

(20)  $A = \int \frac{\sin x}{2\sin^2 x - 3\cos x} dx$

$\sin(-x) = -\sin x$  et  $\cos(-x) = \cos x$

$\sin(\pi+x) = \sin x$      $\cos(\pi+x) = -\cos x$  *chgt en cos*

$\sin(\pi-x) = \sin x$      $\cos(\pi-x) = -\cos x$

*chgt en sin*

$\tan(\pi+x) = \tan x$

*chgt en tan*

$d(-x) = -dx$   
 $d(\pi+x) = dx$   
 $d(\pi-x) = -dx$

$\frac{\sin(-x)}{2\sin^2(-x) - 3\cos(-x)} \quad d(-x) = \frac{\cancel{\sin x}}{2\sin^2 x - 3\cos x} (\cancel{dx})$  *Yes*

*chgt  $t = \cos x$*

$A = \int \frac{\cancel{\sin x}}{2(1-t^2) - 3t (-\cancel{\sin x})} dt = \int \frac{1}{2t^2 + 3t - 2} dt$

$\Delta = 9 + 16 = 25$      $t = \frac{-3 \pm 5}{4} = \begin{cases} -2 \\ \frac{1}{2} \end{cases}$

$\frac{1}{(t+2)(2t-1)} = \frac{a}{t+2} + \frac{b}{2t-1}$

$a = \frac{1}{2t-1} \Big|_{t=-2} = -\frac{1}{5}$  et  $b = \frac{1}{t+2} \Big|_{t=\frac{1}{2}} = \frac{2}{5}$

$\frac{1}{2t^2 + 3t - 2}$   
 $\frac{1}{2(t+2)(t-\frac{1}{2})}$

$\frac{dt}{dx} = -\sin x$

$\Rightarrow dx = \frac{dt}{-\sin x}$

$A = -\frac{1}{5} \int \frac{1}{t+2} dt + \frac{2}{5} \int \frac{1}{2t-1} dt$

$A = -\frac{1}{5} \ln|t+2| + \frac{2}{5} \frac{1}{2} \ln|2t-1| + \text{cste}$

$$B = \int \frac{1}{1+2\cos x} dx$$

$$B = \int \frac{1}{1+2\frac{1-t^2}{1+t^2}} \frac{2}{1+t^2} dt$$

$$= 2 \int \frac{1}{1+t^2+2(1-t^2)} dt$$

$$= 2 \int \frac{1}{3-t^2} dt = 2 \int \frac{1}{(\sqrt{3}-t)(\sqrt{3}+t)} dt$$

$$\frac{1}{(\sqrt{3}-t)(\sqrt{3}+t)} = \frac{a}{\sqrt{3}-t} + \frac{b}{\sqrt{3}+t}$$

$$a = \frac{1}{\sqrt{3}+t} \Big|_{t=\sqrt{3}} = \frac{1}{2\sqrt{3}}$$

$$b = \frac{1}{\sqrt{3}-t} \Big|_{t=-\sqrt{3}} = -\frac{1}{2\sqrt{3}}$$

$$B = \frac{1}{\sqrt{3}} \int \frac{1}{\sqrt{3}-t} dt - \frac{1}{\sqrt{3}} \int \frac{1}{\sqrt{3}+t} dt = -\frac{1}{\sqrt{3}} \ln|\sqrt{3}-t| - \frac{1}{\sqrt{3}} \ln|\sqrt{3}+t|$$

$$B = -\frac{\sqrt{3}}{3} (\ln|\sqrt{3}-t| + \ln|\sqrt{3}+t|) = -\frac{\sqrt{3}}{3} \ln(\sqrt{3}-t)(\sqrt{3}+t) = -\frac{\sqrt{3}}{3} \ln(3-t^2)$$

$$B = -\frac{\sqrt{3}}{3} \ln(3 - \tan^2(\frac{x}{2})) + \text{const}$$

$$\frac{1}{1+2\cos(-x)} d(-x) = \frac{-1}{1+2\cos x} dx \quad \text{NON}$$

$$\frac{1}{1+2\cos(\pi-x)} d(\pi-x) = \frac{-1}{1-2\cos x} dx \quad \text{NON}$$

$$\frac{1}{1+2\cos(\pi+x)} d(\pi+x) = \frac{1}{1-2\cos x} dx \quad \text{NON}$$

SWIFF.

$$\text{left } t = \tan\left(\frac{x}{2}\right) \quad \frac{dt}{dx} = \left(\tan\left(\frac{x}{2}\right)\right)' \cdot \frac{1}{2}$$

