

$$(14) I = \int \frac{x^4 + 3x + 2}{(x+2)(x-1)^2} dx$$

$$\begin{aligned} (x+2)(x-1)^2 &= (x+2)(x^2 - 2x + 1) \\ &= x^3 - 2x^2 + x + 2x^2 - 4x + 2 \\ &= x^3 - 3x + 2 \end{aligned}$$

$$\frac{x^4 + 3x + 2}{(x+2)(x-1)^2} = F(x) + \frac{a}{x+2} + \frac{b}{x-1} + \frac{c}{(x-1)^2}$$

$$\begin{array}{r} x^4 + 3x + 2 \\ -(x^3 - 3x^2 + 2x) \\ \hline 3x^2 + x + 2 \end{array} \quad \begin{array}{l} x^3 - 3x + 2 \\ \text{E(x)} \end{array}$$

$$a = \left. \frac{x^4 + 3x + 2}{(x-1)^2} \right|_{x=-2} = \frac{16 - 6 + 2}{9} = \frac{12}{9} = \frac{4}{3} = a$$

$$c = \left. \frac{x^4 + 3x + 2}{x+2} \right|_{x=1} = \frac{6}{3} = 2 = c$$

• valeur en 0 : $\frac{2}{2} = 0 + \frac{4/3}{2} + \frac{b}{-1} + \frac{2}{1}$

$$1 = \frac{2}{3} - b + 2 \rightarrow b = \frac{2}{3} + 2 - 1 = \frac{2}{3} + 1 = \frac{5}{3} = b$$

$$I = \int \left(x + \frac{4/3}{x+2} + \frac{5/3}{x-1} + \frac{2}{(x-1)^2} \right) dx = \frac{x^2}{2} + \frac{4}{3} \ln|x+2| + \frac{5}{3} \ln|x-1| - \frac{2}{x-1} + \text{cte}$$

rappel : $\int \frac{1}{x^2} dx = \int x^{-2} dx = \frac{x^{-1}}{-1} + \text{cte} = -\frac{1}{x} + \text{cte}$

$$A = \int \frac{x^3}{1-x} dx$$

$$\frac{x^3}{1-x} = E(x) + \frac{a}{1-x}$$

$$a = x^3 \Big|_{x=1} = 1 = a$$

$$\begin{array}{r|l} x^3 & -x+1 \\ \hline -(x^3-x^2) & \boxed{-x^2-x-1} \\ \hline x^2 & \\ -(x^2-x) & \\ \hline x & \\ -(x-1) & \\ \hline 1 & \end{array} \quad E(x)$$

$$A = \int \left(-x^2 - x - 1 + \frac{1}{1-x} \right) dx = \underline{-\frac{x^3}{3} - \frac{x^2}{2} - x - \ln|1-x| + C_0}$$



$$B = \int \frac{dx}{x(x^2+1)}$$

$$\frac{1}{x(x^2+1)} = 0 + \frac{a}{x} + \frac{bx+c}{x^2+1}$$

$c=0$ car $\frac{1}{x(x^2+1)}$ est impaire

$$a = \frac{1}{x^2+1} \Big|_{x=0} = 1$$

ou multi par x

$$\frac{1}{x^2+1} = 1 + \frac{bx^2+cx}{x^2+1}$$

$$B = \int \left(\frac{1}{x} - \frac{x}{x^2+1} \right) dx = \underline{\ln|x| - \frac{1}{2} \ln(x^2+1)}$$

$$x \rightarrow +\infty$$

$$0 = 1 + b \Rightarrow b = -1$$

$$\textcircled{19} \quad A = \int \sin^4(x) \cos(x) dx \quad \text{chyt } X = \sin x \quad \frac{dX}{dx} = \cos x$$

$$\Rightarrow dx = \frac{dX}{\cos(x)} \quad A = \int \sin^4(x) \cos(x) \frac{dX}{\cos(x)} = \int X^4 dX = \frac{X^5}{5} + \text{cte}$$
$$= \frac{\sin^5(x)}{5} + \text{cte}$$

$$B = \int \sin^3(x) \cos^2(x) dx \quad \text{chyt } X = \cos x \quad dx = -\frac{dX}{\sin(x)}$$

$$B = \int \sin^3(x) X^2 \left(-\frac{dX}{\sin(x)} \right) = -\int \sin^2(x) X^2 dX$$
$$= 1 - \cos^2(x) = 1 - X^2$$

$$B = -\int (1 - X^2) X^2 dX = \int (X^4 - X^2) dX = \frac{X^5}{5} - \frac{X^3}{3} + \text{cte}$$

