

⑤ Ensemble de def de  $f(x) = \sqrt{x^2 - 5x + 6}$

Il faut que  $f(x)$  soit  $\geq 0$

$$\Delta = 25 - 24 = 1 \quad x = \frac{5 \pm 1}{2} = \begin{matrix} 3 \\ 2 \end{matrix}$$

x	$-\infty$	2	3	$+\infty$
f(x)	+	$\emptyset$	$\emptyset$	+

$$D_f = ]-\infty, 2] \cup [3, +\infty[$$

⑥  $f(x) = \sin(3x)$

$$f(-x) = \sin(3(-x)) = \sin(-3x) = -\sin(3x) = -f(x)$$

f est impaire

$$g(x) = \sin(x^2)$$

$$g(-x) = \sin((-x)^2) = \sin(x^2) = g(x) \quad \text{g est pair}$$

$$h(x) = \frac{x^2 + 1}{x + 1}$$

$$h(2) = \frac{5}{3}$$

$$h(-2) = \frac{5}{-1} = -5$$

$h(-2) \neq h(2)$  donc  $h$  n'est pas pair

et  $h(-2) \neq -h(2)$ , donc  $h$  n'est pas impair

$$f(x) = \ln\left(\frac{1+x}{1-x}\right)$$

$$f(-x) = \ln\left(\frac{1-x}{1+x}\right) = \ln(1-x) - \ln(1+x)$$

$$= -(\ln(1+x) - \ln(1-x)) = -\ln\left(\frac{1+x}{1-x}\right)$$

$$= -f(x).$$

$f$  est impair

(7)

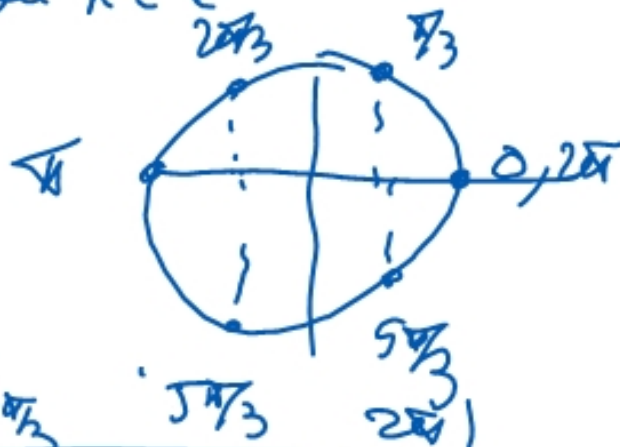
$$\sin x - \cos x = 0 \Leftrightarrow \sin x = \cos x$$

$$x = \frac{\pi}{4} + k\pi \text{ ou } k \in \mathbb{Z}$$



$$|\sin(3x)| = 0 \Leftrightarrow 3x = k\pi \text{ ou } k \in \mathbb{Z}$$

$$\Leftrightarrow x = \frac{k\pi}{3}$$



$x$	0	$\pi/4$	$\pi/3$	$2\pi/3$	$\pi$	$5\pi/4$	$4\pi/3$	$5\pi/3$	$2\pi$
$\sin(x) - \cos(x)$	-	0	+	+	+	0	-	-	-
$ \sin(3x) $	0	+	+	0	+	0	-	0	+
signo de $\frac{\sin(x) - \cos(x)}{ \sin(3x) }$	-	+	-	+	-	+	-	+	-
$\mathcal{J} = ]\frac{\pi}{4}, \frac{\pi}{3}[ \cup ]\frac{2\pi}{3}, \pi[ \cup ]\frac{5\pi}{4}, \frac{4\pi}{3}[ \cup ]\frac{5\pi}{3}, 2\pi[$ modulo $2\pi$									

$$\begin{aligned}
 (11) \quad \sin^3 x \cdot \cos(2x) &= \left( \frac{e^{ix} - e^{-ix}}{2i} \right)^3 \cdot \left( \frac{e^{i2x} + e^{-i2x}}{2} \right) \\
 &= \frac{(e^{ix} - e^{-ix})^3 (e^{i2x} + e^{-i2x})}{8 \cdot i^3 \cdot 2} \quad i^3 = -i
 \end{aligned}$$

$$= -\frac{1}{16i} (e^{i3x} - 3e^{ix} \cdot e^{-ix} + 3e^{ix} e^{-ix} - e^{-i3x}) (e^{i2x} + e^{-i2x})$$

