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$$\textcircled{1} F(x) = \frac{X^4 + 2X + 2}{X^3 + X^2 - 2X} \quad \text{On factorise: } X^3 + X^2 - 2X = X(X^2 + X - 2)$$

$$\Delta = 9 \quad 2 \text{ racines: } \frac{-1 \pm 3}{2} = \begin{cases} 1 \\ -2 \end{cases} \quad \text{donc } X^2 + X - 2 = (X-1)(X+2)$$

$$F(x) = \frac{X^4 + 2X + 2}{X(X-1)(X+2)} = E(x) + \frac{a}{X} + \frac{b}{X-1} + \frac{c}{X+2}$$

$$\begin{array}{r|l} X^4 + 2X + 2 & X^3 + X^2 - 2X \\ - (X^4 + X^3 - 2X^2) & \textcircled{X-1} \\ \hline -X^3 + 2X^2 + 2X + 2 & \text{E(x)} \\ - (-X^3 - X^2 + 2X) & \\ \hline 3X^2 + 2 & \end{array}$$

$$a = \frac{X^4 + 2X + 2}{(X-1)(X+2)} \Big|_{X=0} = \frac{2}{-2} = -1 \quad \underline{a = -1}$$

$$b = \frac{X^4 + 2X + 2}{X(X+2)} \Big|_{X=1} = \frac{5}{3} \quad \underline{b = 5/3}$$

$$c = \frac{X^4 + 2X + 2}{X(X-1)} \Big|_{X=-2} = \frac{16 - 4 + 2}{6} = \frac{14}{6} = \frac{7}{3}$$

$$\frac{X^4 + 2X + 2}{X^3 + X^2 - 2X} = X - 1 - \frac{1}{X} + \frac{5/3}{X-1} + \frac{7/3}{X+2}$$

$$\textcircled{2} f(x) = \frac{x^4 - 2x + 2}{x^4 - 3x^3 + 3x^2 - x} \quad x^4 - 3x^3 + 3x^2 - x = x(x^3 - 3x^2 + 3x - 1) = x(x-1)^3$$

$$f(x) = E(x) + \frac{a}{x} + \left( \frac{b}{x-1} + \frac{c}{(x-1)^2} + \frac{d}{(x-1)^3} \right)$$

$$x^4 - 2x + 2 \mid x^4 - 3x^3 + 3x^2 - x \quad \cdot \quad a = \frac{x^4 - 2x + 2}{(x-1)^3} \Big|_{x=0} = -2 \quad \underline{a = -2}$$

$\dots$  1  $\left( \begin{array}{l} \text{E}(x) \end{array} \right)$

• On multiplie par  $(x-1)^3$ :  $\frac{x^4 - 2x + 2}{x} = 1 \cdot (x-1)^3 - \frac{2}{x}(x-1)^3 + b(x-1)^2 + c(x-1) + d$

valeurs en 1:  $1 = 0 - 0 + 0 + 0 + d \quad \underline{d = 1}$

$$\frac{x^4 - 2x + 2}{x(x-1)^3} = 1 - \frac{2}{x} + \frac{b}{x-1} + \frac{c}{(x-1)^2} + \frac{1}{(x-1)^3}$$

• en  $x=2$ :  $\frac{14}{2} = 1 - 1 + b + c + 1 \quad \underline{b+c=6}$

• en  $x=-1$ :  $\frac{5}{8} = 1 + 2 - \frac{1}{2} + \frac{c}{4} - \frac{1}{8} \Rightarrow \frac{5}{8} + \frac{1}{8} - 3 = -\frac{b}{2} + \frac{c}{4} \Rightarrow -\frac{9}{4} = -\frac{b}{2} + \frac{c}{4}$

$$\Rightarrow \begin{cases} -2b + c = -9 \\ b + c = 6 \end{cases} \quad \begin{array}{l} \text{L}_2 - \text{L}_1: 3b = 6 + 9 = 15 \\ b = \frac{15}{3} = 5 \\ \underline{c = 1} \end{array}$$

$$\frac{x^4 - 2x + 2}{x(x-1)^3} = 1 - \frac{2}{x} + \frac{5}{x-1} + \frac{1}{(x-1)^2} + \frac{1}{(x-1)^3}$$

$$\textcircled{4} \quad x^4 - 16 = (x^2)^2 - 4^2 = (x^2 - 4)(x^2 + 4) = (x - 2)(x + 2)(x^2 + 4)$$

$$\frac{1}{x^4 - 16} = F(x) + \frac{a}{x - 2} + \frac{b}{x + 2} + \frac{cx + d}{x^2 + 4}$$

