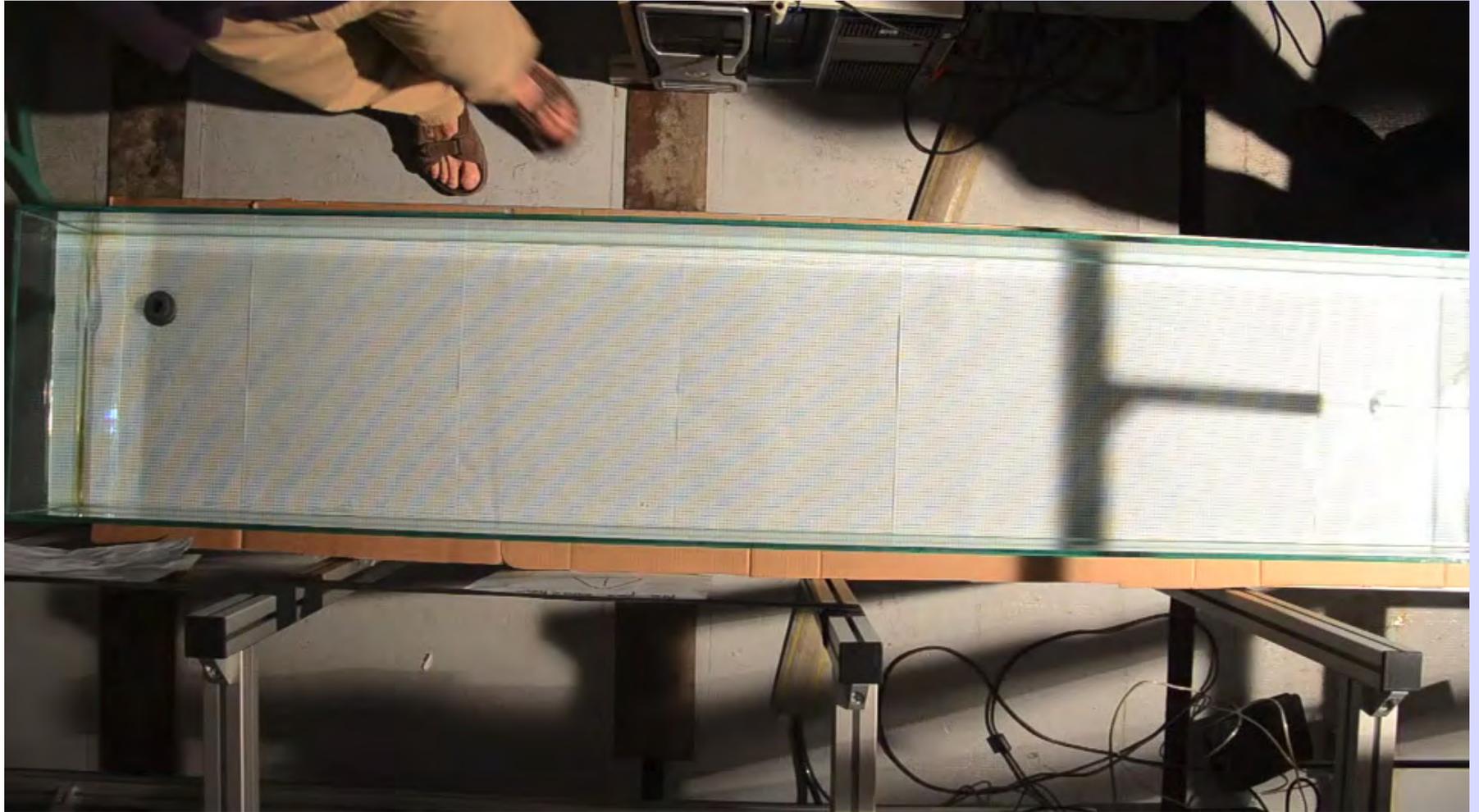


# Cours 3 : Sillage d'un obstacle

1 – Sillage d'un petit objet

