

INSA STRASBOURG 2022

$$\textcircled{1} P(x) = (x - z_1)(x - z_2) \quad z_1 = \frac{\sqrt{3}}{2} + \frac{i}{2} \quad z_2 = \frac{\sqrt{3}}{2} - \frac{i}{2}$$

$$P(x) = x^2 - z_1 x - z_2 x + z_1 z_2 = x^2 - (z_1 + z_2)x + z_1 z_2$$

$$z_1 + z_2 = \frac{\sqrt{3}}{2} + \frac{i}{2} + \frac{\sqrt{3}}{2} - \frac{i}{2} = \sqrt{3}$$

$$z_1 z_2 = \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)\left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right) = \frac{3}{4} - \frac{i\sqrt{3}}{4} + \frac{i\sqrt{3}}{4} + \frac{1}{4} = 1$$

$$P(x) = x^2 - \sqrt{3}x + 1$$

$$\textcircled{2} |z_1| = \sqrt{\frac{3}{4} + \frac{1}{4}} = \sqrt{1} = 1 \quad z_1 = 1\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right)$$

$$|z_1| = 1 \quad \arg(z_1) = \frac{\pi}{6}$$

$$|z_2| = 1$$

$$\arg(z_2) = -\frac{\pi}{6}$$

$$z_1 = 1e^{i\pi/6}$$

$$z_2 = 1e^{-i\pi/6}$$



(3)  $\|\vec{0M_1}\| = |z_1| = 1$  donc  $M_1, e^{i\pi/6}$  sont sur le cercle  
 $\|\vec{0M_2}\| = |z_2| = 1$  de rayon 1

(4)  $z_1 + z_2 = \sqrt{3}$

$$z_1^2 + z_2^2 = (e^{i\pi/6})^2 + (e^{-i\pi/6})^2$$



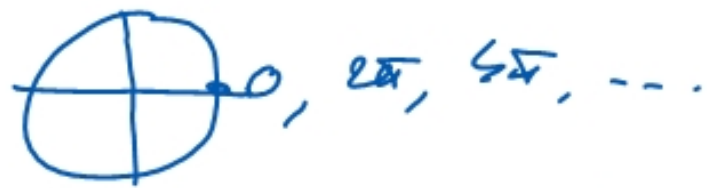
$$= e^{i\pi/3} + e^{-i\pi/3}$$

$$= \frac{1}{2} + i\frac{\sqrt{3}}{2} + \frac{1}{2} - i\frac{\sqrt{3}}{2} = 1$$

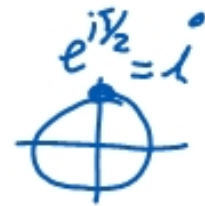
$$z_1^3 + z_2^3 = (e^{i\pi/6})^3 + (e^{-i\pi/6})^3 = e^{i\pi/2} + e^{-i\pi/2}$$
$$= i + (-i) = 0$$



$$\textcircled{1} e^{2i\pi} = e^{i0} = \underline{1}$$



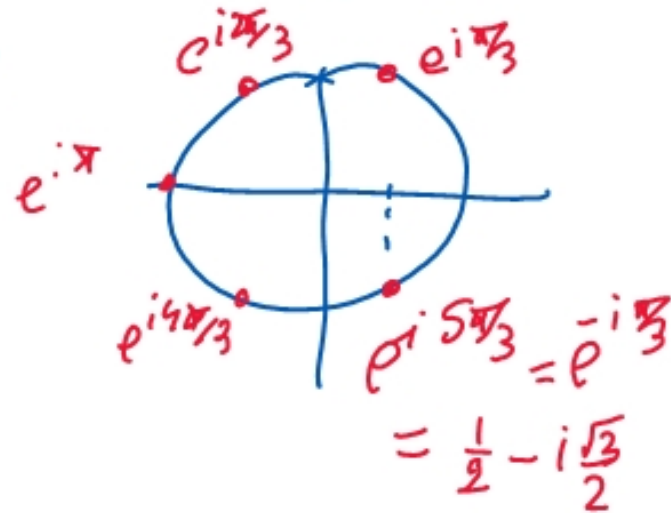
$$e^{i3\pi} = e^{i(3\pi - 2\pi)} = e^{i\pi} = \underline{-1}$$



