

$$\begin{array}{r|l} 2268 & 2 \\ 1134 & 2 \\ 567 & 3 \\ 189 & 3 \\ 63 & 3 \\ 21 & 3 \\ 7 & 7 \\ 1 & \end{array}$$

$$\boxed{2268 = 2^2 \cdot 3^4 \cdot 7}$$

$$(2) \quad a = \frac{1}{3 + \frac{1}{1 + \frac{1}{2+a}}} \Leftrightarrow a = \frac{1}{3 + \frac{1}{\frac{3+a}{2+a}}} \Leftrightarrow a = \frac{1}{3 + \frac{2+a}{3+a}}$$

$$\Leftrightarrow a = \frac{1}{\frac{9+3a+2+a}{3+a}} = \frac{3+a}{4a+11} \Leftrightarrow 4a^2+11a = a+3 \Leftrightarrow \boxed{4a^2+10a-3=0}$$

$$\Delta = 100 + 16 \times 3 = 148 = 4 \times 37 = 2^2 \times 37 = (2\sqrt{37})^2$$

$$a = \frac{10 \pm 2\sqrt{37}}{8} \quad a > 0, \text{ on garde donc } \boxed{a = \frac{5 + \sqrt{37}}{4}}$$

$$(3) \quad \mathcal{L}(1) = 1 - 2 + 20 = 0, \text{ donc } 1 \text{ est racine de } \mathcal{L}$$

$$\begin{array}{r|l} X^3 - 21X + 20 & X-1 \\ \hline -(X^3 - X^2) & \\ \hline X^2 - 21X + 20 & \\ \hline -(X^2 - X) & \\ \hline -20X + 20 & \\ \hline -(-20X + 20) & \\ \hline 0 & \end{array} \quad X^2 + X - 20 = \mathcal{Q}(X)$$

$$\Delta = 1 + 80 = 81 = 9^2$$

$$\text{deux autres racines : } \frac{-1 \pm 9}{2} = \begin{cases} 4 \\ -5 \end{cases}$$

$$P(X) = (X-1)(X-4)(X+5)$$

$$\begin{aligned} P(0) &= 20 > 0 \\ \lim_{x \rightarrow -\infty} \mathcal{L}(x) &= \lim_{x \rightarrow -\infty} x^3 = -\infty \\ \lim_{x \rightarrow +\infty} \mathcal{L}(x) &= \lim_{x \rightarrow +\infty} x^3 = +\infty \\ P(2) &= 8 - 42 + 20 < 0 \end{aligned}$$

x	$-\infty$	-5	0	1	2	4	$+\infty$
signe de $\mathcal{L}(x)$	-	0	+	0	-	0	+

$$\text{donc } P(X) > 0, \text{ donc } \boxed{D_P =]-5; 1[\cup]4; +\infty[}$$

(4) le tronc de fraction impose $x \neq 3$

$$\frac{-6}{x-3} < x+1 \Leftrightarrow 0 < x+1 + \frac{6}{x-3}$$

$$\Leftrightarrow 0 < \frac{(x+1)(x-3)+6}{x-3}$$

$$\Leftrightarrow 0 < \frac{x^2 - 2x + 3}{x-3}$$

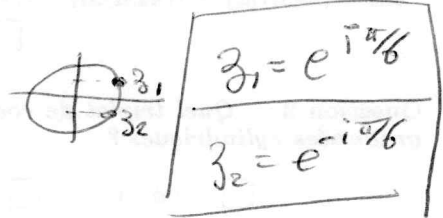
On appelle f la fonction $\frac{x^2 - 2x + 3}{x-3}$. On veut $0 < f(x)$

$$\Delta = 4 - 12 < 0, \text{ donc } x^2 - 2x + 3 > 0$$

f est du signe de $x-3$, donc > 0 pour $x > 3$

$$P =]3; +\infty[$$

(5) $|z_1| = |z_2| = 1$ $\left(\sqrt{\frac{3}{4}} + \frac{1}{4}\right)$



$$\begin{cases} z_1 = e^{i\pi/6} \\ z_2 = e^{-i\pi/6} \end{cases}$$

$$z_1 + z_2 = e^{i\pi/6} + e^{-i\pi/6} = 2\cos\left(\frac{\pi}{6}\right) = 2 \times \frac{\sqrt{3}}{2} = \sqrt{3}$$

$$z_1^2 + z_2^2 = e^{i\pi/3} + e^{-i\pi/3} = 2\cos\left(\frac{\pi}{3}\right) = 2 \times \frac{1}{2} = 1$$