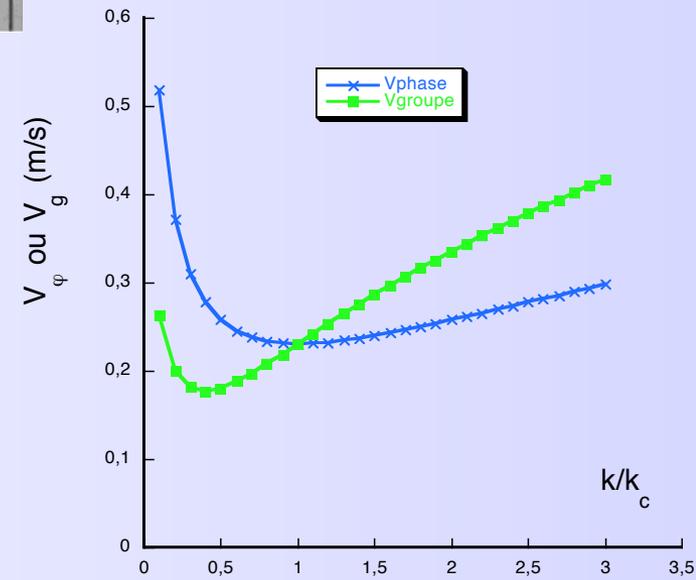
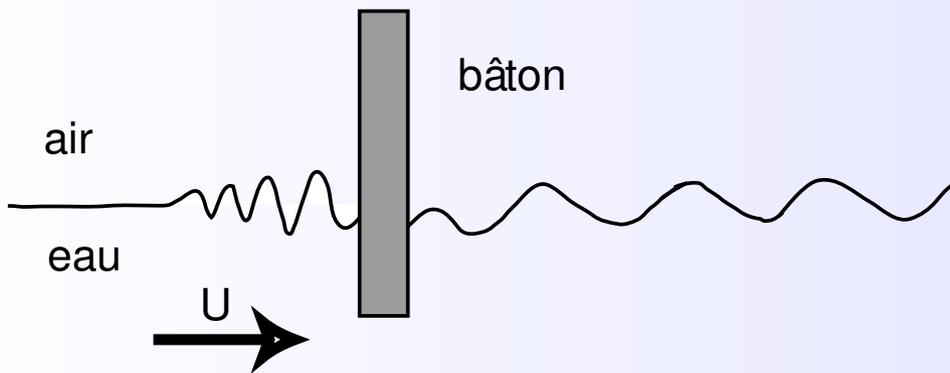
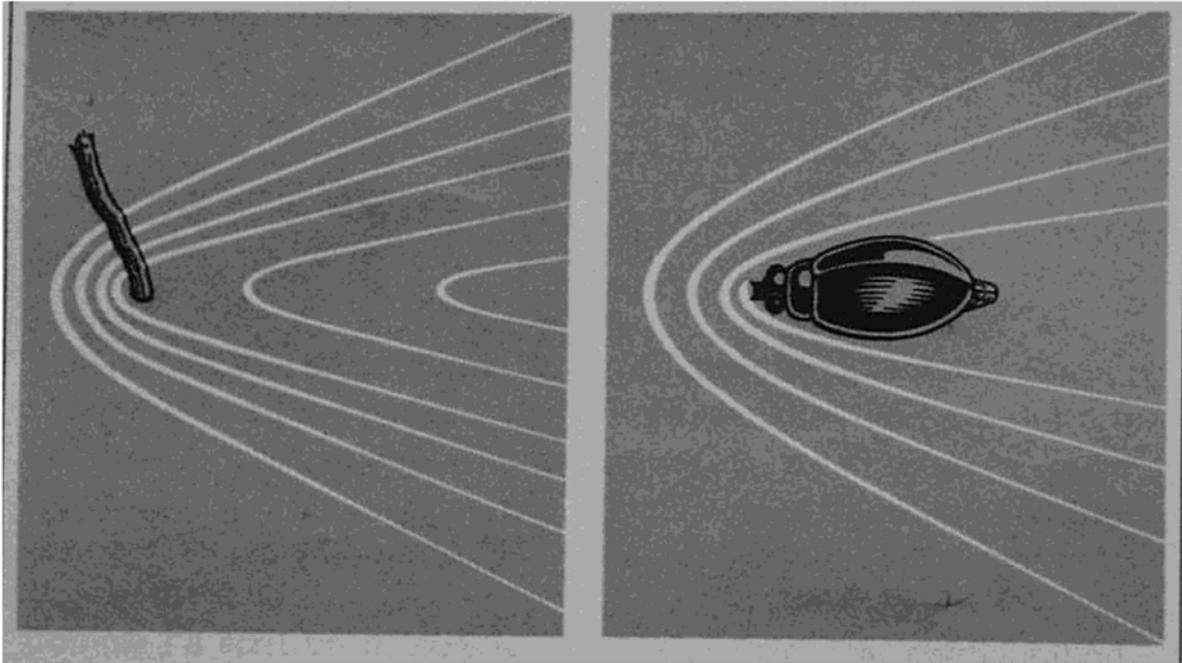
(cas capillaire)

