

$$\textcircled{1} \quad 7 + 8 + 9 + \dots + 37 = \sum_{k=7}^{k=37} k = \frac{(37+7)(37-7+1)}{2}$$

$$= \frac{44 \cdot 31}{2} = 22 \cdot 31 = 682$$

$$\begin{array}{r} 31 \\ 22 \\ \hline 62 \\ 62 \\ \hline 682 \end{array}$$

$$7 + 9 + 11 + \dots + 37 = \sum_{k=3}^{18} (2k+1) = 2 \sum_{k=3}^{18} k + \sum_{k=3}^{18} 1$$

$$= 2 \cdot \frac{(3+18) \cdot (18-3+1)}{2} + \underbrace{18-3+1}_{\text{nombre de termes}}$$

$$= 21 \cdot 16 + 16 = 16(21+1) = 16 \cdot 22$$

$$= 352$$

$$\begin{array}{r} 16 \\ 22 \\ \hline 32 \\ 32 \\ \hline 352 \end{array}$$

$$7 + 10 + 13 + \dots + 37 = \sum_{k=2}^{12} (3k+1) = 3 \sum_{k=2}^{12} k + \sum_{k=2}^{12} 1$$

$$= 3 \frac{(12+2)(12-2+1)}{2} + 12-2+1$$

$$= 3 \cdot 7 \cdot 11 + 11 = 21 \cdot 11 + 11 = 11 \cdot 22 = 242$$

$$\begin{array}{r} 22 \\ 11 \\ \hline 22 \\ 22 \\ \hline 242 \end{array}$$

$$8 + 10 + 12 + \dots + 38 = \sum_{k=4}^{19} 2k = 2 \sum_{k=4}^{19} k = 2 \frac{(4+19)(19-4+1)}{2}$$

$$= 2 \frac{23 \cdot 16}{2} = 9 \cdot 23 \cdot 8 = 9 \cdot 184 = 368$$

$$S = 2^4 + 2^5 + \dots + 2^{10} = \sum_{k=4}^{10} 2^k$$

1^{ère} méthode: $S = \sum_{k=0}^{10} 2^k - \sum_{k=0}^3 2^k$

$$\begin{array}{r} 127 \\ 16 \\ \hline 762 \\ 127 \\ \hline 2032 \end{array}$$

$$\left(S = (1 + 2 + 2^2 + \dots + 2^{10}) - (1 + 2 + 2^2 + 2^3) \right)$$

$$S = \frac{2^{11} - 1}{2 - 1} - \frac{2^4 - 1}{2 - 1} = 2047 - 15 = 2032$$

2^{ème} méthode: $S = 2^4 [1 + 2 + 2^2 + \dots + 2^6] = 16 \cdot \sum_0^6 2^k$

$$= 16 \cdot \frac{2^7 - 1}{2 - 1} = 16 \cdot \frac{127}{1} = 2032$$

$$\begin{aligned}
 (3) \quad \sum_{k=1}^n (k+1)^3 - \sum_{k=1}^n k^3 &= \sum_{k=2}^{n+1} k^3 - \sum_{k=1}^n k^3 \\
 &= \sum_{k=2}^n k^3 + (n+1)^3 - \left(1^3 + \sum_{k=2}^n k^3 \right) \\
 &= (n+1)^3 - 1 = n^3 + 3n^2 \cdot 1 + 3n \cdot 1^2 + 1^3 - 1 \\
 &= \underline{n^3 + 3n^2 + 3n}
 \end{aligned}$$

$$(k+1)^3 = k^3 + 3k^2 + 3k + 1$$

$$\sum_{k=1}^n (k+1)^3 = \sum_{k=1}^n k^3 + 3 \underbrace{\sum_{k=1}^n k^2}_{\sum_n} + 3 \underbrace{\sum_{k=1}^n k}_{\frac{n(n+1)}{2}} + \underbrace{\sum_{k=1}^n 1}_n$$

$$\underbrace{\sum_{k=1}^n (k+1)^3 - \sum_{k=1}^n k^3}_{n^3 + 3n^2 + 3n} = 3 \sum_n + 3 \frac{n(n+1)}{2} + n$$

← 1^{ere} Q.

$$\begin{aligned} \Rightarrow 3 \sum_n &= n^3 + 3n^2 + 3n - 3n \frac{(n+1)}{2} - n \\ &= \frac{2n^3 + 6n^2 + 6n - 3n^2 - 3n - 2n}{2} \\ &= \frac{2n^3 + 3n^2 + n}{2} = \frac{n(2n^2 + 3n + 1)}{2} \end{aligned}$$

$$\Rightarrow \sum_n = \frac{\sum_{k=1}^n k^2}{6} = \frac{n(2n^2 + 3n + 1)}{6}$$

$$\begin{aligned}
 (4) \quad \sum_{k=1}^n \ln\left(\frac{k}{k+1}\right) &= \sum_{k=1}^n \ln(k) - \sum_{k=1}^n \ln(k+1) \\
 &= \ln(1) + \cancel{\ln(2)} + \cancel{\ln(3)} + \dots + \cancel{\ln(n)} \\
 &\quad - (\cancel{\ln(1)} + \cancel{\ln(2)} + \dots + \cancel{\ln(n)} + \ln(n+1)) \\
 &= \ln(1) - \ln(n+1) = -\ln(n+1)
 \end{aligned}$$