$$= -\frac{1}{16i} (e^{i3x} - 3e^{ix} + 3e^{-ix} - e^{-i3x}) (e^{i2x} + e^{-i2x})$$

$$= -\frac{1}{16i} (e^{i5x} - 3e^{i3x} + 3e^{ix} - e^{-ix} + e^{ix} - 3e^{-ix} + 3e^{-i3x} - e^{-i5x})$$

$$= -\frac{1}{16i} \left( (e^{i5x} - e^{-i5x}) - 3(e^{i3x} - e^{-i3x}) + 4(e^{ix} - e^{-ix}) \right)$$

$$\sin^3 x \cos(2x) = -\frac{1}{16i} (2i \sin(5x) - 6i \sin(3x) + 8i \sin(x))$$

$$= -\frac{1}{8} \sin(5x) + \frac{3}{8} \sin(3x) - \frac{1}{2} \sin(x)$$


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$$\sin^2(x) \cos^4(x) = \left( \frac{e^{ix} - e^{-ix}}{2i} \right)^2 \left( \frac{e^{ix} + e^{-ix}}{2} \right)^4$$

$$= \left( \frac{e^{i2x} - 2 + e^{-i2x}}{-4} \right) \left( \frac{e^{i4x} + 4e^{i2x} + 6 + 4e^{-i2x} + e^{-i4x}}{16} \right)$$

$$= -\frac{1}{64} \left( \begin{array}{l} e^{i6x} + 4e^{i4x} + 6e^{i2x} + 4 + e^{-i2x} \\ -2e^{i4x} - 8e^{i2x} - 12 - 8e^{-i2x} - 2e^{-i4x} \\ + e^{i2x} + 4 + 6e^{-i2x} + 4e^{-i4x} + e^{-i6x} \end{array} \right)$$

$$= -\frac{1}{64} (e^{i6x} + 2e^{i4x} - e^{i2x} - 4 - e^{-i2x} + 2e^{-i4x} + e^{-i6x})$$

$$= -\frac{1}{64} (e^{i6x} + e^{-i6x} + 2(e^{i4x} + e^{-i4x}) - (e^{i2x} + e^{-i2x}) - 4)$$

$$= -\frac{1}{64} (2\cos(6x) + 4\cos(4x) - 2\cos(2x) - 4)$$

$$= -\frac{1}{32} \cos(6x) - \frac{1}{16} \cos(4x) + \frac{1}{32} \cos(2x) + \frac{1}{16}$$

$$(5) f(x) = \sqrt{x^2 - 5x + 6}$$

Il faut que  $x^2 - 5x + 6$  soit positif ou nul

$$\Delta = 25 - 24 = 1 \quad x = \frac{5 \pm 1}{2} = \begin{matrix} 3 \\ 2 \end{matrix}$$

