

$$(5a) \quad 2xy' + y = x^{3/2}$$

Hom. $2xy' + y = 0$

$$y_H = Ae^{-\int \frac{1}{2x} dx} = Ae^{-\frac{1}{2} \int \frac{dx}{x}} = Ae^{-\frac{1}{2} \ln x}$$
$$= Ae^{\ln(x^{-1/2})} = \boxed{Ax^{-1/2} = y_H} = \frac{A}{\sqrt{x}}$$

Sol particulière MVC $y_p = A\alpha x^{-1/2}$ $y_p' = A\alpha x^{-1/2} - \frac{1}{2} A\alpha x^{-3/2}$

$$2xy_p' + y_p = 2x A\alpha x^{-1/2} - 2x \frac{1}{2} A\alpha x^{-3/2} + A\alpha x^{-1/2}$$

$$= 2A\alpha x^{1/2} - A\alpha x^{-1/2} + A\alpha x^{-1/2} = 2A\alpha x^{1/2} = x^{3/2}$$

$$\Rightarrow A\alpha = \frac{x^{3/2}}{2x^{1/2}} = \frac{x^{3/2-1/2}}{2} = \frac{x}{2} \Rightarrow A\alpha = \int \frac{x}{2} dx = \frac{1}{2} \int x dx = \frac{x^2}{4}$$

$$\Rightarrow y_p = A\alpha x^{-1/2} = \frac{x^2}{4} x^{-1/2} = \boxed{\frac{x^{3/2}}{4} = y_p}$$

Sols de l'eq complète :

$$\boxed{y = Ax^{-1/2} + \frac{x^{3/2}}{4} \text{ ou } A \in \mathbb{R}}$$