

① $y'' - 4y' = 0$ $P(x) = x^2 - 4x = x(x-4)$

2 racines 0 et 4

$$y_H = A + B e^{4x}$$

$y'' - 4y' + 4y = 0$ $P(x) = x^2 - 4x + 4 = (x-2)^2$
2 est racine double $y_H = (Ax + B) e^{2x}$

$y'' - 4y' + 13y = 0$ $P(x) = x^2 - 4x + 13$

$\Delta = 16 - 52 = -36 = (6i)^2$ 2 racines: $\frac{4 \pm 6i}{2} = 2 \pm 3i$

$$y_H = e^{2x} (A \cos(3x) + B \sin(3x))$$

$$\textcircled{2} \quad y'' + y' - 6y = 2e^{3x}$$

• HOM: $y'' + y' - 6y = 0$ $Q(x) = x^2 + x - 6$ $\Delta = 25$
2 racis: $\frac{-1 \pm 5}{2} = \begin{cases} 2 \\ -3 \end{cases}$ Sol: $y_H = Ae^{2x} + Be^{-3x}$ où $(A, B) \in \mathbb{R}^2$

• Sol particulière: $y_p = ce^{3x}$ $y_p' = 3ce^{3x}$ $y_p'' = 9ce^{3x}$

$$y_p'' + y_p' - 6y_p = 9ce^{3x} + 3ce^{3x} - 6ce^{3x} = 6ce^{3x} = 2e^{3x} \Rightarrow 6c = 2$$

$$\Rightarrow c = \frac{1}{3}$$

$$y_p = \frac{1}{3}e^{3x}$$

• Sol de l'équation complète: $y = Ae^{2x} + Be^{-3x} + \frac{1}{3}e^{3x}$ où $(A, B) \in \mathbb{R}^2$

$$(4) \quad y'' - 4y' + 3y = 6x + 1 + 4e^x + 8e^{-x}$$

HOM: $y'' - 4y' + 3y = 0$ $P(x) = x^2 - 4x + 3$ $\Delta = 16 - 12 = 4$

2 racines: $\frac{4 \pm 2}{2} = \begin{cases} 3 \\ 1 \end{cases}$

$$y_H = Ae^{3x} + Be^x$$

Sol particulier pour 6x+1: $y_1 = ax + b$

$$y_1' = a$$

$$y_1'' = 0$$

$$y_1'' - 4y_1' + 3y_1 = -4a + 3(ax + b) = 3ax + 3b - 4a = 6x + 1$$

$$\Rightarrow \begin{cases} 3a = 6 \\ 3b - 4a = 1 \end{cases} \Rightarrow \begin{cases} a = 2 \\ 3b = 1 + 8 = 9 \Rightarrow b = 3 \end{cases}$$

$$y_1 = 2x + 3$$

Sol part pour $4e^x$: $y_2 = cxe^x$

$$y_2' = ce^x + cxe^x$$

$$y_2'' = ce^x + ce^x + cxe^x$$

$$= 2ce^x + cxe^x$$

$$y_2'' - 4y_2' + 3y_2 = 2ce^x + cxe^x - 4(ce^x + cxe^x) + 3cxe^x = 2ce^x + cxe^x - 4ce^x - 4cxe^x + 3cxe^x = -2ce^x = 4e^x \Rightarrow -2c = 4 \Rightarrow c = -2$$

$$y_2 = -2xe^x$$

Sol part pour $8e^{-x}$: $y_3 = de^{-x}$

$$y_3' = -de^{-x}$$

$$y_3'' = de^{-x}$$

$$y_3'' - 4y_3' + 3y_3 = de^{-x} + 4de^{-x} + 3de^{-x} = 8de^{-x} = 8e^{-x} \Rightarrow d = 1$$

$$y_3 = e^{-x}$$

Sol de l'eq complete: (principe de superposition)

$$y = y_H + y_1 + y_2 + y_3$$

$$y = Ae^{3x} + Be^x + 2x + 3 - 2xe^x + e^{-x}$$

$$(13) \quad x^2 y'' - 2xy' + 2y = 4x^3$$

avec $f(x) = x$ $f'(x) = 1$ $f''(x) = 0$

$$x^2 f''(x) - 2x f'(x) + 2f(x) = x^2 \cdot 0 - 2x \cdot 1 + 2x = 0 \quad \text{OK}$$

donc $f(x) = x$ est sol de l'ED

\Rightarrow les Ax sont aussi solutions.

chtr $y = A(x) \cdot x$ $y'(x) = A'(x)x + A(x)$ $y''(x) = A''(x)x + 2A'(x)$

$$\begin{aligned} x^2 y'' - 2xy' + 2y &= A''(x)x^3 + 2A'(x)x^2 - 2A'(x)x^2 - 2A(x)x + 2A(x)x \\ &= A''(x)x^3 = 4x^3 \quad \Rightarrow \quad A''(x) = 4 \end{aligned}$$