$$= \frac{\cos^5 x}{5} - \frac{\cos^3 x}{3} + \text{cte}$$

$$\begin{aligned}
\textcircled{c} \quad \sin^4 x \cos^4 x &= \left(\frac{e^{ix} - e^{-ix}}{2i} \right)^4 \left(\frac{e^{ix} + e^{-ix}}{2} \right)^4 \\
&= \frac{1}{2^8} \left(\underbrace{e^{i4x}}_{\text{green}} - \underbrace{4e^{i2x}}_{\text{yellow}} + \underbrace{6}_{\text{green}} - \underbrace{4e^{-i2x}}_{\text{yellow}} + \underbrace{e^{-i4x}}_{\text{green}} \right) \left(\underbrace{e^{i4x}}_{\text{green}} + \underbrace{4e^{i2x}}_{\text{yellow}} + \underbrace{6}_{\text{green}} + \underbrace{4e^{-i2x}}_{\text{yellow}} + \underbrace{e^{-i4x}}_{\text{green}} \right) \\
&= \frac{1}{2^8} \left(\underbrace{e^{i8x}}_{\text{green}} + \cancel{4e^{i6x}}_{\text{yellow}} + \underbrace{6e^{i4x}}_{\text{green}} + \cancel{4e^{i2x}}_{\text{yellow}} + 1 \right. \\
&\quad \left. - \cancel{4e^{i6x}}_{\text{yellow}} - \underbrace{16e^{i4x}}_{\text{yellow}} - \cancel{24e^{i2x}}_{\text{yellow}} - 16 - \cancel{4e^{-i4x}}_{\text{yellow}} \right. \\
&\quad \left. + 6e^{i4x} + \cancel{24e^{i2x}}_{\text{yellow}} + 36 + \cancel{24e^{-i2x}}_{\text{yellow}} + 6e^{-i4x} \right. \\
&\quad \left. - \cancel{4e^{i2x}}_{\text{yellow}} - 16 - \cancel{24e^{i2x}}_{\text{yellow}} - \underbrace{16e^{-i4x}}_{\text{yellow}} - \cancel{4e^{-i6x}}_{\text{yellow}} \right. \\
&\quad \left. + 1 + \cancel{4e^{-i2x}}_{\text{yellow}} + \underbrace{6e^{-i4x}}_{\text{green}} + \cancel{4e^{-i6x}}_{\text{yellow}} + \underbrace{e^{-i8x}}_{\text{green}} \right) \\
&= \frac{1}{2^8} \left(\underbrace{e^{i8x}}_{\text{green}} - \underbrace{4e^{i4x}}_{\text{yellow}} + 6 - \underbrace{4e^{-i4x}}_{\text{yellow}} + \underbrace{e^{-i8x}}_{\text{green}} \right) \\
&= \frac{1}{2^8} (2\cos(8x) - 8\cos(4x) + 6) \cdot \text{On intègre : } C = \frac{1}{2^8} \left(2 \frac{\sin(8x)}{8} - \frac{8 \sin(4x)}{4} + 6x \right) + \text{cte} \\
&\quad C = \frac{1}{2^{10}} \sin(8x) - \frac{1}{2^7} \sin(4x) + \frac{3x}{2^7} + \text{cte} \quad 2^{10} = 1024 \text{ et } 2^7 = 128
\end{aligned}$$

$$(14) \quad I = \int \frac{x^4 + 3x + 2}{(x+2)(x-1)^2} dx$$

$$(x+2)(x-1)^2 = (x+2)(x^2 - 2x + 1) = x^3 - 2x^2 + x + 2x^2 - 4x + 2 = x^3 - 3x + 2$$

$$\frac{x^4 + 3x + 2}{(x+2)(x-1)^2} = E(x) + \frac{a}{x+2} + \left(\frac{c}{x-1} + \frac{d}{(x-1)^2} \right)$$

$$\begin{array}{r} x^4 + 3x + 2 \\ - (x^3 - 3x^2 + 2x) \\ \hline 3x^2 + x + 2 \end{array} \quad \begin{array}{l} x^3 - 3x + 2 \\ \hline (x) \\ E(x) \end{array}$$

$$a = \frac{x^4 + 3x + 2}{(x-1)^2} \Big|_{x=-2} = \frac{16 - 6 + 2}{9} = \frac{12}{9} = \frac{4}{3}$$