$$2e^{i\pi/3} \times 3e^{i\pi/6} = 6e^{i(\frac{\pi}{3} + \frac{\pi}{6})} = 6e^{i\frac{3\pi}{6}} = 6e^{i\frac{\pi}{2}} = \underline{6i}$$

$$2e^{i\frac{2\pi}{3}} \cdot 5e^{i\pi} = 10e^{i(\frac{2\pi}{3} + \pi)} = 10e^{i\frac{2\pi + 3\pi}{3}} = 10e^{i\frac{5\pi}{3}}$$

$$= 10e^{i(\frac{5\pi}{3} - 2\pi)} = 10e^{-i\frac{\pi}{3}}$$



$$= 10\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) = \underline{5 - i5\sqrt{3}}$$

$$\textcircled{2}. 3 - 3i = z \quad |z| = \sqrt{9+9} = \sqrt{2 \cdot 9} = 3\sqrt{2}$$

$$3 - 3i = 3\sqrt{2} \left( \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i \right) = 3\sqrt{2} \left( \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i \right)$$



$$= 3\sqrt{2} e^{-i\pi/4}$$

$$\cdot -2e^{i\pi/2} = e^{i\pi} \cdot 2e^{i\pi/2}$$

$$\text{car } -1 = e^{i\pi}$$

$$= 2e^{i\frac{3\pi}{2}} = 2e^{i(\frac{3\pi}{2} - 2\pi)} = 2e^{-i\pi/2}$$

Module 2

argument  $-\pi/2$

$$-2e^{-i\pi/2} = e^{-i\pi} \cdot 2e^{i\pi/2}$$

$$\text{car } -1 = e^{-i\pi}$$

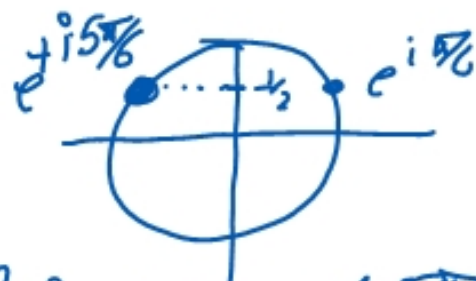
$$= 2e^{-i\pi/2}$$

On préfère que l'argument soit entre  $-\pi$  et  $\pi$

$$(2) \quad -\sqrt{3} + i = z \quad |z| = \sqrt{3+1} = \sqrt{4} = 2$$

$$z = -\sqrt{3} + i = 2 \left( -\frac{\sqrt{3}}{2} + \frac{i}{2} \right)$$

$$= \underline{2 e^{i5\pi/6}}$$



Module 2 argument 5π/6

$$-2e^{i2\pi/3} = e^{-i\pi} 2e^{i2\pi/3} = 2e^{i(\frac{2\pi}{3}-\pi)} = 2e^{i\frac{2\pi-3\pi}{3}} = \underline{2e^{-i\pi/3}}$$

Module 2

argument -π/3

$$(3) \quad -2e^{i\pi/2} \times 2e^{i3\pi/2} = e^{-i\pi} 4e^{i(\frac{\pi}{2} + \frac{3\pi}{2})} = 4e^{i(\pi-\pi)} = \underline{4e^{i0}}$$

$$(5) \quad 2e^{i3\pi/2} = 2 \cdot (-i) = \underline{-2i}$$



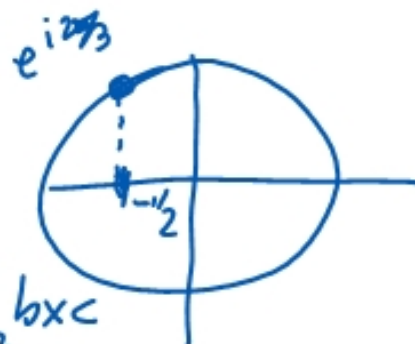
$$e^{2-i\pi/3} = e^2 e^{-i\pi/3} = e^2 \left( \frac{1}{2} - i\frac{\sqrt{3}}{2} \right) = \underline{\frac{e^2}{2} - i\frac{\sqrt{3}e^2}{2}}$$



