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$$\textcircled{1} \cdot \int_0^{0,5} \frac{4x^2 - 2}{x(1-x^2)} dx \quad \text{pb en 0 :}$$

$$\frac{4x^2 - 2}{x(1-x^2)} \underset{0}{\sim} \frac{-2}{x \cdot 1} = -\frac{2}{x}$$

$$\int_0 \frac{1}{x} dx \quad \text{Div (Riemann)} \Rightarrow \int_0 \frac{-2}{x} dx \quad \text{Div}$$

$$\Rightarrow \int_0 F(x) dx \quad \text{Div (Tds, } \mathcal{N})$$

$$\bullet \int_{-1}^{-0,5} F(x) dx \quad \text{pb en } -1$$

$$\frac{4x^2 - 2}{x(1-x^2)} = \frac{4x^2 - 2}{x(1-x)(1+x)} \underset{-1}{\sim} \frac{2}{-1 \cdot -1 \cdot 2 \cdot (1+x)} = \frac{-1}{1+x}$$

$$\int_{-1} \frac{1}{1+x} dx \quad \text{Div (Riemann)} \rightarrow \int_{-1} \frac{-1}{1+x} dx \quad \text{Div}$$

$$\Rightarrow \int_{-1} F(x) dx \quad \text{Div (Tds, } \mathcal{N})$$

- $\int_1^2 F$ pb en 1

$$F(x) = \frac{4x^3 2}{x(1-x)(1+x)} \sim \frac{2}{1 \cdot (1-x) \cdot 2} = \frac{1}{1-x}$$

$$\int_1^2 \frac{1}{1-x} dx \text{ Div (Riemann)} \Rightarrow \underline{\int_1^2 F \text{ Div}} \quad (\nabla \text{ de } \mathcal{N})$$

- $\int_2^3 F$ pas de pb !! On dit que $\int_2^3 F$ CV

- $\int_2^{+\infty} F$ pb en $+\infty$

$$F(x) \underset{+\infty}{\sim} \frac{4x^2}{x \cdot (-x^2)} = -\frac{4}{x}$$

$$\int_2^{+\infty} \frac{1}{x} dx \text{ Div (Riemann)} \Rightarrow \int_2^{+\infty} -\frac{4}{x} dx \text{ Div}$$

$$\Rightarrow \int_2^{+\infty} F \text{ Div} \quad (\nabla \text{ de } \mathcal{N})$$

$$\textcircled{2} \quad F(x) = \frac{2x^2 - x + 2}{x^4 + x^3 - 2x^2} = \frac{2x^2 - x + 2}{x^2(x^2 + x - 2)} = \frac{2x^2 - x + 2}{x^2(x+2)(x-1)}$$

poles en 0; -2; 1 et $\pm\infty$

$$\bullet \quad F(x) \underset{0}{\sim} \frac{2}{x^2 \cdot 2(-1)} = -\frac{1}{x^2} \quad \int_0^1 \frac{1}{x^2} dx \text{ DIV (Riemann)}$$

$$\Rightarrow \int_0^1 -\frac{1}{x^2} dx \text{ DIV} \Rightarrow \underline{\int_0^1 F(x) dx \text{ DIV}} \quad (\text{Tds } \sim)$$

$$\bullet \quad F(x) \underset{-2}{\sim} \frac{8 + 2 + 2}{4(x+2)(-3)} = \frac{-1}{x+2} \quad \int_{-2}^1 \frac{1}{x+2} dx \text{ DIV (Riemann)}$$

$$\Rightarrow \int_{-2}^1 \frac{-1}{x+2} dx \text{ DIV} \Rightarrow \underline{\int_{-2}^1 F(x) dx \text{ DIV}} \quad (\text{Tds } \sim)$$

$$\bullet \quad F(x) \underset{1}{\sim} \frac{2 - 1 + 2}{1 \cdot 3(x-1)} = \frac{1}{x-1} \quad \int_1^{\infty} \frac{1}{x-1} dx \text{ DIV (Riemann)} \Rightarrow \underline{\int_1^{\infty} F dx \text{ DIV}} \quad (\text{Tds } \sim)$$

$$\bullet \quad F(x) \underset{+\infty}{\sim} \frac{2x^2}{x^4} = \frac{2}{x^2} \quad \int_{-\infty}^{+\infty} \frac{1}{x^2} dx \text{ CV} \quad \left(\text{donc } \int_{-\infty}^{+\infty} \frac{1}{x^2} dx \text{ CV} \right) \quad (\text{Riemann})$$

$$\Rightarrow \int_{-\infty}^{+\infty} \frac{2}{x^2} dx \text{ CV} \Rightarrow \underline{\int_{-\infty}^{+\infty} F dx \text{ CV}} \quad (\text{Tds } \sim)$$