•  $F(x) = 0$  car le degré du numérateur est  $<$  degré du dénominateur

$$\bullet \quad a = \frac{1}{(x + 2)(x^2 + 4)} \Big|_{x=2} = \frac{1}{4 \times 8} = \frac{1}{32} \quad \bullet \quad b = \frac{1}{(x - 2)(x^2 + 4)} \Big|_{x=-2} = -\frac{1}{32}$$

• On multiplie par  $x^2 + 4$ :  $\frac{1}{(x - 2)(x + 2)} = \frac{1}{x^2 - 4} = \frac{a(x^2 + 4)}{x - 2} + \frac{b(x^2 + 4)}{x + 2} + cx + d$

valeur en  $i$ :  $\frac{1}{(2i)^2 - 4} = 0 + 0 + c \cdot 2i + d \Rightarrow -\frac{1}{8} = 2ci + d$

$$\Rightarrow \underline{c = 0} \text{ et } \underline{d = -\frac{1}{8}}$$

$$\frac{1}{x^4 - 16} = \frac{1}{32} \frac{1}{x - 2} - \frac{1}{32} \frac{1}{x + 2} - \frac{1/8}{x^2 + 4}$$

$$(6) f(x) = \frac{x^4 + 2x}{x^2 - 3x + 2} = \frac{x^4 + 2x}{(x-2)(x-1)} = E(x) + \frac{a}{x-2} + \frac{b}{x-1}$$

$$\begin{array}{r} x^4 + 2x \quad | \quad x^2 - 3x + 2 \\ -(x^4 - 3x^3 + 2x^2) \quad | \quad x^2 + 3x + 7 \\ \hline 3x^3 - 2x^2 + 2x \quad | \\ -(3x^3 - 9x^2 + 6x) \quad | \\ \hline 7x^2 - 4x \quad | \end{array}$$

$E(x)$

$$a = \left. \frac{x^4 + 2x}{x-1} \right|_{x=2} = \frac{20}{1} = 20$$

$$b = \left. \frac{x^4 + 2x}{x-2} \right|_{x=1} = \frac{3}{-1} = -3$$

$$\frac{x^4 + 2x}{x^2 - 3x + 2} = x^2 + 3x + 7 + \frac{20}{x-2} - \frac{3}{x-1}$$

$$(7) \frac{x^4 + 2x}{(x+1)(x-2)^2} = E(x) = \frac{a}{x+1} + \left( \frac{b}{x-2} + \frac{c}{(x-2)^2} \right) \dots$$

①  $F(x) = \frac{x^4 + 2x + 2}{x^3 + x^2 - 2x}$       On facteur:  $x^3 + x^2 - 2x = x(x^2 + x - 2)$

$\Delta = 9$       2 racis:  $\frac{-1 \pm 3}{2} = \begin{matrix} 1 \\ -2 \end{matrix} \Rightarrow x^2 + x - 2 = (x-1)(x+2)$

$F(x) = \frac{x^4 + 2x + 2}{x(x-1)(x+2)} = E(x) + \frac{a}{x} + \frac{b}{x-1} + \frac{c}{x+2}$

$$\begin{array}{r|l} x^4 + 2x + 2 & x^3 + x^2 - 2x \\ -(x^4 + x^3 - 2x^2) & \textcircled{x-1} \\ \hline -x^3 + 2x^2 + 2x + 2 & E(x) \\ \cdot & \\ \cdot & \end{array}$$

$a = \frac{x^4 + 2x + 2}{(x-1)(x+2)} \Big|_{x=0} = \frac{2}{-2} = -1$        $a = -1$

$b = \frac{x^4 + 2x + 2}{x(x+2)} \Big|_{x=1} = \frac{5}{3}$

$c = \frac{x^4 + 2x + 2}{x(x-1)} \Big|_{x=-2} = \frac{16 - 4 + 2}{6} = \frac{14}{6} = \frac{7}{3}$

$$\frac{x^4 + 2x + 2}{x^3 + x^2 - 2x} = x - 1 - \frac{1}{x} + \frac{5/3}{x-1} + \frac{7/3}{x+2}$$

$$\textcircled{2} \quad F(x) = \frac{x^4 - 2x + 2}{x^4 - 3x^3 + 3x^2 - x} \quad x^4 - 3x^3 + 3x^2 - x = x(x^3 - 3x^2 + 3x - 1) = x(x-1)^3$$

$$F(x) = E(x) + \frac{a}{x} + \left( \frac{b}{x-1} + \frac{c}{(x-1)^2} + \frac{d}{(x-1)^3} \right)$$

$$x^4 - 2x + 2 \mid x^4 - 3x^3 + 3x^2 - x \quad a = \frac{x^4 - 2x + 2}{(x-1)^3} \Big|_{x=0} = -2 \quad \underline{a = -2}$$

