

(16) (1) $z^4 - z^2 - 2 = 0$ du pose $Z = z^2$ (1) $Z^2 - Z - 2 = 0$

$\Delta = 1 + 8 = 9$

$Z = \frac{1 \pm 3}{2} = \begin{cases} 2 \\ -1 \end{cases}$

les quatre solutions sont: $\boxed{\pm \sqrt{2}e^{i\pi/4} \text{ et } \pm i}$

(2) $z^6 = -8 = 8e^{i\pi}$ il y a 6 sols: $z_k = 8^{1/6} e^{i(\frac{\pi}{6} + \frac{2k\pi}{6})} = 2^{2/3} e^{i(\frac{\pi+2k\pi}{6})} = \sqrt{2} e^{i(\frac{\pi+2k\pi}{6})}$

pour $k=0$: $z_0 = \sqrt{2} e^{i\pi/6}$

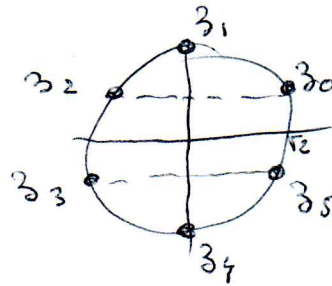
$k=1$: $z_1 = \sqrt{2} e^{i3\pi/6} = \sqrt{2} e^{i\pi/2} = \sqrt{2} i$

$k=2$: $z_2 = \sqrt{2} e^{i5\pi/6}$

$k=3$: $z_3 = \sqrt{2} e^{i7\pi/6}$

$k=4$: $z_4 = \sqrt{2} e^{i9\pi/6} = \sqrt{2} e^{i3\pi/2} = -\sqrt{2} i$

$k=5$: $z_5 = \sqrt{2} e^{i11\pi/6} = \sqrt{2} e^{-i\pi/6}$



(3) $z^4 = -\frac{1}{2} + i\frac{\sqrt{3}}{2} = e^{i2\pi/3}$

il y a 4 sols: $z_k = e^{i(\frac{2\pi}{4} + \frac{2k\pi}{4})}$ ou $k=0, 1, 2$ ou 3

$= e^{i(\frac{\pi}{2} + \frac{k\pi}{2})} = e^{i(\frac{\pi+3k\pi}{2})}$

pour $k=0$:

$z_0 = e^{i\pi/2}$

pour $k=1$: $z_1 = e^{i3\pi/2} = e^{-i\pi/2}$

$k=2$

$z_2 = e^{i5\pi/2}$

$k=3$: $z_3 = e^{i7\pi/2} = e^{-i\pi/2}$