↓

$$dx = \frac{2}{1+t^2} dt$$

$$dx \cos x = \frac{1-t^2}{1+t^2}$$

$$C = \int \frac{dx}{2 + \cos x}$$

pas d'invariants

chgt  $t = \tan \frac{x}{2}$

$$\frac{dt}{dx} = \left(1 + \tan^2 \frac{x}{2}\right) \cdot \frac{1}{2}$$

$$C = \int \frac{1}{2 + \frac{1-t^2}{1+t^2}} \cdot \frac{2}{1+t^2} dt = 2 \int \frac{1}{2(1+t^2) + 1-t^2} dt$$

$$\Downarrow$$
$$\boxed{\frac{dx}{1+t^2} = \frac{2}{1+t^2} dt}$$

$$C = 2 \int \frac{1}{t^2 + 3} dt$$

$$t^2 + 3 = 3 \left( \frac{t^2}{3} + 1 \right) = 3 \left( \frac{t^2}{\sqrt{3}^2} + 1 \right) = 3 \left( \left( \frac{t}{\sqrt{3}} \right)^2 + 1 \right)$$

chgt  $T = \frac{t}{\sqrt{3}}$      $t = \sqrt{3} T$      $dt = \sqrt{3} dT$

$$C = 2 \int \frac{1}{3 \left( \frac{t^2}{3} + 1 \right)} \sqrt{3} dT = \frac{2\sqrt{3}}{3} \operatorname{arctan}(T) + \text{cte} = \frac{2\sqrt{3}}{3} \operatorname{arctan} \left( \frac{t}{\sqrt{3}} \right) + \text{cte}$$

$$\underline{C = \frac{2\sqrt{3}}{3} \operatorname{arctan} \left( \frac{\tan(x/2)}{\sqrt{3}} \right) + \text{cte}}$$

$$D = \int \frac{\cos^3 x - \sin x \cos x}{\sin^3 x - 3 \sin x + 2} dx$$

$$A(-x) = \frac{\cos^3 x + \sin x \cos x}{-\sin^3 x + 3 \sin x + 2} (-dx) \quad \text{NON}$$

$$A(\pi - x) = \frac{-\cos^3 x + \sin x \cos x}{\sin^3 x - 3 \sin x + 2} (-dx) = \frac{\cos^3 x - \sin x \cos x}{3 \sin^3 x - 3 \sin x + 2} dx$$

On pose  $X = \sin(x)$

$$dX = \cos x dx$$

YES

$$D = \int \frac{\cos x (\cos^2 x - \sin x)}{X^3 - 3X + 2} \frac{dX}{\cos x}$$

$$\cos^2 x = 1 - \sin^2 x = 1 - x^2$$

$$D = \int \frac{1 - x^2 - x}{x^3 - 3x + 2} dx$$

On factorise  $x^3 - 3x + 2$ .

$$\text{On trouve } x^3 - 3x + 2 = (x-1)^2 (x+2)$$

On réduit en fractions simples.

$$\text{On trouve } \frac{1 - x^2 - x}{x^3 - 3x + 2} = \frac{-8/9}{x-1} + \frac{-1/3}{(x-1)^2} + \frac{-1/9}{x+2}$$

On intègre

$$\int \frac{1}{x-1} dx = \ln|x-1| ; \int \frac{1}{(x-1)^2} = -\frac{1}{x-1}$$

$$\textcircled{20} A = \int \frac{\sin x}{2\sin^2 x - 3\cos x} dx$$

$f(x)$ .

$$f(-x) = \frac{-\sin x}{2\sin^2 x - 3\cos x} (-dx)$$

$$= f(x) \quad \text{Yes}$$

$$\text{choix } x = \cos x$$

$$\frac{dx}{dx} = -\sin x \Rightarrow dx = \frac{dx}{-\sin x}$$

$$A = \int \frac{\cancel{\sin x}}{2(1-\cos^2 x) - 3x} \frac{dx}{-\cancel{\sin x}}$$

$$= - \int \frac{dx}{2(1-x^2) - 3x} = \int \frac{1}{2x^2 + 3x - 2} dx$$

On résout en Ets simples

$$2x^2 + 3x - 2 = 2(x+2)(x-\frac{1}{2})$$

$$\Delta = 9 + 16 = 25 \quad x = \frac{-3 \pm 5}{4} = \frac{-2}{4}$$

$$A = \int \frac{1}{2(x+2)(x-\frac{1}{2})} dx = \frac{1}{2} \int \frac{1}{(x+2)(x-\frac{1}{2})} dx$$

