

$$\begin{aligned}
 \textcircled{4} \quad \sum_{k=1}^n \ln\left(\frac{k}{k+1}\right) &= \sum_{k=1}^n \left[\ln(k) - \ln(k+1) \right] \\
 &= \sum_{k=1}^n \ln(k) - \sum_{k=1}^n \ln(k+1) \\
 &= \sum_{k=1}^n \ln(k) - \sum_{k=2}^{n+1} \ln(k) \quad (\text{decalage}) \\
 &= \left(\sum_{k=2}^n \ln(k) + \ln(1) \right) - \left(\sum_{k=2}^n \ln(k) + \ln(n+1) \right) \\
 &= \ln(1) - \ln(n+1) = \underline{\underline{-\ln(n+1)}}
 \end{aligned}$$