$$\Rightarrow A'(x) = 4x + C \quad \Rightarrow \quad A(x) = 2x^2 + Cx + D$$

$$y = 2x^3 + (x^2 + D)x \quad \text{où } C, D \in \mathbb{R}$$

$$\textcircled{1} \quad y'' - 4y' = 0 \quad P(x) = x^2 - 4x = x(x-4)$$

2 racines 0 et 4

$$y_H = A e^{0x} + B e^{4x}$$

$$y_H = A + B e^{4x} \text{ où } A, B \in \mathbb{R}$$

$$y'' - 4y' + 4y = 0 \quad P(x) = x^2 - 4x + 4 = (x-2)^2$$

une racine double: 2

$$y_H = (Ax + B)e^{2x} \text{ où } (A, B) \in \mathbb{R}^2$$

$$y'' - 4y' + 13y = 0$$

2 racines:

$$\frac{4 \pm 6i}{2} = 2 \pm 3i$$

$$\Delta = 16 - 52 = -36 = (6i)^2$$

$$y_H = e^{2x} (A \cos(3x) + B \sin(3x))$$

$$(2) \quad y'' + y' - 6y = \underline{2e^{3x}}$$

Hom: $y'' + y' - 6y = 0$ $P(x) = x^2 + x - 6$ $\Delta = 1 + 24 = 25$

2 racis. $-\frac{-1 \pm 5}{2} = \begin{cases} -3 \\ 2 \end{cases}$

$$y_H = A e^{-3x} + B e^{2x}$$

Sol particulière: $y_p = a e^{3x}$ $y_p' = 3a e^{3x}$ $y_p'' = 9a e^{3x}$

$$y_p'' + y_p' - 6y_p = 9a e^{3x} + 3a e^{3x} - 6a e^{3x} = 6a e^{3x} = \underline{2e^{3x}}$$

$$\Rightarrow 6a = 2 \quad \Rightarrow \quad a = \frac{2}{6} = \frac{1}{3}$$

$$y_p = \frac{1}{3} e^{3x}$$

Solu de l'éq complète:

$$y = A e^{-3x} + B e^{2x} + \frac{1}{3} e^{3x} \quad \text{où } (A, B) \in \mathbb{R}^2$$

$$(4) \quad y'' - 4y' + 3y = 6x + 1 + 4e^x + 8e^{-x}$$

HOM $\cdot y'' - 4y' + 3y = 0$ $P(x) = x^2 - 4x + 3$ $\Delta = 16 - 12 = 4$

2 racines: $\frac{4 \pm 2}{2} = \begin{matrix} 1 \\ 3 \end{matrix}$

$$y_H = Ae^x + Be^{3x}$$

Sol part pour le second membre $6x+1$: $y_1 = ax + b$ $y_1' = a$ $y_1'' = 0$

$$y_1'' - 4y_1' + 3y_1 = 0 - 4a + 3(ax + b) = 3ax + 3b - 4a = 6x + 1$$

$$\Rightarrow \begin{cases} 3a = 6 \\ 3b - 4a = 1 \end{cases} \Rightarrow a = 2 \Rightarrow 3b - 8 = 1 \Rightarrow 3b = 9 \Rightarrow b = 3$$

$$y_1 = 2x + 3$$

Sol part pour $4e^x$: $y_2 = axe^x$ $y_2' = ae^x + axe^x$ $y_2'' = ae^x + ae^x + axe^x = 2ae^x + axe^x$

$$y_2'' - 4y_2' + 3y_2 = (2ae^x + axe^x) - 4(ae^x + axe^x) + 3axe^x = -2ae^x = 4e^x \Rightarrow -2a = 4 \Rightarrow a = -2$$

$$y_2 = -2xe^x$$

Sol part pour $8e^{-x}$ $y_3 = ae^{-x}$ $y_3' = -ae^{-x}$ $y_3'' = ae^{-x}$

$$y_3'' - 4y_3' + 3y_3 = ae^{-x} + 4ae^{-x} + 3ae^{-x} = 8ae^{-x} = 8e^{-x} \Rightarrow a = 1$$

$$y_3 = e^{-x}$$

Sol de l'équation complète: (principe de superposition) $y = Ae^x + Be^{3x} + 2x + 3 - 2xe^x + e^{-x}$

$$(13) \quad x^2 y'' - 2xy' + 2y = 4x^3$$

$$f(x) = x \quad f'(x) = 1 \quad f''(x) = 0$$

$$x^2 f''(x) - 2x f'(x) + 2f(x) = 0 - 2x \cdot 1 + 2x = 0. \text{ donc } f \text{ est solution de HOM}$$

donc tous les Ax sont aussi solution

on fait varier la constante A . C'est à dire on pose $y = A(x)x$

$$y' = A'(x)x + A(x) \cdot 1 = A'(x)x + A(x), \quad y'' = A''(x)x + A'(x) \cdot 1 + A'(x) = A''(x)x + 2A'(x)$$

$$\begin{aligned} x^2 y'' - 2xy' + 2y &= x^2 (A''(x)x + 2A'(x)) - 2x (A'(x)x + A(x)) + 2A(x)x \\ &= A''(x)x^3 + 2x^2 A'(x) - 2x^2 A'(x) - 2x A(x) + 2A(x)x \end{aligned}$$

$$A''(x)x^3 = 4x^3 \Rightarrow A''(x) = 4 \Rightarrow A'(x) = 4x + C \Rightarrow \underline{A = 2x^2 + (x+D)}$$

$$\Rightarrow \boxed{y = 2x^3 + (x^2 + D)x \quad \text{où } (C, D) \in \mathbb{R}^2}$$