$$\textcircled{6} \quad A = \left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^{2019}$$

$$= \left(e^{i\frac{2\pi}{3}}\right)^{2019}$$

$$= e^{i\frac{2\pi}{3} \times 2019} = e^{i\frac{2\pi \times 2019}{3}} = e^{i\frac{4038\pi}{3}}$$



appel:  $(a^b)^c = a^{b \times c}$

$$\begin{array}{r} 4038 \\ \underline{-3} \\ 10 \\ \underline{-9} \\ 13 \\ \underline{18} \\ 0 \end{array} \quad \begin{array}{r} 3 \\ \hline 1346 \end{array}$$

$$A = e^{i\pi 1346} = e^{i0} = 1$$

$$\begin{array}{r} 4040 \\ \underline{10} \\ 14 \\ \underline{20} \\ 2 \end{array} \quad \begin{array}{r} 3 \\ \hline 1346 \end{array}$$

$$4040 = 3 \times 1346 + 2 \Rightarrow \frac{4040}{3} = 1346 + \frac{2}{3}$$

$$B = \left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^{2020} = e^{i\pi \frac{4040}{3}} = e^{i(1346\pi + \frac{2\pi}{3})}$$

$$B = e^{i\frac{2\pi}{3}}$$

$$\textcircled{1} \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{\sqrt{3} \pm i}{2} \Rightarrow b = -\sqrt{3} \quad a = 1$$

et  $\sqrt{\Delta} = i$ , donc  $\Delta = -1$

$$b^2 - 4ac = -1 \Rightarrow 3 - 4c = -1 \Rightarrow c = \frac{-4}{-4} = 1$$

$$P(x) = X^2 - \sqrt{3}X + 1$$

$$\textcircled{2} z_1 = \frac{\sqrt{3} + i}{2} \quad |z_1| = \sqrt{\frac{3}{4} + \frac{1}{4}} = \sqrt{\frac{4}{4}} = \sqrt{1} = 1 \quad \text{Modulus } \underline{1}$$

$$z_1 = \frac{\sqrt{3}}{2} + i \frac{1}{2} = e^{i\pi/6} \quad (\text{cercle trigonométrique}) \quad \text{argument } \underline{\frac{\pi}{6}}$$

$$z_1 = \cos \theta + i \sin \theta = e^{i\theta}$$

$$z_2 = \frac{\sqrt{3} - i}{2} \quad |z_2| = 1$$

Modulus 1

$$z_2 = \frac{\sqrt{3}}{2} - i \frac{1}{2} = e^{-i\pi/6}$$

argument  $-\frac{\pi}{6}$

$\textcircled{3}$  modulus 1, donc sur le cercle de centre 0 de rayon 1

$$\textcircled{4} \quad z_1 + z_2 = \frac{\sqrt{3}+i}{2} + \frac{\sqrt{3}-i}{2} = \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} = \underline{\underline{\sqrt{3}}}$$

$$z_1^2 + z_2^2 = (e^{i\pi/6})^2 + (e^{-i\pi/6})^2 = e^{i\pi/3} + e^{-i\pi/3}$$

$$= \frac{1}{2} + i\frac{\sqrt{3}}{2} + \frac{1}{2} - i\frac{\sqrt{3}}{2} = 1$$

$$z_1^3 + z_2^3 = (e^{i\pi/6})^3 + (e^{-i\pi/6})^3 = e^{i\pi/2} + e^{-i\pi/2} = i + (-i) = \underline{\underline{0}}$$

