2} + \mathcal{O}(x^2)\right)}{\ln\left(1 - \frac{4x^2}{2} + \mathcal{O}(x^2)\right)} =: f(x)$$

$$\ln(1-x) = -x + \mathcal{O}(x)$$

$$f(x) = \frac{-\frac{9x^2}{2} + \mathcal{O}(x^2)}{-\frac{4x^2}{2} + \mathcal{O}(x^2)} = \frac{\cancel{x^2} \left(-\frac{9}{2} + \mathcal{O}(1)\right)}{\cancel{x^2} \left(-2 + \mathcal{O}(1)\right)}$$

$$\xrightarrow{0} \frac{9}{4}$$



$$\textcircled{1a} \quad 2n + \frac{4}{n+2} \rightarrow +\infty \quad \left( +\infty + \frac{4}{\infty} = +\infty + 0 = +\infty \right)$$

$$\textcircled{1b} \quad \frac{n^3}{3} \xrightarrow{+\infty} +\infty$$

$$\frac{3n^2 - 5n}{n-1} = \frac{n^2 \left(3 - \frac{5}{n}\right)}{n \left(1 - \frac{1}{n}\right)} = n \frac{\left(3 - \frac{5}{n}\right)}{1 - \frac{1}{n}} \xrightarrow{+\infty} +\infty \times \frac{3}{1} = +\infty$$

$$\textcircled{1c} \quad n + (-1)^n \rightarrow ?$$

$$-1 \leq (-1)^n \leq 1$$

$\Downarrow$

$$n-1 \leq n + (-1)^n$$

$\downarrow$

$$+\infty \quad \text{donc } n + (-1)^n \rightarrow +\infty \quad (\text{T des gendarmes})$$

$$(d) \quad x^2 + x \sin x = x^2 \left(1 + \frac{\sin x}{x}\right)$$

$$-1 \leq \sin x \leq 1 \quad \Rightarrow \quad \forall x > 0$$

$$\begin{array}{ccc} -\frac{1}{x} \leq \frac{\sin x}{x} \leq \frac{1}{x} \\ x \rightarrow +\infty \downarrow & & x \rightarrow +\infty \downarrow \\ 0 & & 0 \end{array}$$

$$\text{donc} \quad \lim_{x \rightarrow +\infty} \frac{\sin x}{x} = 0 \quad (\text{T des gendarmes})$$

$$\text{donc} \quad x^2 \left(1 + \frac{\sin x}{x}\right) \rightarrow +\infty \quad (+\infty \times (1+0) = +\infty \times 1 = +\infty)$$

$$(e) \quad \frac{3x-5}{x+1} - \frac{\sin x}{x} \qquad \frac{3x-5}{x+1} = \frac{x(3-\frac{5}{x})}{x(1+\frac{1}{x})} \rightarrow 3$$

↓

3

$x \rightarrow +\infty$   
↓

0 (T des gendarmes)

$$\rightarrow 3 - 0 = 3$$

$$\textcircled{p} \quad \sqrt{x^2+3x} - x = \sqrt{x^2\left(1+\frac{3}{x}\right)} - x = f(x)$$

$$\left| \begin{array}{c} \downarrow \\ +\infty \\ \text{F.I.} \end{array} \right| \begin{array}{c} -\infty \\ \downarrow \\ 1 \end{array} \right| = x \sqrt{1+\frac{3}{x}} - x$$

$\downarrow \quad \downarrow$   
 $+\infty \quad 1 \quad -\infty \quad \text{F. I.}$

DL de  $\sqrt{1+x}$ :  $\sqrt{1+x} = 1 + \frac{1}{2}x + o(x)$

$$f(x) = x \left( 1 + \frac{1}{2} \frac{3}{x} + o\left(\frac{3}{x}\right) \right) - x = \cancel{x} + \frac{3}{2} + o(3) - \cancel{x}$$

$$\rightarrow \frac{3}{2} + 0 = \underline{\underline{\frac{3}{2}}}$$

$$\textcircled{2a} \quad \sin(2x)e^{-3x} = f(x)$$

$$\sin(x) = x - \frac{x^3}{6} + \mathcal{O}(x^3)$$

$$\sin(2x) = 2x - \frac{8x^3}{6} + \mathcal{O}(x^3) = 2x - \frac{4x^3}{3} + \mathcal{O}(x^3)$$