$\therefore$   $\textcircled{1}$   
 $E(x)$

• An Multipl. mit  $(x-1)^3$ :  $\frac{x^4 - 2x + 2}{x} = 1 \cdot (x-1)^3 - \frac{2}{x}(x-1)^3 + b(x-1)^2 + c(x-1) + d$

Wähle  $x=1$ :  $1 = 0 + 0 + 0 + 0 + d \quad \underline{d = 1}$

$$\frac{x^4 - 2x + 2}{x(x-1)^3} = 1 - \frac{2}{x} + \frac{b}{x-1} + \frac{c}{(x-1)^2} + \frac{1}{(x-1)^3}$$

• an  $x=2$ :  $\frac{14}{2} = 1 - 1 + b + c + 1 \quad \underline{b+c = 6}$

• an  $x=-1$ :  $\frac{5}{8} = 1 + 2 - \frac{b}{2} + \frac{c}{4} - \frac{1}{8} \Rightarrow \frac{5}{8} + \frac{1}{8} - 3 = -\frac{b}{2} + \frac{c}{4} \Rightarrow -\frac{9}{4} = -\frac{b}{2} + \frac{c}{4}$

$$\Rightarrow \begin{cases} -2b + c = -9 \\ b + c = 6 \end{cases} \quad |2-4|: 3b = 6 + 9 = 15 \quad \underline{b = 5} \Rightarrow c = 6 - 5 = 1 \quad \underline{c = 1}$$

$$\frac{x^4 - 2x + 2}{x(x-1)^3} = 1 - \frac{2}{x} + \frac{5}{x-1} + \frac{1}{(x-1)^2} + \frac{1}{(x-1)^3}$$

$$\textcircled{4} \quad \frac{1}{x^4-16} = \frac{1}{(x^2-4)(x^2+4)} = \frac{1}{(x-2)(x+2)(x^2+4)} = E(x) + \frac{a}{x-2} + \frac{b}{x+2} + \frac{cx+d}{x^2+4}$$

$E(x) = 0$  car le degré du numérateur est plus petit que le degré du dénominateur

$$\bullet \quad a = \frac{1}{(x+2)(x^2+4)} \Big|_{x=2} = \frac{1}{32} \quad \bullet \quad b = \frac{1}{(x-2)(x^2+4)} \Big|_{x=-2} = -\frac{1}{32}$$

$$\bullet \quad \text{On multiplie par } x^2+4 : \frac{1}{(x-2)(x+2)} = \frac{1}{x^2-4} = \frac{\frac{1}{32}(x^2+4)}{x-2} - \frac{\frac{1}{32}(x^2+4)}{x+2} + cx + d$$

valeur en  $i$  : 
$$\frac{1}{-8} = 0 + 0 + c \cdot 2^0 + d$$

$$\Rightarrow \underline{c=0} \text{ et } \underline{d = -\frac{1}{8}}$$

$$\frac{1}{x^4-16} = \frac{1/32}{x-2} - \frac{1/32}{x+2} - \frac{1/8}{x^2+4}$$

$$\textcircled{6} \quad \frac{x^4 + 2x}{x^2 - 3x + 2} = \frac{x^4 + 2x}{(x-1)(x-2)} = E(x) + \frac{a}{x-1} + \frac{b}{x-2}$$

$$\begin{array}{r} x^4 + 2x \\ -(x^2 - 3x + 2) \\ \hline 3x^3 - 2x^2 + 2x \\ -(3x^3 - 9x^2 + 6x) \\ \hline 7x^2 - 4x \end{array} \quad \begin{array}{l} x^2 - 3x + 2 \\ \hline x^2 + 3x + 7 \\ \hline \end{array}$$

$E(x)$

$$a = \left. \frac{x^4 + 2x}{x-2} \right|_{x=1} = \frac{3}{-1} = -3$$

$$b = \left. \frac{x^4 + 2x}{x-1} \right|_{x=2} = \frac{20}{1} = 20$$

$$\frac{x^4 + 2x}{x^2 - 3x + 2} = x^2 + 3x + 7 - \frac{3}{x-1} + \frac{20}{x-2}$$