

~~correction~~

$$(15a) \quad z^2 - 5z + 9 = 0$$

$$\Delta = 25 - 36 = -11 = (i\sqrt{11})^2$$

$$2 \text{ racines } \frac{5 \pm i\sqrt{11}}{2}$$

$$\text{factorisation: } z^2 - 5z + 9 = \left(z - \frac{5+i\sqrt{11}}{2}\right) \left(z - \frac{5-i\sqrt{11}}{2}\right)$$

$$(15b) \quad 2z^2 - 3z + 3 - i = 0$$

$$\Delta = 9 - 4 \times 2 \times (3-i) = 9 - 24 + 8i = -15 + 8i$$

On cherche les racines de Δ c'est les $z = a+ib$ tels que $z^2 = \Delta$

$$-15 + 8i = (a+ib)^2 = a^2 - b^2 + i2ab$$

$$\begin{cases} a^2 - b^2 = -15 \\ 2ab = 8 \end{cases}$$

$$a^2 + b^2 = \sqrt{15^2 + 8^2} = \sqrt{289} = 17$$

$$L_3 + L_1: 2a^2 = 2 \Rightarrow a^2 = 1 \Rightarrow a = \pm 1$$

$$L_3 - L_1: 2b^2 = 17 + 15 = 32 \Rightarrow b^2 = 16$$

$$\Rightarrow b = \pm 4$$

$$L_2: a \text{ et } b \text{ ont le m\^eme signe}$$

racines de Δ

$$\sqrt{\Delta} = \pm (1+4i)$$

$$\text{2 racines: } \frac{3 \pm (1+4i)}{4} = \begin{cases} \frac{4+4i}{4} = 1+i \\ \frac{2-4i}{4} = \frac{1}{2}-i \end{cases}$$

$$\equiv 2z^2 - 3z + 3 - i = 2(z - 1 - i)(z - \frac{1}{2} + i)$$

(16) $z^4 - z^2 - 2 = 0$ on pose $Z = z^2$

$$Z^2 - Z - 2 = 0 \quad \Delta = 1 - 4 \cdot (-2) = 1 + 8 = 9$$

$$Z = \frac{1 \pm 3}{2} = \begin{cases} \frac{4}{2} = 2 = z^2 \\ -\frac{2}{2} = -1 = z^2 \end{cases} \Rightarrow \underline{z = \pm \sqrt{2}}$$

$$\underline{z = \pm i}$$

$$z^4 - z^2 - 2 = (z - \sqrt{2})(z + \sqrt{2})(z - i)(z + i)$$

$$(6) \quad -8 = 8e^{i\pi}$$

les racines 6^{ième} de $8e^{i\pi}$ sont les $z_k = 8^{\frac{1}{6}} e^{i(\frac{\pi}{6} + \frac{2k\pi}{6})}$

$$z_0 = \sqrt{2} e^{i\frac{\pi}{6}} \quad \text{ou } k=0, 1, \dots, 5$$

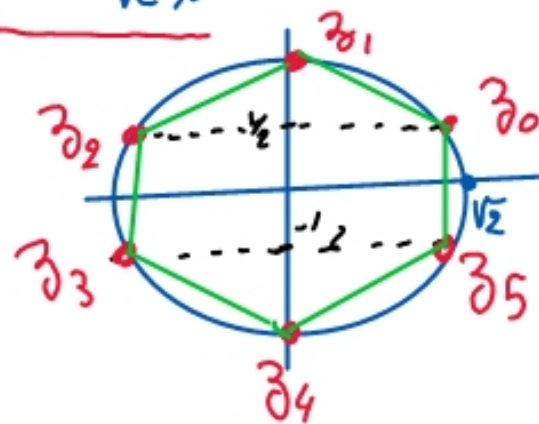
$$z_1 = \sqrt{2} e^{i(\frac{\pi}{6} + \frac{2\pi}{6})} = \sqrt{2} e^{i\frac{3\pi}{6}} = \sqrt{2} e^{i\frac{\pi}{2}} = \sqrt{2}i \quad 8^{\frac{1}{6}} = (2^3)^{\frac{1}{6}} = 2^{\frac{3 \times \frac{1}{6}}{1}} = \sqrt{2}$$

$$z_2 = \sqrt{2} e^{i(\frac{\pi}{6} + \frac{4\pi}{6})} = \sqrt{2} e^{i\frac{5\pi}{6}}$$

