

14)  $x^3 y'' + 4x^2 y' + 2xy = 1$

$y = \frac{1}{x}$  ;  $y' = -\frac{1}{x^2}$  ;  $y'' = \frac{2}{x^3}$

$x^3 y'' + 4x^2 y' + 2xy = 2 - 4 + 2 = 0$  Donc  $\frac{1}{x}$  est sol de l'hom

On fait le chgt  $y(x) = \frac{A(x)}{x} = A(x)x^{-1}$  ;  $y'(x) = A'(x)x^{-1} - A(x)x^{-2}$   
 $y''(x) = A''(x)x^{-1} - 2A'(x)x^{-2} + 2A(x)x^{-3}$   
 $x^3 y'' + 4x^2 y' + 2xy = A''(x)x^2 - 2A'(x)x + 2A(x) + 4A(x)x - 4A(x) + 2A(x) = 1$   
 $= A''(x)x^2 + 2A'(x)x = 1$  . On pose  $z(x) = A'(x)$

$z' x^2 + 2z x = 1$

Hom:  $z_H = Ae^{-\int \frac{2x dx}{x^2}} = Ae^{-2 \ln x} = Ax^{-2}$

Sol partic: MVC:  $z_P = Cx^{-2}$  ;  $z_P' = Cx^{-3} - 2Cx^{-3}$

$z_P' x^2 + 2z_P x = Cx^{-1} - 2Cx^{-1} + 2Cx^{-1} = 1 \Rightarrow Cx^{-1} = 1 \Rightarrow Cx = x$

$\Rightarrow z_P = \frac{1}{x}$

Sols de l'eq complète:  $z = Cx^{-2} + x$

On intègre z pour trouver A:  $A(x) = C \int x^{-2} dx + \int x dx$   
 $= -Cx^{-1} + \frac{1}{2}x^2 + B$

$y(x) = \frac{A(x)}{x}$ , donc  $y = \frac{B}{x} - \frac{C}{x^2} + \frac{1}{2}x = \frac{Bx - C + \frac{1}{2}x^2}{x^2}$