

resol

①  $y' + 2y = f(x)$       HOM:  $y_H = A e^{-2x}$       où  $A \in \mathbb{R}$

• Si  $f(x) = x^2 + x + 1$ , alors une sol part est de la forme:  $y_p = ax^2 + bx + c$

ici,  
je  
préfère  
la  
M  
V  
C

• Si  $f(x) = e^x(x+1)$ , ...  $y_p = (ax+b)e^x$

• Si  $f(x) = x e^x$  ...  $y_p = (ax+b)e^x$

• Si  $f(x) = e^{-2x}(x+1)$  ...  $y_p = x(ax+b)e^{-2x} = (ax^2+bx)e^{-2x}$

• Si  $f(x) = e^x$  ...  $y_p = ae^x$

• Si  $f(x) = (x^2+x+1)e^{-2x}$  ...  $y_p = x(ax^2+bx+c)e^{-2x} = (ax^3+bx^2+cx)e^{-2x}$

• Si  $f(x) = 3\sin x + 4\cos x$  ...  $y_p = a\sin x + b\cos x$

• Si  $f(x) = 2\sin x$  ...  $y_p = a\sin x + b\cos x$

HOBT

$$y' + 2y = e^{-2x}(x+1)$$

$$y_H = A e^{-2x} \text{ où } A \in \mathbb{R}$$

Sol particulière :

première méthode :  $y_P = (ax^2 + bx) e^{-2x}$

$$y_P' = (2ax + b) e^{-2x} - 2(ax^2 + bx) e^{-2x}$$

$$y_P' + 2y_P = e^{-2x}(x+1)$$

$$(u \cdot v)' = u'v + uv'$$

$$\Leftrightarrow \underbrace{(2ax + b) e^{-2x}}_{y_P'} - \underbrace{2(ax^2 + bx) e^{-2x}}_{+ 2y_P} + 2(ax^2 + bx) e^{-2x} = e^{-2x}(x+1)$$

$$(2ax + b) e^{-2x} = e^{-2x}(x+1) \rightarrow \begin{cases} 2a = 1 \\ b = 1 \end{cases} \Rightarrow \begin{matrix} a = 1/2 \\ b = 1 \end{matrix}$$

$$y_P = \left(\frac{1}{2}x^2 + x\right) e^{-2x}$$

Sols de l'éq complète :  $y = A e^{-2x} + \left(\frac{1}{2}x^2 + x\right) e^{-2x}$  où  $A \in \mathbb{R}$

$$y = \left(A + \frac{1}{2}x^2 + x\right) e^{-2x}$$

2<sup>ème</sup> méthode avec la M.V.C. (c'est aussi l'exo 3)

$$\textcircled{3} \quad y' + 2y = e^{-2x}(x+1)$$

HOM:  $y_H = A e^{-2x}$  où  $A \in \mathbb{R}$

Sol part: M.V.C. On pose  $y_p = A(x) e^{-2x}$

$$y_p' = A'(x) e^{-2x} - 2A(x) e^{-2x}$$

$$\begin{aligned} y_p' + 2y_p &= A'(x) e^{-2x} - 2A(x) e^{-2x} + 2A(x) e^{-2x} \\ &= (x+1) e^{-2x} \Rightarrow A'(x) = x+1 \Rightarrow A(x) = \int (x+1) dx = \frac{x^2}{2} + x \end{aligned}$$

donc  $y_p = \left(\frac{x^2}{2} + x\right) e^{-2x}$

Sol de l'eq complète:  $y = A e^{-2x} + \left(\frac{x^2}{2} + x\right) e^{-2x}$

$$y = \left(A + \frac{x^2}{2} + x\right) e^{-2x} \quad \text{où } A \in \mathbb{R}$$

$$(2) \quad y' + 2y = 3 \sin x + 4 \cos x$$

• HOM :  $y' + 2y = 0$        $y_H = \underline{A e^{-2x}}$     où  $A \in \mathbb{R}$