$$z_3 = \sqrt{2} e^{i(\frac{\pi}{6} + \frac{6\pi}{6})} = \sqrt{2} e^{i\frac{7\pi}{6}} = \sqrt{2} e^{-i\frac{5\pi}{6}}$$

$$z_4 = \sqrt{2} e^{i(\frac{\pi}{6} + \frac{8\pi}{6})} = \sqrt{2} e^{i\frac{9\pi}{6}} = \sqrt{2} e^{i\frac{3\pi}{2}} = \sqrt{2} e^{-i\frac{\pi}{2}} = -\sqrt{2}i$$

$$z_5 = \sqrt{2} e^{i(\frac{\pi}{6} + \frac{10\pi}{6})} = \sqrt{2} e^{i\frac{11\pi}{6}} = \sqrt{2} e^{-i\frac{\pi}{6}}$$



$$⑥ \quad z^4 = -\frac{1}{2} + i\frac{\sqrt{3}}{2} = e^{i\frac{2\pi}{3}}$$

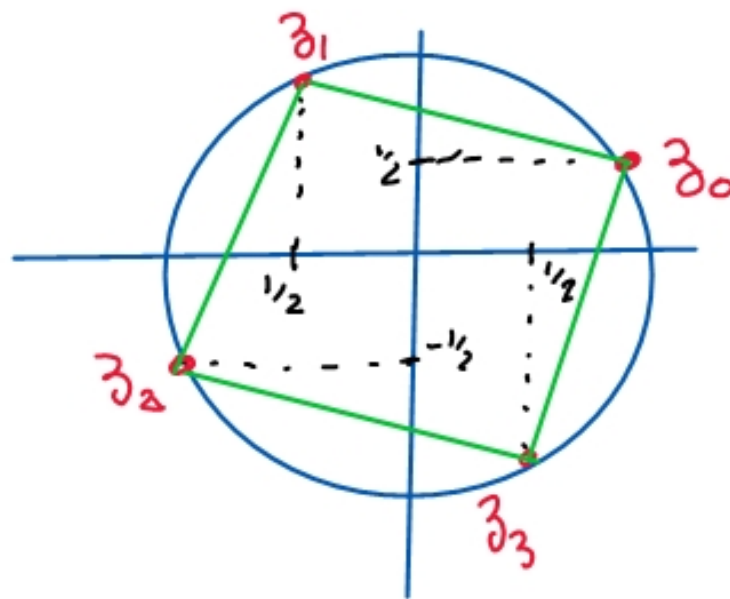
Les racines 4^{ième} de $e^{i\frac{2\pi}{3}}$ sont les $e^{i\left(\frac{2\pi}{12} + \frac{2k\pi}{4}\right)} = e^{i\left(\frac{\pi}{6} + \frac{k\pi}{2}\right)} = z_k$
 où $k = 0, 1, 2$ ou 3

$$z_0 = e^{i\frac{\pi}{6}}$$

$$z_1 = e^{i\left(\frac{\pi}{6} + \frac{\pi}{2}\right)} = e^{i\frac{4\pi}{6}} = e^{i\frac{2\pi}{3}}$$

$$z_2 = e^{i\left(\frac{\pi}{6} + \pi\right)} = e^{i\frac{7\pi}{6}} = e^{-i\frac{5\pi}{6}}$$

$$z_3 = e^{i\left(\frac{\pi}{6} + \frac{3\pi}{2}\right)} = e^{i\frac{10\pi}{6}} = e^{i\frac{5\pi}{3}} = e^{-i\frac{\pi}{3}}$$



$$(17) \quad z^2 + (2+2i)z + 3-2i = 0$$

$$\Delta = (2+2i)^2 - 4(3-2i) = 4+8i-4-12+8i \\ = -12+16i = (a+ib)^2$$

$$\begin{cases} a^2 - b^2 = -12 \\ 2ab = 16 \\ a^2 + b^2 = \sqrt{12^2 + 16^2} = \sqrt{400} = 20 \end{cases} \quad L_3 + L_1: 2a^2 = 8 \Rightarrow a^2 = 4 \Rightarrow a = \pm 2$$

$$L_3 - L_1: 2b^2 = 32 \Rightarrow b^2 = 16 \Rightarrow b = \pm 4$$

$$L_2: a \text{ et } b \text{ ont le m\^eme signe} \quad \text{donc } \sqrt{\Delta} = \pm (2+4i)$$

$$\text{deux racines: } \frac{-(2+2i) \pm (2+4i)}{2} = \begin{cases} \frac{2i}{2} = i \\ \frac{-4-6i}{2} = -2-3i \end{cases}$$