$$(3) \int_0^1 \frac{e^{\sqrt{x}} - 1}{\sin x} dx \quad \text{pb en } 0$$

$$\frac{e^{\sqrt{x}} - 1}{\sin x} = \frac{1 + \sqrt{x} + o(\sqrt{x}) - 1}{x + o(x)} = \frac{\sqrt{x} + o(\sqrt{x})}{x + o(x)} \underset{0}{\sim} \frac{\sqrt{x}}{x} = \frac{1}{\sqrt{x}} = \frac{1}{x^{1/2}}$$

$$\int_0^1 \frac{1}{x^{1/2}} dx \text{ CV (Riemann)} \rightarrow \int_0^1 \frac{e^{\sqrt{x}} - 1}{\sin x} dx \text{ CV (T des N)}$$

$$\int_0^1 \frac{\ln x}{1-x} dx \quad \text{pb en } 0 \text{ et en } 1$$

$$\text{pb en } 0: \frac{\ln x}{1-x} \underset{0}{\sim} \frac{\ln(x)}{1} = \ln(x)$$

$$\int_0^1 \ln(x) dx \text{ CV (Cesaro)} \\ \Rightarrow \int_0^1 \frac{\ln x}{1-x} dx \text{ CV (T des N)}$$

$$\text{pb en } 1: \text{chgt } X = x-1. \quad \frac{\ln x}{1-x} = \frac{\ln(1+X)}{-X} = \frac{X + o(X)}{-X} = -1 + o(1)$$

$\frac{\ln x}{1-x}$ ne tend pas vers l'infini en 1. Donc pas de problème pour $\int_{1/2}^1 \frac{\ln x}{1-x} dx$
 $\int_0^1 \frac{\ln x}{1-x} dx$ CV

ex 5: $f(x) = \int_1^{+\infty} \frac{1}{(t^2-1)^x} dt$

phs au let en $+\infty$

• phs au $+\infty$ $\frac{1}{(t^2-1)^x} \sim \frac{1}{t^{2x}}$

$\int_1^{+\infty} \frac{1}{t^{2x}} dt$ $\left\{ \begin{array}{l} \text{CV qd } x > 1 \\ \text{Div qd } x \leq 1 \end{array} \right.$ $\left. \begin{array}{l} \text{câd } x > \frac{1}{2} \\ \text{câd } x \leq \frac{1}{2} \end{array} \right\}$
 (Riemann) *

• phs au 1 $\frac{1}{(t^2-1)^x} = \frac{1}{(t-1)^x(t+1)^x} \sim \frac{1}{(t-1)^x \cdot 2^x}$

$\int_1^{+\infty} \frac{1}{(t-1)^x} dt$ $\left\{ \begin{array}{l} \text{CV qd } x < 1 \\ \text{Div qd } x \geq 1 \end{array} \right.$ (Riemann)

donc $\int_1^{+\infty} \frac{1}{(t^2-1)^x} dt$ $\left\{ \begin{array}{l} \text{CV qd } x < 1 \\ \text{Div qd } x \geq 1 \end{array} \right.$ (T des \sim)

* donc $\int_1^{+\infty} \frac{1}{(t^2-1)^x} dt$ $\left\{ \begin{array}{l} \text{CV qd } x > \frac{1}{2} \\ \text{Div qd } x \leq \frac{1}{2} \end{array} \right.$ (T des \sim)

donc $\int_1^{+\infty} \frac{1}{(t^2-1)^x} dt$ $\left\{ \begin{array}{l} \text{CV qd } x \in]\frac{1}{2}, 1[\\ \text{Div sinon} \end{array} \right.$

$\mathcal{D}_f =]\frac{1}{2}, 1[$

$$\textcircled{BF} \quad F = \int_{-1}^1 x \sqrt{\frac{1+x}{1-x}} dx \quad \text{pb en 1}$$

$$x \sqrt{\frac{1+x}{1-x}} \underset{1}{\sim} 1 \sqrt{\frac{2}{1-x}} = \frac{\sqrt{2}}{\sqrt{1-x}}$$

$$\int_{-1}^1 \frac{1}{\sqrt{1-x}} dx \quad \text{CV} \quad (\text{Riemann avec } \alpha > \frac{1}{2})$$

$$\Rightarrow \int_{-1}^1 x \sqrt{\frac{1+x}{1-x}} dx \quad \text{CV}$$

$$\text{chgt } X = \sqrt{\frac{1+x}{1-x}}$$

on cherche x en fonction de X

$$\textcircled{1} \quad F(x) = \frac{4x^2 - 2}{x(1-x^2)} = \frac{4x^2 - 2}{x(1-x)(1+x)}$$