$$d = \frac{x^4 + 3x + 2}{x+2} \Big|_{x=1} = \frac{6}{3} = 2$$

Walew e 0: $\frac{2}{2 \times 1} = 0 + \frac{4/3}{2} + \frac{c}{-1} + 2 \Rightarrow 1 = \frac{2}{3} - c + 2 \Rightarrow c = \frac{2}{3} + 1 = \frac{5}{3}$

$$I = \int \left(x + \frac{4/3}{x+2} + \frac{5/3}{x-1} + \frac{2}{(x-1)^2} \right) dx = \frac{x^2}{2} + \frac{4}{3} \ln|x+2| + \frac{5}{3} \ln|x-1| - \frac{2}{x-1} + C$$

$$A = \int \frac{x^3}{1-x} dx$$

$$\frac{x^3}{1-x} = E(x) + \frac{a}{1-x}$$

$$\begin{array}{r} x^3 \\ -x+1 \\ \hline -x^2-x-1 \\ \textcircled{1} \end{array}$$

$$x^3 = (1-x)(-x^2-x-1) + \textcircled{1} \quad \text{reste}$$

$$\frac{x^3}{1-x} = -x^2-x-1 + \frac{1}{1-x}$$

← on a divisé par 1-x

$$A = -\frac{x^3}{3} - \frac{x^2}{2} - x - \ln|1-x| + \text{cte}$$



$$B = \int \frac{dx}{x(x^2+1)}$$

$$\frac{1}{x(x^2+1)} = \frac{a}{x} + \frac{bx+c}{x^2+1}$$

$$a = \frac{1}{x^2+1} \Big|_{x=0} = 1$$

on multiplie par x^2+1 :

$$\frac{1}{x} = \frac{1}{x}(x^2+1) + bx+c$$

valeurs a i:

$$\frac{1}{i} = 0 + bi+c$$

$$-i = bi+c \Rightarrow b=-1 \text{ et } c=0$$

$$B = \int \frac{1}{x} dx - \int \frac{x}{x^2+1} dx$$

$$= \ln|x| - \frac{1}{2} \int \frac{2x}{x^2+1} dx$$

$$B = \ln|x| - \frac{1}{2} \ln(x^2+1) + \text{cte}$$

$$(19) A = \int \sin^4 x \cos x \, dx$$

$$\text{chgt } X = \sin(x)$$

$$\frac{dX}{dx} = \cos x,$$

\Downarrow

$$A = \int X^4 \cancel{\cos x} \frac{dX}{\cancel{\cos x}} = \frac{X^5}{5} + cte = \frac{\sin^5(x)}{5} + cte$$

$$dx = \frac{dX}{\cos(x)}$$

$$B = \int \sin^3 x \cos^2 x \, dx$$

$$\text{chgt } X = \cos(x)$$

$$\frac{dX}{dx} = -\sin x$$

\Downarrow

$$dx = -\frac{dX}{\sin x}$$

$$B = \int \sin^3 x X^2 \left(-\frac{dX}{\sin x}\right) = -\int \sin^2 x X^2 dX$$

minus also

$$\sin^2(x) + \cos^2(x) = 1$$

$$\Rightarrow \sin^2(x) = 1 - \cos^2(x) = 1 - X^2$$

$$B = -\int (1 - X^2) X^2 dX = \int (X^4 - X^2) dX$$

$$= \frac{X^5}{5} - \frac{X^3}{3} + cte = \frac{\cos^5(x)}{5} - \frac{\cos^3(x)}{3} + cte$$