$x$	$-\infty$	$0$	$2$	$3$	$+\infty$
$x^2 - 5x + 6$	$+$	$0$	$0$	$-$	$+$

$$\mathcal{D}_f = ]-\infty, 2] \cup [3, +\infty[$$

$$(6) f(x) = \sin(3x) ; f(-x) = \sin(3(-x)) = \sin(-3x) = -\sin(3x) = -f(x)$$

donc  $f$  est impaire

$$g(x) = \sin(x^2)$$

$$g(-x) = \sin((-x)^2) = \sin(x^2) = g(x). \quad g \text{ est paire}$$

$$f(x) = \frac{x^2+1}{x+1}$$

$$f(-x) = \frac{(-x)^2+1}{-x+1} = \frac{x^2+1}{-x+1} \neq f(x)$$

$$\triangle \quad -x+1 = -\frac{x^2+1}{x-1} \neq -f(x)$$

$$f(2) = \frac{5}{3}$$

$$f(-2) = \frac{5}{-1} = -5$$

$$f(-2) \neq f(2) \quad \text{donc } f \text{ n'est pas paire}$$

$$f(-2) \neq -f(2) \quad \text{impair}$$

ni paire ni impaire



$$f(x) = \ln\left(\frac{1+x}{1-x}\right)$$

$$f(-x) = \ln\left(\frac{1-x}{1+x}\right) = \ln(1-x) - \ln(1+x)$$

$$= -(\ln(1+x) - \ln(1-x)) = -\ln\frac{1+x}{1-x} = -f(x),$$

$f$  est impair

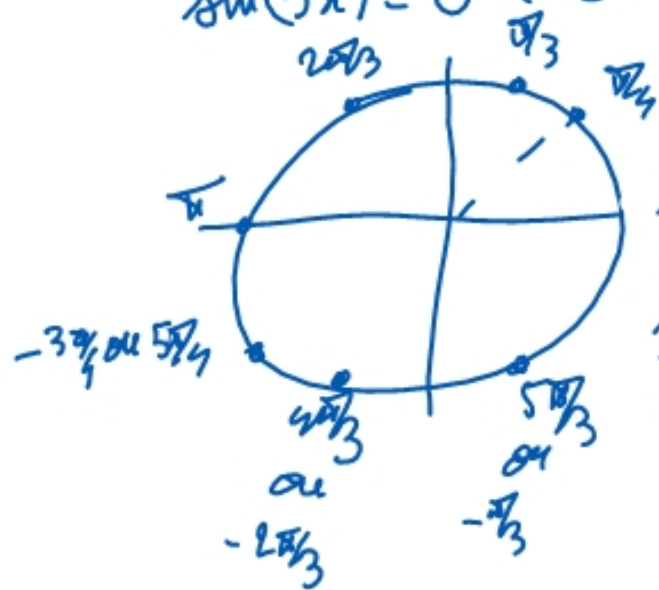
$$(7) f(x) = \frac{\sin x - \cos x}{\sin(3x)}$$

On cherche les  $x$  tels que  $\sin x - \cos x = 0$   
et les  $x$  tels que  $\sin(3x) = 0$

puis, on fait un tableau de signes pour  $\sin x - \cos x$  et  $\sin(3x)$   
sur  $[-\pi; \pi]$

$$\sin x - \cos x = 0 \Leftrightarrow \sin x = \cos x \Leftrightarrow x = \frac{\pi}{4} + k\pi \text{ ou } k \in \mathbb{Z}$$

$$|\sin(3x)| = 0 \Leftrightarrow 3x = k\pi \text{ ou } k \in \mathbb{Z} \Leftrightarrow x = \frac{k\pi}{3} \text{ ou } k \in \mathbb{Z}$$



$x$	$-\pi$	$-\frac{3\pi}{4}$	$-\frac{2\pi}{3}$	$-\frac{\pi}{3}$	$0$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	$\pi$
$\sin x - \cos x$	+	○	-	-	-	○	+	+	+
$ \sin(3x) $	○	-	-	○	+	○	+	○	+
$\frac{\sin x - \cos x}{ \sin(3x) }$	-	+	-	+	-	+	-	+	-

$$J = ]-\frac{3\pi}{4}; -\frac{2\pi}{3}[ \cup ]-\frac{\pi}{3}; 0[ \cup ]\frac{\pi}{4}; \frac{\pi}{3}[ \cup ]\frac{2\pi}{3}; \pi[ \text{ modulo } 2\pi$$

$$\sin^3(x) \cos(2x) = \left( \frac{e^{ix} - e^{-ix}}{2i} \right)^3 \left( \frac{e^{i2x} + e^{-i2x}}{2} \right) \quad i^3 \cdot i^2 = -i$$

$$= \frac{(e^{ix} - e^{-ix})^3 (e^{i2x} + e^{-i2x})}{(2i)^3 \cdot 2} = \frac{1}{-16i} \left( \quad \right)^3 \left( \quad \right)$$

$$= -\frac{1}{16i} \left( e^{i3x} - 3e^{ix}e^{-i2x} + 3e^{ix}e^{-i2x} - e^{-i3x} \right) (e^{i2x} + e^{-i2x})$$

$$= -\frac{1}{16i} \left( e^{i3x} - 3e^{ix} + 3e^{-ix} - e^{-i3x} \right) (e^{i2x} + e^{-i2x})$$

$$= -\frac{1}{16i} \left[ \begin{array}{l} e^{i5x} - 3e^{i3x} + 3e^{ix} - e^{-ix} \\ + e^{ix} - 3e^{-ix} + 3e^{-i3x} - e^{-i5x} \end{array} \right]$$

$$= -\frac{1}{16i} \left[ (e^{i5x} - e^{-i5x}) - 3(e^{i3x} - e^{-i3x}) + 4(e^{ix} - e^{-ix}) \right]$$

$$= -\frac{1}{16i} \left[ 2i \sin(5x) - 6i \sin(3x) + 8i \sin(x) \right]$$

$$= -\frac{1}{8} \sin(5x) + \frac{3}{8} \sin(3x) - \frac{1}{2} \sin(x)$$

$$\sin^2(x) \cos^4(x) =$$