# Close to minimum velocity



1,5 mm cylinder moving at 22 cm/s

# 5 – Sillage des bateaux ou d'un obstacle



Atténuation des ondes :

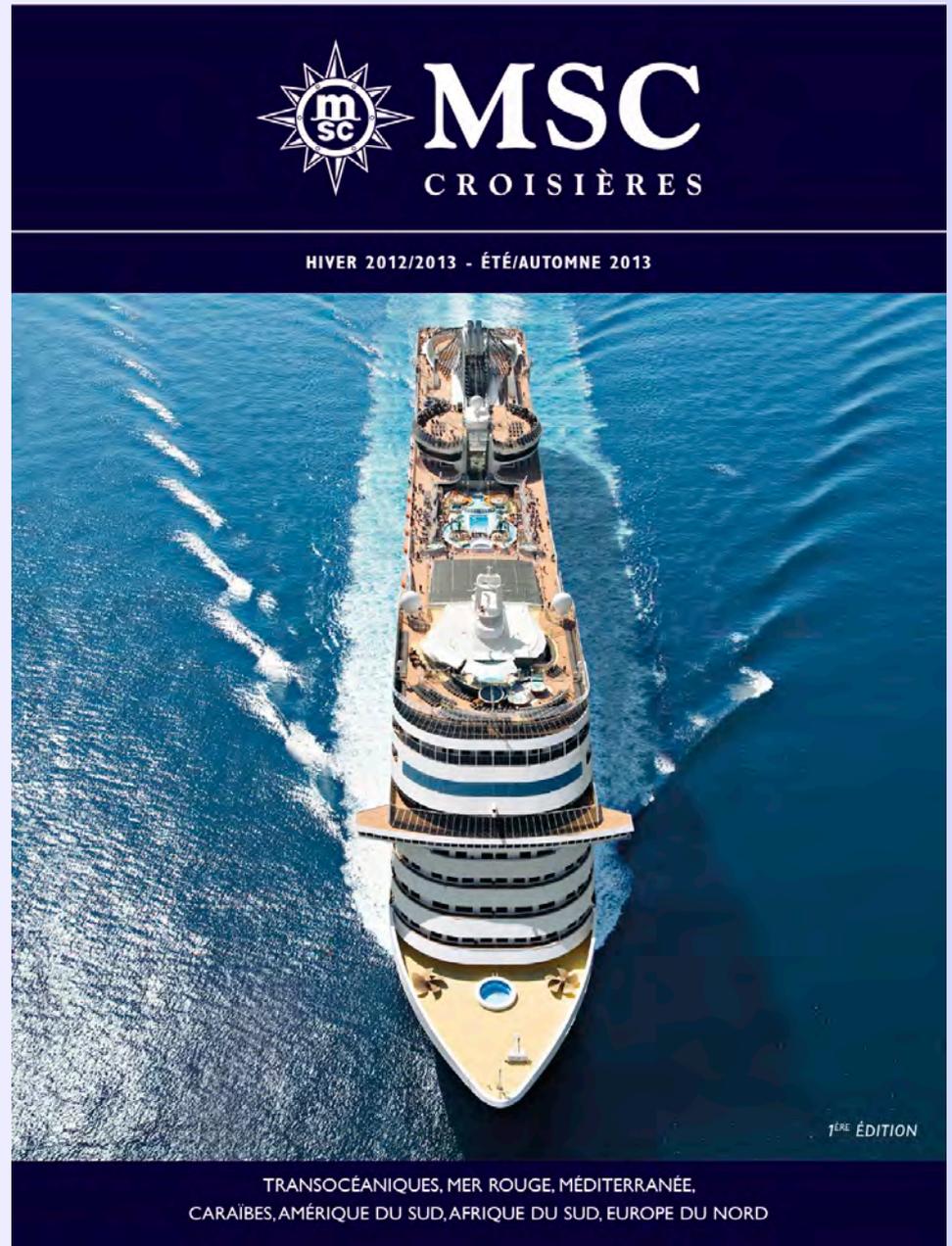
$$\zeta(t) = \zeta_0 \exp(-2\nu k^2 t).$$

Dans l'approximation linéaire :

$\lambda$ (m)	L (m)	$\tau$
1 mm	12 mm	0,01 s
1 cm	30 cm	1 s
1 m	23 km	3,5 heures
10 m	7500 km	14 jours
100 m	60 fois le tour de la Terre !	4 ans !

