

$$\textcircled{2} \quad 2^4 + 2^5 + \dots + 2^{10} = 2^4 [1 + 2 + \dots + 2^6] = 2^4 \cdot \frac{2^7 - 1}{2 - 1} = 2^4 (2^7 - 1) = 2^{11} - 2^4$$

$$= 2048 - 16 = 2032$$

$$\textcircled{3} \quad \sum_{k=1}^n (k+1)^3 - \sum_{k=1}^n k^3 = \sum_{k=2}^{n+1} k^3 - \sum_{k=1}^n k^3$$

$$= \left(\sum_{k=2}^{n+1} k^3 + (n+1)^3 \right) - \left(1^3 + \sum_{k=2}^n k^3 \right) = (n+1)^3 - 1$$

$$= n^3 + 3n^2 + 3n$$

• $(k+1)^3 = k^3 + 3k^2 + 3k + 1$, donc $\sum_{k=1}^n (k+1)^3 = \sum_{k=1}^n k^3 + 3 \sum_{k=1}^n k^2 + 3 \sum_{k=1}^n k + \sum_{k=1}^n 1$

$$\Rightarrow \sum_{k=1}^n (k+1)^3 = \sum_{k=1}^n k^3 + 3S_n + 3 \frac{n(n+1)}{2} + n$$

$$\Rightarrow \sum_{k=1}^n (k+1)^3 - \sum_{k=1}^n k^3 = 3S_n + \frac{3n^2 + 3n + 2n}{2} = (n+1)^3 - 1 \quad (01)$$

$$\Rightarrow 3S_n = n^3 + 3n^2 + 3n - \frac{3n^2 + 5n}{2} = \frac{2n^3 + 6n^2 + 6n - 3n^2 - 5n}{2}$$

$$= \frac{2n^3 + 3n^2 + n}{2}$$

$$\Rightarrow \boxed{S_n = \frac{2n^3 + 3n^2 + n}{6}}$$