

TD DLs

(12a)  $\lim_{x \rightarrow e} \frac{\ln(\ln x)}{x^2 - 3ex + e^2}$  chgt  $X = x - e$ . Alors, quand  $x \rightarrow e$ ,  $X \rightarrow 0$

$$f(x) = \frac{\ln(\ln x)}{x^2 - 3ex + e^2} = \frac{\ln(\ln(X+e))}{(X+e)^2 - 3e(X+e) + e^2} = \frac{\ln(\ln(e(1+\frac{X}{e})))}{X^2 + 2eX + e^2 - 3eX - 3e^2 + e^2}$$

← on a mis le + gden facteur  
← on a développé

$$= \frac{\ln(\ln e + \ln(1+\frac{X}{e}))}{X^2 - eX} = \frac{\ln(1 + \frac{X}{e} + o(x))}{-eX + X^2} = \frac{\frac{X}{e} + o(x)}{-eX + X^2} = \frac{X(\frac{1}{e} + o(1))}{X(-e+X)} \rightarrow \boxed{-\frac{1}{e^2}}$$

(12b)  $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\frac{1}{\cos^2 x} - 2\cot x}{1 + \cos 4x}$  chgt  $X = x - \frac{\pi}{4} \Rightarrow x = \frac{\pi}{4} + X$

Maintenant, on utilise les formules de Lupa avec  $\cos(a+b)$  et  $\sin(a+b)$

$$f(x) = \frac{\frac{1}{\cos^2 x} - 2\cot x}{1 + \cos 4x} = \frac{\frac{1}{\cos^2(\frac{\pi}{4}+X)} - 2\frac{\sin(\frac{\pi}{4}+X)}{\cos(\frac{\pi}{4}+X)}}{1 + \cos(4X + \pi)}$$

$$f(x) = \frac{\frac{1}{(\frac{\sqrt{2}}{2}\cos X + \frac{\sqrt{2}}{2}\sin X)^2} - \frac{2(\frac{\sqrt{2}}{2}\sin X + \frac{\sqrt{2}}{2}\cos X)}{\frac{\sqrt{2}}{2}\cos X - \frac{\sqrt{2}}{2}\sin X}}{1 - \cos 4X}$$

← on a multiplié en haut et en bas par  $(\cos X - \sin X)^2$

$$= \frac{1 - \cos(4X)}{2 - 2(\sin X + \cos X)(\cos X - \sin X)}$$

$$= \frac{1 - \cos(4X)}{(\cos X - \sin X)^2 (1 - \cos 4X)}$$

$$= \frac{2 - 2(\cos^2 X - \sin^2 X)}{(\cos X - \sin X)^2 (1 - \cos 4X)}$$

double produit

$$\star 2 - 2\cos^2 X + 2\sin^2 X = 2 - 2(1 - X^2 + o(X^2)) + 2(X^2 + o(X^2)) = 2 - 2(1 - X^2 + o(X^2)) + 2X^2 + o(X^2)$$

$$= 4X^2 + o(X^2)$$

$$\star (\cos X - \sin X)^2 = (1 - X + o(X))^2 = 1 - 2X + o(X)$$

$$\star (1 - \cos 4X) = 1 - (1 - \frac{(4X)^2}{2} + o(X^2)) = 8X^2 + o(X^2)$$

Donc  $f(x) = \frac{4X^2 + o(X^2)}{(1 - 2X + o(X))(8X^2 + o(X^2))} = \frac{X^2(4 + o(1))}{X^2(1 - 2X + o(X))(8 + o(1))} \rightarrow \frac{4}{1 \times 8} = \boxed{\frac{1}{2}}$

(13)  $\lim_{x \rightarrow 1} \frac{\ln(x)}{2x-1}$  chgt  $X = x - \frac{1}{2} \Rightarrow x = \frac{1}{2} + X$

$$\frac{\ln(x)}{2x-1} = \frac{\ln(1+2X)}{1+2X-1} = \frac{\ln(1+2X)}{2X} = \frac{2X + o(X)}{2X} = \frac{2 + o(1)}{2} \rightarrow \boxed{1}$$