

(2a) $f(x) = \sin(2x) e^{-3x}$ à l'ordre 3 en 0

$$\sin(2x) = 2x - \frac{(2x)^3}{6} + o(x^3) = 2x - \frac{8x^3}{6} + o(x^3) = 2x - \frac{4}{3}x^3 + o(x^3)$$

$$e^{-3x} = 1 - 3x + \frac{(-3x)^2}{2!} + \frac{(-3x)^3}{6} + o(x^3) = 1 - 3x + \frac{9x^2}{2} - \frac{9x^3}{2} + o(x^3)$$

$$f(x) = \left(2x - \frac{4}{3}x^3 + o(x^3) \right) \left(1 - 3x + \frac{9x^2}{2} - \frac{9x^3}{2} + o(x^3) \right)$$

$$= 2x - 6x^2 + 9x^3 - \frac{4}{3}x^3 + o(x^3) = 2x - 6x^2 + \frac{23}{3}x^3 + o(x^3)$$

(2b) $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + o(x^4)$

$$\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} + o(x^4)$$

$\begin{array}{r} x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + o(x^4) \\ - (x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + o(x^4)) \\ \hline -x^2 - \frac{x^4}{2} + o(x^4) \\ - (-x^2 - \frac{x^3}{2} - \frac{x^4}{3} + \dots) \\ \hline \frac{x^3}{2} - \frac{x^4}{6} + o(x^4) \\ - (\frac{x^3}{2} + \frac{x^4}{4} + o(x^4)) \\ \hline \frac{5x^4}{12} + o(x^4) \end{array}$	$\begin{array}{r} -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} + o(x^4) \\ \hline -1 + x + \frac{x^2}{2} + \frac{5x^3}{12} + o(x^3) \end{array}$ $-\frac{1}{2} + \frac{1}{3} = \frac{-3+2}{6}$ $-\frac{1}{6} - \frac{1}{4} = \frac{-2-3}{12} = \frac{-5}{12}$
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$$\frac{\ln(1+x)}{\ln(1-x)} = -1 + x - \frac{x^2}{2} + \frac{5}{12}x^3 + o(x^3)$$