

$$(14) \cdot A_n = \prod_{k=0}^n e^{2k} = e^{\sum_{k=0}^n 2k} \quad \text{can } e^a \cdot e^b = e^{a+b}$$

$$= e^{2 \sum_0^n k} = e^{2 \frac{n(n+1)}{2}} = e^{n(n+1)} = e^{n^2+n}$$

$$\cdot B_n = \sum_{1 \leq i_1 < i_2 < \dots < i_n} \ell = \sum_{i=1}^n \binom{n}{i} \ell = \sum_{i=1}^n \ell \binom{n}{i} = \sum_{i=1}^n \ell (n-i+1)$$

$$= \ell \left(\sum_{i=1}^n n - \sum_{i=1}^n i + \sum_{i=1}^n 1 \right) = \ell n \cdot n - \frac{n(n+1)}{2} + n$$

$$= \frac{4n^2 - (n^2+n) + 2n}{2} = \frac{3n^2+n}{2}$$

$$\cdot C_n = \sum_{k=0}^n \sum_{j=k}^n 2^j = \sum_{k=0}^n (2^k + 2^{k+1} + \dots + 2^n)$$

$$= \sum_{k=0}^n 2^k (1 + 2 + \dots + 2^{n-k}) = \sum_{k=0}^n 2^k \frac{2^{n-k+1} - 1}{2-1} = \sum_{k=0}^n (2^{n+1} - 2^k)$$

$$= \sum_{k=0}^n 2^{n+1} - \sum_{k=0}^n 2^k = 2^{n+1} \sum_{k=0}^n 1 - \sum_{k=0}^n 2^k$$

$$= 2^{n+1} \cdot (n+1) - \frac{2^{n+1} - 1}{2-1} = n \cdot 2^{n+1} + 2^{n+1} - (2^{n+1} - 1)$$

$$= n \cdot 2^{n+1} + 1$$