• Sol part : nous le pose  $y_2 = a \sin x + b \cos x$

$$y_2' = a \cos x - b \sin x$$

$$\begin{aligned} y_2' + 2y_2 &= a \cos x - b \sin x + 2a \sin x + 2b \cos x = (2a - b) \sin x + (2b + a) \cos x \\ &= 3 \sin x + 4 \cos x \end{aligned}$$

$$\begin{cases} 2a - b = 3 \\ b + a = 4 \end{cases} \Leftrightarrow \times 2 \begin{cases} 4a - 2b = 6 \\ b + a = 4 \end{cases} \Leftrightarrow \begin{matrix} L_1 + L_2 \\ \end{matrix} \begin{cases} 5a = 10 \\ b + a = 4 \end{cases}$$

$$\Leftrightarrow \begin{cases} a = 2 \\ b = 4 - a = 2 \end{cases} \Leftrightarrow \begin{cases} a = 2 \\ b = 1 \end{cases} \quad \underline{y_2 = 2 \sin x + \cos x}$$

• Sols de l'eq complète :  $\boxed{y = A e^{-2x} + 2 \sin x + \cos x}$     où  $A \in \mathbb{R}$

• Le sol tel  $y(0) = 4$  :  $y(0) = A \cdot 1 + 2 \cdot 0 + 1 = 4 \Rightarrow A = 3$

$$\boxed{y = 3e^{-2x} + 2 \sin x + \cos x}$$

5a  $2x y' + y = x^{3/2}$

Hom:  $2x y' + y = 0$

$a \ln(b) = \ln(b^a)$

$$y_H = A e^{-\int \frac{1}{2x} dx} = A e^{-\frac{1}{2} \int \frac{1}{x} dx} = A e^{-\frac{1}{2} \ln x}$$

$$y_H = A e^{\ln(x^{-1/2})}$$

$$y_H = A x^{-1/2} \quad \text{où } A \in \mathbb{R}$$

Sol part: MVC

$$y_p = A(x) x^{-1/2}$$

$$y_p' = A'(x) x^{-1/2} + A(x) \cdot \left[-\frac{1}{2} x^{-3/2}\right]$$

$$2x y_p' + y_p = 2x \left( A'(x) x^{-1/2} - \frac{1}{2} A(x) x^{-3/2} \right) + A(x) x^{-1/2}$$

$$= 2 A'(x) x^{1/2} - \cancel{A(x) x^{-1/2}} + \cancel{A(x) x^{1/2}} = x^{3/2}$$

$$2 A'(x) x^{1/2} = x^{3/2} \Rightarrow A'(x) = \frac{1}{2} \frac{x^{3/2}}{x^{1/2}} = \frac{1}{2} x^{3/2-1/2} = \frac{1}{2} x$$

$$A'(x) = \frac{1}{2} x \Rightarrow A(x) = \int \frac{1}{2} x dx = \frac{1}{2} \int x dx = \frac{1}{2} \frac{x^2}{2} = \frac{x^2}{4} = A$$

$$y_p = \frac{x^2}{4} \cdot x^{-1/2} = \frac{x^{3/2}}{4} = y_p$$

Sols de l'éq complète :

$$y = A x^{-1/2} + \frac{x^{3/2}}{4}$$

4a  $y' + 2y = 4x + (4x-11)e^{2x}$

• HOM  $y' + 2y = 0$   $y_H = Ae^{-2x}$  où  $A \in \mathbb{R}$

• Sol part pour l'éq  $y' + 2y = 4x$   $y_1 = ax + b$   $y_1' = a$   
 $y_1' + 2y_1 = a + 2(ax + b) = 2ax + a + 2b = 4x$   $\Rightarrow \begin{cases} 2a = 4 \\ a + 2b = 0 \end{cases} \Rightarrow \begin{cases} a = 2 \\ b = -1 \end{cases}$

$y_1 = 2x - 1$

• Sol part pour  $y' + 2y = (4x-11)e^{2x}$ :  $y_2 = (ax+b)e^{2x}$  ou MVC  
 $y_2' = a e^{2x} + 2(ax+b)e^{2x}$   
 $y_2' + 2y_2 = a e^{2x} + (2ax+2b)e^{2x} + (2ax+2b)e^{2x} = e^{2x}(4ax + a + 4b)$   
 $= (4x-11)e^{2x}$   $\begin{cases} 4a = 4 \\ a + 4b = -11 \end{cases} \Leftrightarrow \begin{cases} a = 1 \\ 4b = -12 \end{cases} \Rightarrow \begin{cases} a = 1 \\ b = -3 \end{cases}$

$y_2 = (x-3)e^{2x}$

• Sols de l'éq complète:  $y = y_H + y_1 + y_2$  Principe de Superposition  
 $y = Ae^{-2x} + 2x - 1 + (x-3)e^{2x}$