• $\int_0^{0,5} F$ pb en 0 car $\lim_{x \rightarrow 0} F(x) = \infty$

$$\frac{4x^2 - 2}{x(1-x)(1+x)} \underset{0}{\sim} \frac{-2}{x \cdot 1 \cdot 1} = \frac{-2}{x} \quad \int_0^{\frac{1}{2}} \frac{1}{x} dx \text{ Div (Riemann: } \alpha=1)$$

$$\Rightarrow \int \frac{-2}{x} dx \text{ Div} \Rightarrow \underline{\int_0^{(0,5)} F(x) dx \text{ Div (T ds } \sim)}$$

• $\int_{-1}^{-0,5} F$ pb en -1 $\frac{4x^2 - 2}{x(1-x)(1+x)} \underset{-1}{\sim} \frac{4-2}{-1 \cdot 2(1+x)} = -\frac{1}{1+x}$

$$\int_{-1}^{-0,5} \frac{1}{1+x} dx \text{ Div (Riemann, } \alpha=1) \Rightarrow \int_{-1}^{-0,5} \frac{-1}{1+x} dx \text{ Div} \Rightarrow \underline{\int_{-1}^{-0,5} F(x) dx \text{ Div (T ds } \sim)}$$

• $\int_1^2 F$ pb en 1 $\frac{4x^2 - 2}{x(1-x)(1+x)} \underset{1}{\sim} \frac{2}{1 \cdot (1-x)(2)} = \frac{1}{1-x}$

$$\int_1^2 \frac{1}{1-x} dx \text{ Div (Riemann, } \alpha=1) \Rightarrow \underline{\int_1^2 F \text{ Div (T ds } \sim)}$$

• $\int_2^3 F$ pas de pb. On doit faire l'intégral CV

• $\int_2^{+\infty} F$ pb en $+\infty$ $\frac{4x^2-2}{x(1-x^2)} \underset{+\infty}{\sim} \frac{4x^2}{x(-x^2)} = -\frac{4}{x}$

$\int \frac{1}{x} dx$ DIV (Riemann, $\alpha=1$) $\Rightarrow \int \frac{-4}{x} dx$ DIV $\Rightarrow \int \underline{F(x) dx}$ DIV (Tds \mathcal{U})

② $F(x) = \frac{2x^2-x+2}{x^4+x^3-2x^2} = \frac{2x^2-x+2}{x^2(x^2+x-2)} = \frac{2x^2-x+2}{x^2(x-1)(x+2)}$ pb en 0; 1; -2; $\pm\infty$

• pb en 0: $F(x) \underset{0}{\sim} \frac{2}{x^2(-1)2} = -\frac{1}{x^2}$ $\int_0 \frac{1}{x^2} dx$ DIV (Riemann; $\alpha=2$)

$\Rightarrow \int_0 -\frac{1}{x^2} dx$ DIV $\Rightarrow \int_0 \underline{F(x) dx}$ DIV (Tds \mathcal{U})

• pb en 1 $F(x) \underset{1}{\sim} \frac{3}{1(x-1)^3} = \frac{1}{x-1}$ $\int_1 \frac{1}{x-1} dx$ DIV (Riemann; $\alpha=1$) $\Rightarrow \int_1 \underline{F} dx$ DIV (Tds \mathcal{U})

• pb en -2. $F(x) \underset{-2}{\sim} \frac{12}{4 \cdot (-3)(x+2)} = -\frac{1}{x+2}$ $\int_{-2} \frac{1}{x+2} dx$ DIV (Riem.) $\Rightarrow \int_{-2} -\frac{1}{x+2} dx$ DIV $\Rightarrow \int_{-2} \underline{F} dx$ DIV (Tds \mathcal{U})

• pb en $+\infty$ $F(x) \underset{+\infty}{\sim} \frac{2x^2}{x^4} = \frac{2}{x^2}$ $\int \frac{1}{x^2} dx$ CV (Riemann; $\alpha=2$) $\Rightarrow \int \frac{2}{x^2} dx$ CV $\Rightarrow \int \underline{F(x) dx}$ CV (Tds \mathcal{U})