$$A = \frac{1}{2} \int \frac{2}{5} \frac{1}{x+2} + \frac{2}{5} \frac{1}{x-\frac{1}{2}} dx$$

$$= -\frac{1}{5} \ln|x+2| + \frac{1}{5} \ln|x-\frac{1}{2}| + \text{cte}$$

$$A = -\frac{1}{5} \ln|\cos x + 2| + \frac{1}{5} \ln|\cos x - \frac{1}{2}| + \text{cte}$$

$$\frac{1}{(x+2)(x-\frac{1}{2})} = \frac{a}{x+2} + \frac{b}{x-\frac{1}{2}}$$

$$0 = \frac{1}{x-\frac{1}{2}} \Big|_{x=-2} = -\frac{2}{5}$$

$$b = \frac{1}{x+2} \Big|_{x=\frac{1}{2}} = \frac{2}{5}$$

$$B = \int \frac{1}{1+2\cos x} dx \quad f(x)$$

$$f(-x) = \frac{1}{1+2\cos x} (-dx) \neq f(x) \text{ NON}$$

$$f(\pi-x) = \frac{1}{1-2\cos x} (-dx) \neq f(x) \text{ NON}$$

$$f(\pi+x) = \frac{1}{1-2\cos x} (dx) \neq f(x) \text{ NON}$$

$$B = \int \frac{1}{1+2\frac{1-t^2}{1+t^2}} \cdot \frac{2dt}{1+t^2}$$

$$B = 2 \int \frac{1}{4t^2 + 2(1+t^2)} dt = 2 \int \frac{1}{3-t^2} dt$$

SMIFF.

$$\text{Chgt } t = \tan\left(\frac{x}{2}\right)$$

$$\frac{dt}{dx} = (1+\tan^2\left(\frac{x}{2}\right)) \cdot \frac{1}{2}$$

↓

$$dx = \frac{2dt}{1+t^2}$$

$$\cos x = \frac{1-t^2}{1+t^2}$$

$$B = 2 \int \frac{1}{(\sqrt{3}-t)(\sqrt{3}+t)} dt = 2 \left( \frac{a}{\sqrt{3}-t} + \frac{b}{\sqrt{3}+t} \right) dt$$

$$a = \frac{1}{\sqrt{3}+t} \Big|_{t=\sqrt{3}} = \frac{1}{2\sqrt{3}}$$

$$b = \frac{1}{\sqrt{3}-t} \Big|_{t=-\sqrt{3}} = \frac{1}{2\sqrt{3}}$$

$$B = \frac{1}{\sqrt{3}} \int \frac{1}{\sqrt{3}-t} dt + \frac{1}{\sqrt{3}} \int \frac{1}{\sqrt{3}+t} dt = -\frac{1}{\sqrt{3}} \ln|t-\sqrt{3}| + \frac{1}{\sqrt{3}} \ln|t+\sqrt{3}| + c_0$$

$$B = -\frac{1}{\sqrt{3}} \ln|\tan\left(\frac{x}{2}\right) - \sqrt{3}| + \frac{1}{\sqrt{3}} \ln|\tan\left(\frac{x}{2}\right) + \sqrt{3}| + c_0$$

$$C = \int \frac{1}{2 + \cos x} dx$$

Pas d'invariants.

dyt  $t = \tan\left(\frac{x}{2}\right)$

$$C = \int \frac{1}{2 + \frac{1-t^2}{1+t^2}} \frac{2}{1+t^2} dt$$

$$dx = \frac{2}{1+t^2} dt$$

$$\cos x = \frac{1-t^2}{1+t^2}$$

$$C = 2 \int \frac{1}{2(1+t^2) + (1-t^2)} dt = 2 \int \frac{1}{t^2 + 3} dt$$

$$C = 2 \int \frac{1}{3\left(\frac{t^2}{3} + 1\right)} dt = \frac{2}{3} \int \frac{1}{\left(\frac{t}{\sqrt{3}}\right)^2 + 1} dt$$

dyt

$$T = \frac{t}{\sqrt{3}}$$

$$dt = \sqrt{3} dT$$

$$C = \frac{2}{3} \int \frac{1}{T^2 + 1} \sqrt{3} dT = \frac{2\sqrt{3}}{3} \operatorname{arctan}(T) + \text{cte}$$

$$C = \frac{2\sqrt{3}}{3} \operatorname{arctan}\left(\frac{t}{\sqrt{3}}\right) + \text{cte}$$

$$C = \frac{2\sqrt{3}}{3} \operatorname{arctan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right) + \text{cte}$$