

$$(7) \quad A_n = \sum_{1 \leq i \leq j \leq n} i = \sum_{i=1}^n \left(\sum_{j=i}^n i \right)$$

$$= \sum_{i=1}^n i \left(\sum_{j=i}^n 1 \right) = \sum_{i=1}^n i (n-i+1)$$

$$= \sum_{i=1}^n i n - \sum_{i=1}^n i^2 + \sum_{i=1}^n i$$

$$= n \sum_{i=1}^n i - \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$$

$$= n \left(\frac{n(n+1)}{2} \right) - \frac{2n^3 + 3n^2 + n}{6} + \frac{n(n+1)}{2}$$

$$= \frac{3n^3 + 3n^2}{6} - \frac{2n^3 + 3n^2 + n}{6} + \frac{3n^2 + 3n}{6}$$

$$= \frac{3n^3 + 3n^2 - 2n^3 - 3n^2 - n + 3n^2 + 3n}{6}$$

$$A_n = \frac{n^3 + 3n^2 + 2n}{6}$$

$$B_n = \sum_{1 \leq i < j \leq n} j = \sum_{i=1}^n \left(\sum_{j=i}^n j \right) = \sum_{i=1}^n \frac{(n-i+1)(n+i)}{2}$$

$$= \frac{1}{2} \sum_{i=1}^n (n^2 + ni - n^2 - i^2 + n + i)$$

$$= \frac{1}{2} \left(n^2 \sum_{i=1}^n 1 - \sum_{i=1}^n i^2 + n \sum_{i=1}^n 1 + \sum_{i=1}^n i \right)$$

$$= \frac{1}{2} \left(n^2 \cdot n - \frac{n(n+1)(2n+1)}{6} + n^2 + \frac{n(n+1)}{2} \right)$$

$$= \frac{1}{2} \left(\frac{6n^3 - 2n^3 - 2n^2 - n + 6n^2 + 3n + 3n}{6} \right)$$

$$= \frac{1}{2} \frac{4n^3 + 6n^2 + 2n}{6} = \frac{2n^3 + 3n^2 + n}{6}$$