$$\text{factorisation: } z^2 + (2+2i)z + 3-2i = (z-i)(z+2+3i)$$

$$(15a) \quad z^2 - 5z + 9 = 0 \quad \Delta = 25 - 36 = -11 = (i\sqrt{11})^2$$

$$z = \frac{5 \pm i\sqrt{11}}{2}$$

Factorisation: $z^2 - 5z + 9 = \left(z - \frac{5+i\sqrt{11}}{2}\right) \left(z - \frac{5-i\sqrt{11}}{2}\right)$

$$(15b) \quad 2z^2 - 3z + 3 - i = 0$$

$$\Delta = 9 - 4 \times 2 \times (3 - i) = 9 - 24 + 8i = -15 + 8i$$

On cherche " $\sqrt{\Delta}$ ", c'est-à-dire $a + ib$ tel que $(a + ib)^2 = -15 + 8i$

$$a^2 - b^2 + i2ab = -15 + 8i$$

$$\left. \begin{array}{l} a^2 - b^2 = -15 \\ 2ab = 8 \end{array} \right\} \begin{array}{l} \text{module} \\ \text{de } \Delta \end{array} \quad a^2 + b^2 = \sqrt{(-15)^2 + 8^2} = \sqrt{289} = 17$$

$$L_3 + L_1: 2a^2 = 2 \Rightarrow a^2 = 1$$

$$\underline{a = \pm 1}$$

$$L_3 - L_1: 2b^2 = 32 \Rightarrow b^2 = 16$$

$$\underline{b = \pm 4}$$

L_2 : a et b ont le même signe, donc $\underline{\underline{\sqrt{\Delta} = \pm (1 + 4i)}}$

Les deux racines du polynôme $2z^2 - 3z + 3 - i$ sont :

$$\frac{3 \pm (1+4i)}{4} = \begin{cases} \frac{4+4i}{4} = 1+i \\ \frac{2-4i}{4} = \frac{1}{2} - i \end{cases}$$

Factorisation : $2z^2 - 3z + 3 - i = 2(z - 1 - i)(z - \frac{1}{2} + i)$

⑩ $z^4 - z^2 - 2 = 0 \Leftrightarrow Z^2 - Z - 2 = 0$ où $Z = z^2$

$\Delta = 1 + 8 = 9$ $Z = \frac{1 \pm 3}{2} = \begin{cases} 2 = z^2 \Rightarrow z = \pm\sqrt{2} \\ -1 = z^2 \Rightarrow z = \pm i \end{cases}$

$\mathcal{S} = \{-\sqrt{2}; \sqrt{2}; i; -i\}$

Factorisation : $z^4 - z^2 - 2 = (z + \sqrt{2})(z - \sqrt{2})(z - i)(z + i)$

$$(16) z^6 = -8$$

$$-8 = 8 \times (-1) = 8 e^{i\pi}$$

les racines 6^{ième} de $8 e^{i\pi}$ sont les $z_k = 8^{1/6} e^{i(\frac{\pi}{6} + \frac{2k\pi}{6})}$

où $k = 0, 1, 2, \dots, 5$

$$z_0 = \underline{\sqrt{2} e^{i\frac{\pi}{6}}}$$

$$z_1 = \sqrt{2} e^{i(\frac{\pi}{6} + \frac{2\pi}{6})} = \sqrt{2} e^{i\frac{3\pi}{6}} = \underline{\sqrt{2} e^{i\frac{\pi}{2}}} = \underline{\sqrt{2} i}$$

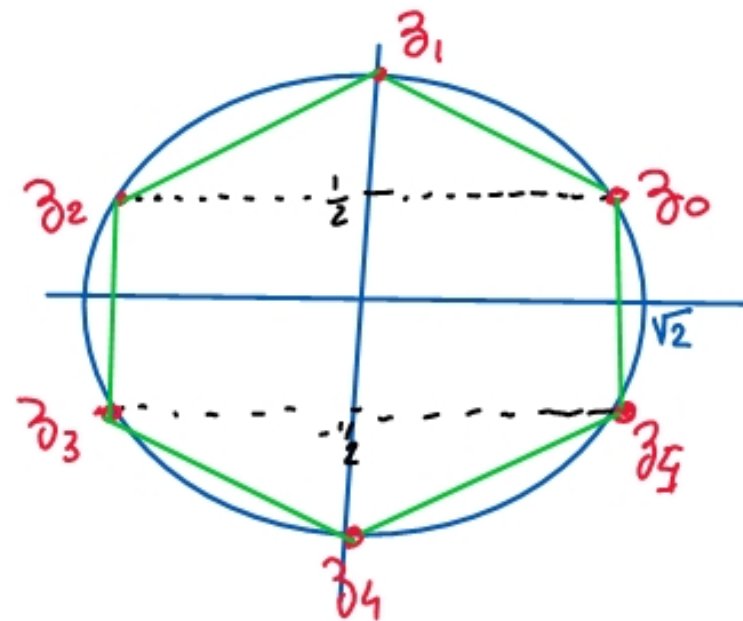
$$z_2 = \sqrt{2} e^{i(\frac{\pi}{6} + \frac{4\pi}{6})} = \underline{\sqrt{2} e^{i\frac{5\pi}{6}}}$$