$$\begin{aligned}
\textcircled{c} \quad \sin^4 x \cos^4 x &= \left(\frac{e^{ix} - e^{-ix}}{2i} \right)^4 \left(\frac{e^{ix} + e^{-ix}}{2} \right)^4 \\
&= \frac{1}{2^8} \left(\underbrace{e^{i4x}}_{\text{green}} - \underbrace{4e^{i2x}}_{\text{yellow}} + \underbrace{6}_{\text{blue}} - \underbrace{4e^{-i2x}}_{\text{yellow}} + \underbrace{e^{-i4x}}_{\text{green}} \right) \left(e^{i4x} + 4e^{i2x} + 6 + 4e^{-i2x} + e^{-i4x} \right) \\
&= \frac{1}{2^8} \left(\begin{aligned}
&\underbrace{e^{i8x}}_{\text{green}} + \cancel{4e^{i6x}}_{\text{yellow}} + \underbrace{6e^{i4x}}_{\text{blue}} + \cancel{4e^{i2x}}_{\text{yellow}} + 1 \\
&\cancel{-4e^{i0x}}_{\text{yellow}} - \underbrace{16e^{i4x}}_{\text{yellow}} - \cancel{24e^{i2x}}_{\text{yellow}} - 16 - \cancel{4e^{-i0x}}_{\text{yellow}} \\
&+ \underbrace{6e^{i4x}}_{\text{blue}} + \cancel{24e^{i2x}}_{\text{yellow}} + 36 + \cancel{24e^{-i2x}}_{\text{yellow}} + \underbrace{6e^{-i4x}}_{\text{blue}} \\
&\cancel{-4e^{-i2x}}_{\text{yellow}} - 16 - \cancel{24e^{-i2x}}_{\text{yellow}} - \underbrace{16e^{-i4x}}_{\text{yellow}} - \cancel{4e^{-i6x}}_{\text{yellow}} \\
&+ 1 + \cancel{4e^{-i0x}}_{\text{yellow}} + \underbrace{6e^{-i4x}}_{\text{blue}} + \cancel{4e^{-i6x}}_{\text{yellow}} + \underbrace{e^{-i8x}}_{\text{green}}
\end{aligned} \right) \\
&= \frac{1}{2^8} \left(\underbrace{e^{i8x}}_{\text{green}} - \underbrace{4e^{i4x}}_{\text{yellow}} + 6 - \underbrace{4e^{-i4x}}_{\text{yellow}} + \underbrace{e^{-i8x}}_{\text{green}} \right) \\
&= \frac{1}{2^8} \left(2\cos(8x) - 8\cos(4x) + 6 \right) \quad \text{On intègre: } C = \frac{1}{2^8} \left(2 \frac{\sin(8x)}{8} - \frac{8\sin(4x)}{4} + 6x \right) + \text{cte} \\
&\quad \left(C = \frac{1}{2^{10}} \sin(8x) - \frac{1}{2^7} \sin(4x) + \frac{3}{2^7} x + \text{cte} \right) \quad 2^{10} = 1024 \text{ et } 2^7 = 128
\end{aligned}$$

$$(14) \quad D = \int \frac{x+1}{x^2+4x+8} dx \quad \Delta = -16 < 0$$

c'est l'écrasé simple

on se ramène à X^2+1

$$x^2+4x+8 = (x+2)^2 + 4 = 4 \left[\left(\frac{x+2}{2}\right)^2 + 1 \right]$$

chgt $t = \frac{x+2}{2}$

$$\Downarrow \frac{dt}{dx} = \frac{1}{2}$$

$$\Downarrow dx = 2 dt$$

$$\Downarrow 2t = x+2$$

$$\Downarrow \boxed{x = 2t - 2}$$

mince

$$D = \int \frac{x+1}{4[t^2+1]} 2 dt$$

$$D = \int \frac{2t-2+1}{4[t^2+1]} dt = \frac{1}{2} \int \frac{2t-1}{t^2+1} dt = \frac{1}{2} \int \frac{2t}{t^2+1} dt - \frac{1}{2} \int \frac{1}{t^2+1} dt$$

$$= \frac{1}{2} \ln(t^2+1) - \frac{1}{2} \operatorname{arctan}(t) = \frac{1}{2} \ln\left(\left(\frac{x+2}{2}\right)^2 + 1\right) - \frac{1}{2} \operatorname{arctan}\left(\frac{x+2}{2}\right) + cte$$

$$(14) \quad E = \int \frac{dx}{x^2 - x + 1} \quad \Delta < 0$$

$$x^2 - x + 1 = \left(x - \frac{1}{2}\right)^2 + \frac{3}{4} = \frac{3}{4} \left[\frac{4}{3} \left(x - \frac{1}{2}\right)^2 + 1 \right]$$

$$= \frac{3}{4} \left[\left(\frac{2}{\sqrt{3}} \left(x - \frac{1}{2}\right)\right)^2 + 1 \right] = \frac{3}{4} \left[\left(\frac{2x-1}{\sqrt{3}}\right)^2 + 1 \right]$$

$$\text{Let } t = \frac{2x-1}{\sqrt{3}} \quad dt = \frac{2}{\sqrt{3}} dx$$

$$E = \int \frac{\frac{\sqrt{3}}{2} dt}{\frac{3}{4} (t^2 + 1)} = \frac{\sqrt{3}}{2} \frac{4}{3} \arctan(t) + c_1$$

$$= \frac{2\sqrt{3}}{3} \arctan\left(\frac{2x-1}{\sqrt{3}}\right) + c_1$$