③ • $\int_0^1 \frac{e^{\sqrt{x}} - 1}{\sin x} dx$ pb en 0 $\frac{e^{\sqrt{x}} - 1}{\sin x} \underset{0}{\sim} ?$

$$\frac{e^{\sqrt{x}} - 1}{\sin x} = \frac{1 + \sqrt{x} + o(\sqrt{x}) - 1}{x + o(x)} = \frac{\sqrt{x} + o(\sqrt{x})}{x + o(x)} \underset{0}{\sim} \frac{\sqrt{x}}{x} = \frac{1}{\sqrt{x}}$$

$$\int_0^1 \frac{1}{\sqrt{x}} dx \text{ CV (Riemann: } \kappa = \frac{1}{2} < 1) \Rightarrow \int_0^1 \frac{e^{\sqrt{x}} - 1}{\sin x} dx \text{ CV (TdsR)}$$

• $\int_0^1 \frac{\ln x}{1-x} dx$ pas 0 et en 1

pb en 0: $\frac{\ln x}{1-x} \underset{0}{\sim} \frac{\ln(x)}{1} = \ln(x)$ $\int_0^1 \ln(x) dx \text{ CV (cours)} \Rightarrow \int_0^{1/2} \frac{\ln x}{1-x} dx \text{ CV (TdsR)}$

pb en 1: On se ramène en 0 avec le dtpt $X = x - 1$

$$\frac{\ln x}{1-x} = \frac{\ln(1+X)}{-X} = \frac{X + o(X)}{-X} = -1 + o(1) \underset{\substack{x \rightarrow 0 \\ x \rightarrow 1}}{\sim} -1$$

$$\int_0^{-1} -1 dx \text{ CV (pas de problèmes!)} \Rightarrow \int_{1/2}^1 \frac{\ln x}{1-x} dx \text{ CV (TdsR)}$$

$\int_0^{1/2}$ et $\int_{1/2}^1$ CV, donc $\int_0^1 \frac{\ln x}{1-x} dx \text{ CV}$

⑤ $f(x) = \int_1^{+\infty} \frac{1}{(t^2-1)^x} dt$ problèmes en 1 et en $+\infty$

pb en $+\infty$: $\frac{1}{(t^2-1)^x} \underset{t \rightarrow +\infty}{\sim} \frac{1}{t^{2x}}$ $\int_1^{+\infty} \frac{1}{t^{2x}} dt$ $\left\{ \begin{array}{l} \text{CV qd } 2x > 1 \\ \text{Div sinon} \end{array} \right.$ c.à.d. $x > \frac{1}{2}$ (Riemann)

donc $\int_1^{+\infty} \frac{1}{(t^2-1)^x} dt$ CV qd $x > \frac{1}{2}$ (∇ da \sim)

pb en 1: $\frac{1}{(t^2-1)^x} = \frac{1}{(t-1)^x (t+1)^x} \underset{t \rightarrow 1}{\sim} \frac{1}{(t-1)^x \cdot 2^x}$

$\int_1 \frac{1}{(t-1)^x} dt$ $\left\{ \begin{array}{l} \text{CV qd } x < 1 \\ \text{Div qd } x \geq 1 \end{array} \right.$ (Riemann) $\Rightarrow \int_1 \frac{1}{(t-1)^{2x}} dt$ CV qd $x < 1$

$\Rightarrow \int_1 \frac{1}{(t^2-1)^x} dt$ CV qd $x < 1$ (∇ da \sim)

$\int_1^{+\infty} \frac{1}{(t^2-1)^x} dt$ CV qd $x \in]\frac{1}{2}; 1[$

$\mathcal{D}_f =]\frac{1}{2}; 1[$

$$\textcircled{8} F = \int_{-1}^1 x \sqrt{\frac{1+x}{1-x}} dx \quad \text{pb en 1}$$

$$x \sqrt{\frac{1+x}{1-x}} \underset{z \rightarrow 1}{\sim} 1 \sqrt{\frac{2}{1-x}} = \frac{\sqrt{2}}{\sqrt{1-x}} \quad \int_{-1}^1 \frac{1}{\sqrt{1-x}} dx \text{ CV (Riemann : } \alpha = \frac{1}{2} < 1)$$

$$\Rightarrow \int_{-1}^1 \frac{\sqrt{2}}{\sqrt{1-x}} dx \text{ CV} \Rightarrow \int_{-1}^1 x \sqrt{\frac{1+x}{1-x}} dx \text{ CV (} \forall dz, \nu)$$

Calcul de F: chgt de variable $X = \sqrt{\frac{1+x}{1-x}}$. On cherche x en fonction de X

$$\Rightarrow X^2 = \frac{1+x}{1-x} \Rightarrow (1-x)X^2 = 1+x \Rightarrow X^2 - xX^2 = 1+x \Rightarrow X^2 - 1 = xX^2 + x = x(X^2 + 1)$$

$$\Rightarrow x = \frac{X^2 - 1}{X^2 + 1}$$

on est conduit à l'intégrale d'une fraction rationnelle.