$$z_3 = \sqrt{2} e^{i(\frac{\pi}{6} + \frac{6\pi}{6})} = \sqrt{2} e^{i\frac{7\pi}{6}} = \underline{\sqrt{2} e^{-i\frac{5\pi}{6}}}$$

$$z_4 = \sqrt{2} e^{i(\frac{\pi}{6} + \frac{8\pi}{6})} = \sqrt{2} e^{i\frac{9\pi}{6}} = \sqrt{2} e^{i\frac{3\pi}{2}} = \underline{-\sqrt{2} i}$$

$$z_5 = \sqrt{2} e^{i(\frac{\pi}{6} + \frac{10\pi}{6})} = \sqrt{2} e^{i\frac{11\pi}{6}} = \underline{\sqrt{2} e^{-i\frac{\pi}{6}}}$$

$$8^{1/6} = (2^3)^{1/6} = 2^{3 \times \frac{1}{6}} = 2^{\frac{1}{2}} = \sqrt{2}$$



$$(16) \quad z^4 = -\frac{1}{2} + i\frac{\sqrt{3}}{2} = e^{i\frac{2\pi}{3}}$$

Les racines 4^{ième} de $e^{i\frac{2\pi}{3}}$ sont les

$$z_k = e^{i\left(\frac{2\pi}{12} + 2\frac{k\pi}{4}\right)} = e^{i\left(\frac{\pi}{6} + \frac{k\pi}{2}\right)}$$

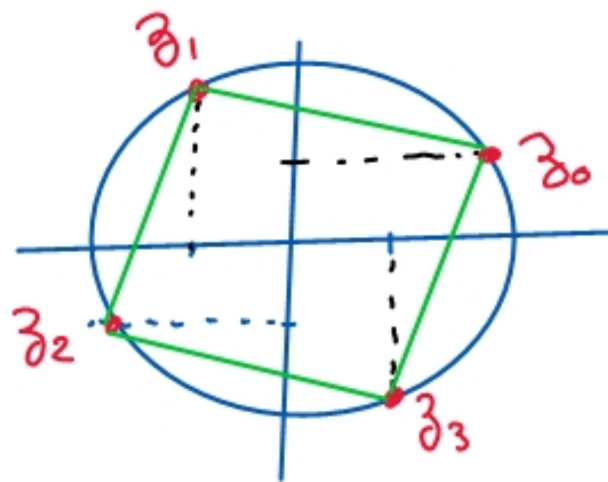
où $k=0, 1, 2$ ou 3

$$z_0 = \underline{e^{i\pi/6}}$$

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$$z_2 = e^{i\left(\frac{\pi}{6} + \pi\right)} = e^{i\frac{7\pi}{6}} = \underline{e^{-i\frac{5\pi}{6}}}$$

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$$(17) \quad z^2 + (2+2i)z + 3-2i = 0$$

$$\begin{aligned} \Delta &= (2+2i)^2 - 4(3-2i) = 4+8i-4-12+8i = -12+16i \\ &= (a+ib)^2 \\ &= a^2 - b^2 + i2ab \end{aligned}$$

$$|\Delta| \begin{cases} a^2 - b^2 = -12 \\ 2ab = 16 \\ a^2 + b^2 = \sqrt{12^2 + 16^2} = \sqrt{144 + 256} = \sqrt{400} \\ \qquad \qquad \qquad = 20 \end{cases}$$

$$\begin{array}{r} 16 \\ 16 \\ \hline 32 \\ 16 \\ \hline 256 \end{array} 3$$

$$\therefore \begin{cases} 2a^2 = 8 \Rightarrow a^2 = 4 \Rightarrow a = \pm 2 \end{cases}$$