## 2 – Sillage d'un navire

(cas gravitaire)

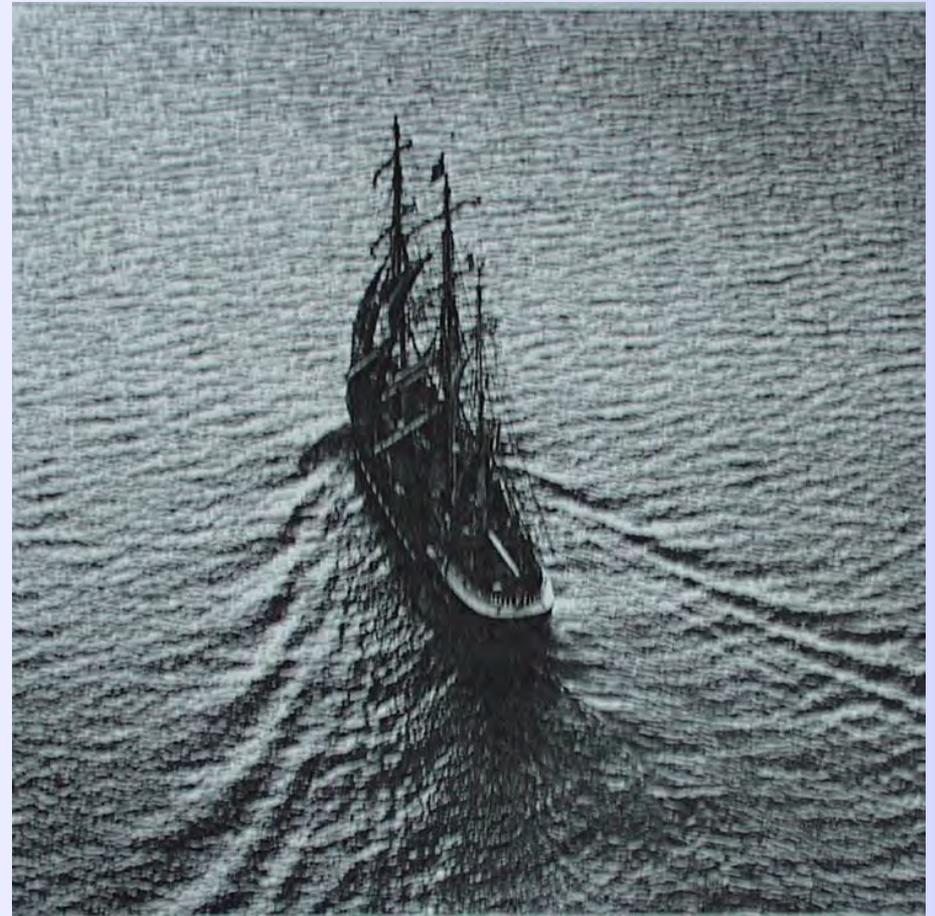


# Sillage de Kelvin





*Sillage de Kelvin  
à  $19^{\circ}28'$*

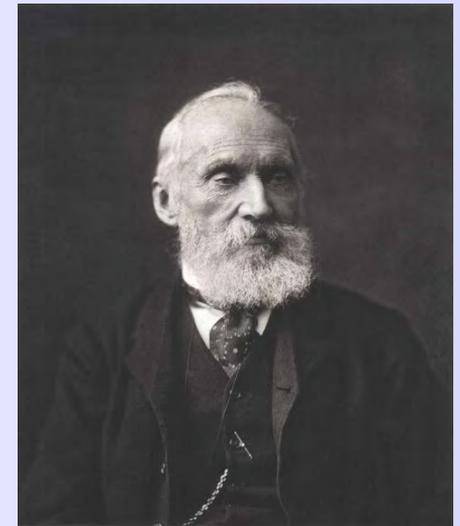




# Sillage de Kelvin

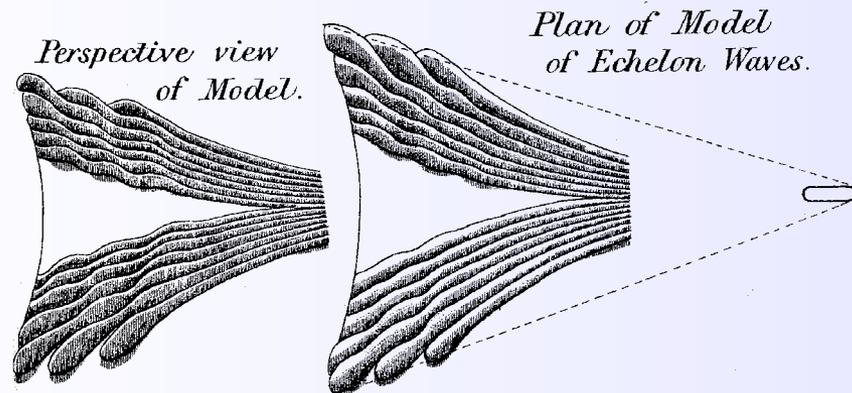
