

$$\begin{aligned}
 \textcircled{5} \quad A_n &= \sum_{k=0}^{2n} \min(k, n) = \sum_{k=0}^n \min(k, n) + \sum_{k=n+1}^{2n} \min(k, n) \\
 &= \sum_{k=0}^n k + \sum_{k=n+1}^{2n} n = \frac{n(n+1)}{2} + n \sum_{n+1}^{2n} 1 \\
 &= \frac{n(n+1)}{2} + n \cdot (2n - (n+1) + 1) = \frac{n(n+1)}{2} + n \cdot n \\
 &= \frac{n^2 + n + 2n^2}{2} = \frac{3n^2 + n}{2}
 \end{aligned}$$

$$\begin{aligned}
 B_n &= \sum_{k=1}^{2n} (-1)^k k = \sum_{\substack{k \text{ pair} \\ \text{car } (-1)^k = 1}} k - \sum_{\substack{k \text{ impair} \\ \text{et } (-1)^k = -1}} k \\
 &= \sum_{p=1}^n 2p - \sum_{p=1}^n (2p-1) = \sum_{p=1}^n 2p - \sum_{p=1}^n 2p + \sum_{p=1}^n 1 \\
 &= n
 \end{aligned}$$

$$\textcircled{5} C_n = \sum_{k=1}^n (-1)^k k^2$$

$$= \sum_{k \text{ pairs}} k^2 - \sum_{k \text{ unpairs}} k^2$$

$$= \sum_{p=1}^n \binom{2p}{2}^2 - \sum_{p=1}^n \binom{2p-1}{1}^2$$

$$= 4 \sum_{p=1}^n p^2 - \sum_{p=1}^n (4p^2 - 4p + 1)$$

$$= 4 \sum_{p=1}^n p^2 - 4 \sum_{p=1}^n p^2 + 4 \sum_{p=1}^n p - \sum_{p=1}^n 1$$

$$= 4 \frac{n(n+1)}{2} - n = 2n^2 + 2n - n = \underline{2n^2 + n}$$