$$z_1^3 + z_2^3 = e^{i\pi/2} + e^{-i\pi/2} = i - i = 0$$

$$(z_1)^{2023} = e^{i\frac{\pi}{6} \times 2023}$$

$$\begin{array}{r} 2023 \quad | \quad 6 \\ -18 \quad | \quad 337 \\ \hline 22 \\ -18 \\ \hline 43 \\ -42 \\ \hline 1 \end{array}$$

$$2023 = 6 \times 337 + 1 \Rightarrow \frac{2023\pi}{6} = 337\pi + \frac{\pi}{6}$$

$$z_1^{2023} = e^{i\left(337\pi + \frac{\pi}{6}\right)} = e^{i\left(\pi + \frac{\pi}{6}\right)} = -e^{i\pi/6}$$

$$z_1^{2023} = -\frac{\sqrt{3}}{2} - \frac{i}{2}$$

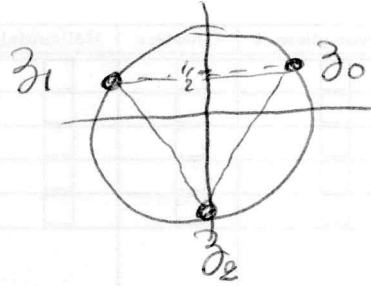
⑥ Racines cubiques de $i = e^{i\pi/2}$

les racines cubiques de $e^{i\pi/2}$ sont les $e^{i\left(\frac{\pi/2}{3} + \frac{2k\pi}{3}\right)} = e^{i\left(\frac{\pi}{6} + \frac{2k\pi}{3}\right)}$ où $k=0, 1$ ou 2

$$z_0 = e^{i\pi/6}$$

$$z_1 = e^{i\left(\frac{\pi}{6} + \frac{2\pi}{3}\right)} = e^{i\frac{5\pi}{6}}$$

$$z_2 = e^{i\left(\frac{\pi}{6} + \frac{4\pi}{3}\right)} = e^{i\frac{9\pi}{6}} = e^{i\frac{3\pi}{2}} = -i$$



on voit apparaître le triangle équilatéral

Les Racines 6^{èmes} de $-8 = 8e^{i\pi}$ sont les

$$z_k = 8^{1/6} e^{i\left(\frac{\pi}{6} + \frac{2k\pi}{6}\right)} = \sqrt{2} e^{i\left(\frac{\pi}{6} + \frac{2k\pi}{6}\right)} \quad \text{car } 8 = 2^3$$

$$z_0 = \sqrt{2} e^{i\pi/6}$$

$$z_1 = \sqrt{2} e^{i3\pi/6} = \sqrt{2} e^{i\pi/2} = \sqrt{2} i$$

$$z_2 = \sqrt{2} e^{i5\pi/6}$$

$$z_3 = \sqrt{2} e^{i7\pi/6} = \sqrt{2} e^{-i5\pi/6}$$

$$z_4 = \sqrt{2} e^{i9\pi/6} = \sqrt{2} e^{i3\pi/2} = -\sqrt{2} i$$

$$z_5 = \sqrt{2} e^{i11\pi/6} = \sqrt{2} e^{-i\pi/6}$$

⑦ $P(x) = i x^2 + (-4+3i)x - 5-i$

$$\Delta = (4+3i)^2 + 4i(5+i) = 16 - 24i - 9 + 20i - 4 = 3 - 4i = (a+ib)^2$$

$$a^2 - b^2 = 3$$

$$2ab = -4$$

$$a^2 + b^2 = \sqrt{9+16} = 5$$

$$L_3 + L_1: 2a^2 = 8 \Rightarrow a^2 = 4 \Rightarrow a = \pm 2$$

$$L_3 - L_1: 2b^2 = 2 \Rightarrow b^2 = 1 \Rightarrow b = \pm 1$$

L_2 : a et b ont des signes opposés.

$$\text{donc } \sqrt{\Delta} = \pm (2-i)$$

les deux racines sont:
$$\frac{4-3i \pm (2-i)}{2i} = \left/ \begin{array}{l} \frac{6-4i}{2i} = \frac{3-2i}{i} = \frac{3i+2}{-1} = -2-3i \\ \frac{2-2i}{2i} = \frac{1-i}{i} = \frac{i+1}{-1} = -1-i \end{array} \right.$$

$$P(x) = i (x+1+i)(x+2+3i)$$

$$(8) \sum_{k=0}^{n-1} R e^{i k \left(\frac{\theta}{n} + \frac{2k\pi}{n} \right)} = \sum_{k=0}^{n-1} R e^{i \frac{\theta}{n}} e^{i \frac{2k\pi}{n}}$$