$$\textcircled{1} \quad e^{i2\pi} = e^0 = 1$$

$$e^{i3\pi} = e^{i(3\pi - 2\pi)} = e^{i\pi} = -1$$

$$2e^{i\pi/3} \times 3e^{i\pi/6} = 6e^{i(\frac{\pi}{3} + \frac{\pi}{6})} = 6e^{i\frac{2\pi + \pi}{6}} = 6e^{i\frac{3\pi}{6}} = 6e^{i\frac{\pi}{2}} = \underline{\underline{6i}}$$

$$2e^{i2\pi/3} \times 5e^{i\pi} = 10e^{i(\frac{2\pi}{3} + \pi)} = 10e^{i\frac{5\pi}{3}} = 10e^{-i\pi/3} = 10\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) = \underline{\underline{5 - 5\sqrt{3}i}}$$



$$\textcircled{2} z_1 = 3-3i \quad |z_1| = \sqrt{9+9} = \sqrt{2 \times 9} = 3\sqrt{2} \quad \text{module } 3\sqrt{2}$$

$$z_1 = 3-3i = 3\sqrt{2} \left( \frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}} \right) = 3\sqrt{2} \left( \frac{\sqrt{2}}{2} - i \frac{\sqrt{2}}{2} \right) = 3\sqrt{2} e^{-i\pi/4}$$

argument:  $- \pi/4$

$$z_2 = -2 e^{i\pi/2}$$

module 2

$$z_2 = e^{-i\pi} \cdot 2 e^{i\pi/2} = 2 e^{i(\pi/2 - \pi)} = 2 e^{-i\pi/2} \quad \text{argument } -\frac{\pi}{2}$$

$$z_3 = -\sqrt{3} + i \quad |z_3| = \sqrt{3+1} = \sqrt{4} = 2$$

$$z_3 = 2 \left( -\frac{\sqrt{3}}{2} + \frac{1}{2}i \right) = 2 e^{i5\pi/6}$$

$$z_4 = -2 e^{i2\pi/3} = e^{-i\pi} \cdot 2 e^{i2\pi/3} = 2 e^{i(\frac{2\pi}{3} - \pi)} = 2 e^{-i\pi/3}$$

module 2

argument  $-\frac{\pi}{3}$

$$\begin{aligned} \textcircled{3} \quad -2e^{i\pi/2} \times 2e^{i3\pi/2} &= e^{-i\pi} \times 2e^{i\pi/2} \times 2e^{i3\pi/2} \\ &= 4e^{i(-\pi + \pi/2 + 3\pi/2)} = \underline{4e^{i\pi}} = -4 \end{aligned}$$

$$\textcircled{5} \quad 2e^{i3\pi/2} = 2(-i) = -2i$$

$$e^{2-i\pi/3} = e^2 \cdot e^{-i\pi/3} = e^2 \left[ \frac{1}{2} - i\frac{\sqrt{3}}{2} \right] = \underline{\frac{e^2}{2} - i\frac{\sqrt{3}e^2}{2}}$$

$$\textcircled{6} A = \left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^{2019} = e^{i\frac{2\pi}{3} \cdot 2019}$$

$$\begin{array}{r} \overline{2019} \\ -18 \\ \hline 21 \\ -21 \\ \hline 9 \\ -9 \\ \hline 0 \end{array} \Bigg| \begin{array}{r} 3 \\ \hline 673 \end{array}$$

$$2019 = 3 \times 673$$

$$A = e^{i2\pi \cdot 673} = e^{i0} = 1$$

$$B = \left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^{2020} = e^{i\frac{2\pi}{3} \cdot 2020}$$

$$\begin{array}{r} 2020 \\ \vdots \\ \vdots \\ 1 \end{array} \Bigg| \begin{array}{r} 3 \\ \hline 673 \end{array}$$

$$2020 = 3 \times 673 + 1$$

$$\frac{2020}{3} = 673 + \frac{1}{3}$$

$$\begin{aligned} B &= e^{i2\pi \left[673 + \frac{1}{3}\right]} \\ &= e^{i\left(2\pi \cdot 673 + \frac{2\pi}{3}\right)} = e^{i\frac{2\pi}{3}} \\ &= -\frac{1}{2} + i\frac{\sqrt{3}}{2} \end{aligned}$$