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \mathcal{O}(x^3)$$

$$e^{-3x} = 1 - 3x + \frac{9x^2}{2} - \frac{27x^3}{6} + \mathcal{O}(x^3) = 1 - 3x + \frac{9}{2}x^2 - \frac{9}{2}x^3 + \mathcal{O}(x^3)$$

$$\begin{aligned} \sin(2x)e^{-3x} &= \left(2x - \frac{4x^3}{3} + \mathcal{O}(x^3)\right) \left(1 - 3x + \frac{9}{2}x^2 - \frac{9}{2}x^3 + \mathcal{O}(x^3)\right) \\ &= 2x - 6x^2 + 9x^3 - \frac{4x^3}{3} + \mathcal{O}(x^3) \end{aligned}$$

$$= 2x - 6x^2 + \frac{23x^3}{3} + \mathcal{O}(x^3)$$

$$(2b) \ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \mathcal{O}(x^4)$$

$$\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} + \mathcal{O}(x^4)$$

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \mathcal{O}(x^4)$$


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$$- (x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \mathcal{O}(x^4))$$

$$\frac{-x^2 \quad -\frac{x^4}{2} + \mathcal{O}(x^4)}{-(-x^2 - \frac{x^3}{2} - \frac{x^4}{3} + \mathcal{O}(x^4))}$$

$$\frac{\frac{x^3}{2} - \frac{x^4}{6} + \mathcal{O}(x^4)}{- (\frac{x^3}{2} + \frac{x^4}{4} + \mathcal{O}(x^4))}$$

$$\frac{-\frac{5}{12}x^4 + \mathcal{O}(x^4)}{-(-\frac{5}{12}x^4 + \mathcal{O}(x^4))}$$

$$\frac{0 + \mathcal{O}(x^4)}{0 + \mathcal{O}(x^4)}$$

$$-x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} + \mathcal{O}(x^4)$$

$$-1 + x - \frac{x^2}{2} + \frac{5}{12}x^3 + \mathcal{O}(x^3)$$

$$-\frac{x^4}{4} - \frac{x^4}{4} = -\frac{2x^4}{4} = -\frac{x^4}{2}$$

$$-\frac{1}{2} + \frac{1}{3} = \frac{-3+2}{6} = -\frac{1}{6}$$

$$-\frac{1}{6} - \frac{1}{4} = \frac{-2-3}{12} = -\frac{5}{12}$$

(4a) DL en 0 à l'ordre 2 de  $\cos x$

$$\lim_{0} \frac{\cos x - 1}{x^2} = \frac{0}{0} \quad \text{F.I.}$$

$$= \lim_{0} \frac{1 - \frac{x^2}{2} + o(x^2) - 1}{x^2} = \lim_{0} \frac{x^2(-\frac{1}{2} + o(1))}{x^2}$$

$$= -\frac{1}{2}$$

(4b)

$$f(x) = \frac{\cos(x) - 1}{x \ln(1+x)} \quad \leftarrow \text{ordre 2}$$

$$\quad \leftarrow \text{ordre 2}$$

$$f(x) = \frac{1 - \frac{x^2}{2} + o(x^2) - 1}{x(x + o(x))} = \frac{-\frac{x^2}{2} + o(x^2)}{x^2 + o(x^2)} = \frac{x^2(-\frac{1}{2} + o(1))}{x^2(1 + o(1))} \xrightarrow{x \rightarrow 0} \frac{-\frac{1}{2} + 0}{1 + 0} = -\frac{1}{2}$$

$$(4c) \lim_{x \rightarrow 0} \frac{x - \sin x}{x(1 - \cos x)} = \frac{0}{0} \quad \text{die}$$

$$f(x) = \frac{x - \sin x}{x(1 - \cos x)} = \frac{x - (x - \frac{x^3}{6} + o(x^3))}{x(1 - (1 - \frac{x^2}{2} + o(x^2)))} = \frac{\frac{x^3}{6} + o(x^3)}{\frac{x^3}{2} + o(x^3)}$$

$$= \frac{\cancel{x^3} [\frac{1}{6} + o(1)]}{\cancel{x^3} [\frac{1}{2} + o(1)]} \rightarrow \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{6} \times \frac{2}{1} = \frac{1}{3}$$

$$(6a) \frac{\ln(\cos 3x)}{\ln(\cos 2x)} = \frac{\ln(1 - \frac{9x^2}{2} + o(x^2))}{\ln(1 - \frac{4x^2}{2} + o(x^2))} \quad \ln(1-x)$$

$$= \frac{\frac{9x^2}{2} + o(x^2)}{\frac{4x^2}{2} + o(x^2)} \rightarrow \frac{9/2}{4/2} = \frac{9}{4}$$