$$\begin{aligned}
 (5) \quad A_n &= \sum_{k=0}^{2n} \min(k, n) = \sum_{k=0}^n \underbrace{\min(k, n)}_k + \sum_{k=n+1}^{2n} \underbrace{\min(k, n)}_n \\
 &= \sum_{k=0}^n k + \sum_{k=0}^n n = \frac{n(n+1)}{2} + n \left(\sum_{i=0}^n 1 \right) \\
 &= \frac{n(n+1)}{2} + n \uparrow (n+1)
 \end{aligned}$$

$$\textcircled{1} \quad 7 + 8 + 9 + \dots + 37 = \sum_{k=7}^{k=37} k = \sum_{k=1}^{37} k - \sum_{k=1}^6 k$$

$$= \frac{37 \cdot 38}{2} - \frac{6 \cdot 7}{2}$$

$$= 37 \cdot 19 - 3 \cdot 7 = 703 - 21$$

$$= \underline{682}$$

$$\begin{array}{r} 37 \\ 19 \\ \hline 333 \\ 37 \\ \hline 703 \end{array}$$

$$\begin{array}{r} 16 \\ 22 \\ \hline 38 \\ 32 \\ \hline 352 \end{array}$$

$$7 + 9 + 11 + \dots + 37 = \sum_{k=3}^{k=18} (2k+1) = 2 \sum_{k=3}^{18} k + \sum_{k=3}^{18} 1$$

$$= 2 \left(\frac{3+18}{2} (18-3+1) \right) + (18-3+1)$$

$$= 21 \cdot 16 + 16 = 16[21+1] = 16 \cdot 22 = \underline{352} \quad \text{↑} \quad \text{numero de termos}$$

$$\begin{aligned}
7 + 10 + 13 + \dots + 37 &= \sum_{k=2}^{k=12} (3k + 1) = \sum (3k) + \sum 1 \\
&= 3 \sum_{k=2}^{12} k + \sum_{k=2}^{12} 1 = 3 \left(\frac{14 \cdot 11}{2} \right) + 11 \\
&= 3 \cdot 7 \cdot 11 + 11 = 21 \cdot 11 + 11 = 11(21 + 1) \\
&= 11 \cdot 22 = 242
\end{aligned}$$

$$\begin{aligned}
8 + 10 + 12 + \dots + 38 &= \sum_{k=4}^{k=19} 2k = 2 \sum_{k=4}^{19} k \\
&= 2 \frac{(4+19)(19-4+1)}{2} = 23 \cdot 16 = 368
\end{aligned}$$

$$\begin{array}{r}
23 \\
16 \\
\hline
138 \\
23 \\
\hline
368
\end{array}$$

$$\textcircled{2} \quad S = 2^4 + 2^5 + \dots + 2^{10} = \sum_{k=4}^{10} 2^k$$

1^{er} méthode : $S = \sum_{k=0}^{10} 2^k - \sum_{k=0}^3 2^k$

$$= \frac{2^{11} - 1}{2 - 1} - \frac{2^4 - 1}{2 - 1} = 2^{11} - (2^4 - 1)$$
$$= 2048 - 16 = 2032$$

2^{er} méthode : $2^4 + 2^5 + \dots + 2^{10} = 2^4 [1 + 2 + 2^2 + \dots + 2^6]$

$$= 16 \frac{2^7 - 1}{2 - 1} = 16 \cdot 127 = 2032$$

$$(3) \cdot \sum_{k=1}^{n+1} (k-1)^3 - \sum_{k=1}^n k^3 = \sum_{k=2}^{n+1} k^3 - \sum_{k=1}^n k^3$$

$$= \left(\sum_{k=2}^n k^3 + (n+1)^3 \right) - \left(1^3 + \sum_{k=2}^n k^3 \right)$$

$$= \underline{(n+1)^3 - 1} = n^3 + 3n^2 + 3n + 1 - 1 = \underline{n^3 + 3n^2 + 3n} = n(n^2 + 3n + 3)$$

• $(k+1)^3 = k^3 + 3k^2 + 3k + 1 \leftarrow$ fórmula de Binômio.

$$\sum_{k=1}^n (k+1)^3 = \sum_{k=1}^n k^3 + 3 \left(\sum_{k=1}^n k^2 \right) + 3 \left(\sum_{k=1}^n k \right) + \left(\sum_{k=1}^n 1 \right)$$

$$+ 3 S_n + 3 \frac{n(n+1)}{2} + n$$

$$3\sum_n = \underbrace{\sum (k+1)^3 - \sum k^3}_{n^3 + 3n^2 + 3n} - 3 \frac{n(n+1)}{2} - n$$

$$3\sum_n = n^3 + 3n^2 + 3n - 3 \frac{(n^2+n)}{2} - n$$

$$= \frac{2n^3 + 6n^2 + 6n - 3n^2 - 3n - 2n}{2}$$

$$= \frac{2n^3 + 3n^2 + n}{2} = \frac{n(2n^2 + 3n + 1)}{2}$$

$$\sum_n = \frac{n(n+1)(2n+1)}{6} = \sum_{k=1}^n k^2$$

④

$$A_n = \sum_{k=1}^n \ln\left(\frac{k}{k+1}\right) = \sum_{k=1}^n (\ln(k) - \ln(k+1))$$

$$= \sum_{k=1}^n \ln(k) - \sum_{k=1}^n \ln(k+1)$$

$$= \sum_{k=1}^n \ln(k) - \sum_{k=2}^{n+1} \ln(k)$$

$$= \left(\ln(1) + \sum_{k=2}^n \cancel{\ln(k)} \right) - \left(\sum_{k=2}^n \cancel{\ln(k)} + \ln(n+1) \right)$$

$$= \ln(1) - \ln(n+1) = \underline{\underline{-\ln(n+1)}}$$