$$= R e^{i \frac{\theta}{n}} \sum_{k=0}^{n-1} e^{i \frac{2k\pi}{n}} \quad \text{on a sorti les constantes}$$

$$= R e^{i \frac{\theta}{n}} \sum_{k=0}^{n-1} \left(e^{i \frac{2\pi}{n}} \right)^k \quad \text{on reconnaît la somme géométrique de raison } e^{i \frac{2\pi}{n}}$$

$$= R e^{i \frac{\theta}{n}} \frac{\left(e^{i \frac{2\pi}{n}} \right)^n - 1}{e^{i \frac{2\pi}{n}} - 1} = R e^{i \frac{\theta}{n}} \frac{e^{i 2\pi} - 1}{e^{i \frac{2\pi}{n}} - 1} = 0$$

$$(9) xy' + y = \ln(x)$$

$$\text{MOT: } xy' + y = 0$$

$$y = A e^{-\int \frac{1}{x} dx} = A e^{-\ln x} = A e^{\ln(x^{-1})} = \frac{A}{x}$$

$$\boxed{y_H = \frac{A}{x} \text{ où } A \in \mathbb{R}}$$

$$\text{Solution particulière: M.V.C. } y_P = \frac{A(x)}{x} \quad y'_P = \frac{A'(x)x - A(x)}{x^2}$$

$$xy'_P + y_P = \frac{A'(x)x^2 - A(x)x}{x^2} + \frac{A(x)}{x} = A'(x) = \ln(x)$$

$$\Rightarrow A(x) = \int \ln(x) dx = \int 1 \ln(x) dx \quad \text{I.P.P. } \begin{cases} u = \ln(x) & u' = \frac{1}{x} \\ v' = 1 & v = x \end{cases}$$

$$= [x \ln(x)] - \int x \frac{1}{x} dx = x \ln(x) - x$$

$$\Rightarrow \boxed{y_P = \ln(x) - 1}$$

$$\text{Sols de l'eq complète: } \boxed{y = \frac{A}{x} + \ln(x) - 1 \text{ où } A \in \mathbb{R}}$$

⑩ $y' + y = 2e^{-x}$ $f(0) = 3$

H.M.T: $y_H = Ae^{-x}$ où $A \in \mathbb{R}$

Sol particulière de la forme $y_p = axe^{-x}$; $y_p' = ae^{-x} - axe^{-x}$

$y_p' + y_p = ae^{-x} - axe^{-x} + axe^{-x} = 2e^{-x} \Rightarrow a = 2$ $y_p = 2xe^{-x}$

Sols de l'éq complète $y = Ae^{-x} + 2xe^{-x}$ où $A \in \mathbb{R}$

La sol telle que $f(0) = 3$: $f(0) = A \Rightarrow A = 3$ $f(x) = 3e^{-x} + 2xe^{-x}$

$f(x) = (3 + 2x)e^{-x}$

⑪ $y'' + 2y' + y = x$ $f(0) = f'(0) = 0$

H.M.T: $X^2 + 2X + 1 = (X+1)^2$ -1 est racine double

$y_H = (Ax+B)e^{-x}$ où $A, B \in \mathbb{R}$

Sol particulière : $y_p = ax + b$ $y_p' = a$ $y_p'' = 0$

$y_p'' + 2y_p' + y_p = 0 + 2a + ax + b = ax + 2a + b = 1x + 0$

$\begin{cases} a = 1 \\ 2a + b = 0 \end{cases} \Leftrightarrow \begin{cases} a = 1 \\ b = -2 \end{cases}$

$y_p = (Ax+B)e^{-x} + x - 2$

Sols de l'éq complète $y = (Ax+B)e^{-x} + x - 2$

La solution f telle que $f(0) = f'(0) = 0$: $f(x) = (Ax+B)e^{-x} + x - 2$

$f(0) = B - 2 = 0 \Rightarrow B = 2$ $f'(0) = A + (Ax+B)e^{-x} + 1$

$f'(0) = A + 2 + 1 = 0 \Rightarrow A = -3$

$f(x) = (-3x+2)e^{-x} + x - 2$

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$$y'' - 4y' + 13y = 0$$

C'est une équation homogène

$$P(x) = x^2 - 4x + 13$$

$$\Delta = 16 - 4 \times 13 = 16 - 52 = -36 = (6i)^2$$

$$\text{deux racines : } \frac{4 \pm 6i}{2} = 2 \pm 3i$$

$$y = e^{2x} (A \cos(3x) + B \sin(3x)) \text{ où } A, B \in \mathbb{R}$$