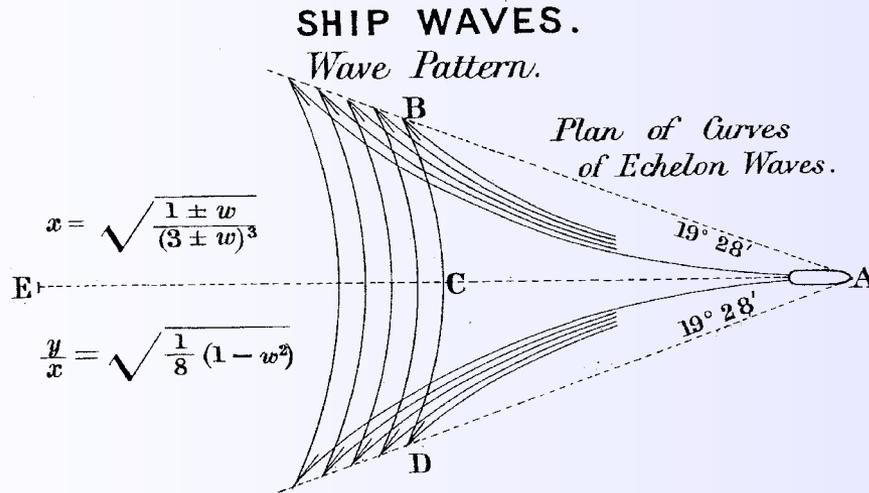


Lalla Rookh



Lord Kelvin  
(William  
Thomson)  
1824-1907

# Sillage de Kelvin expliqué en 1887

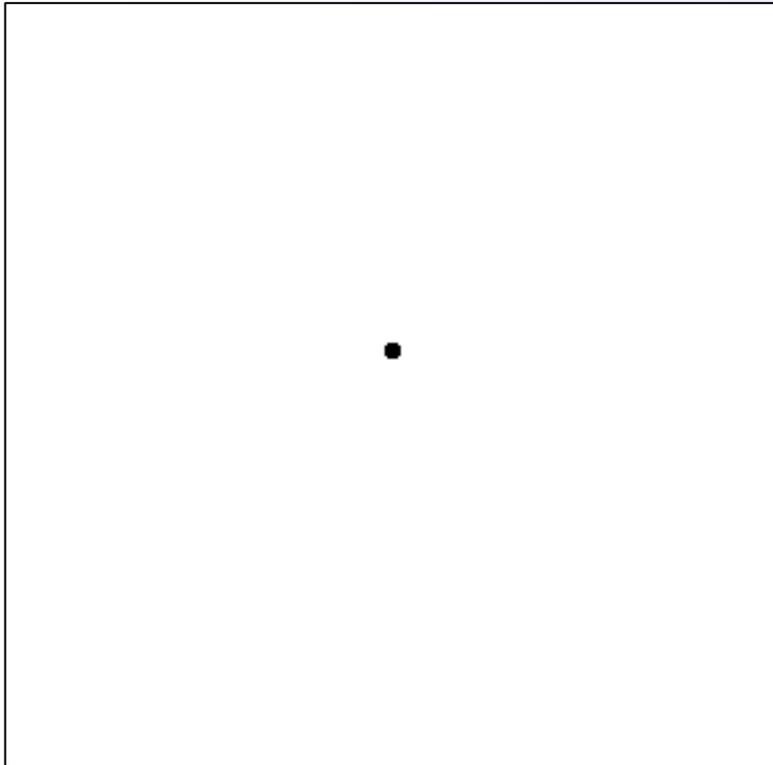


$$\alpha = 19.47^{\circ}$$

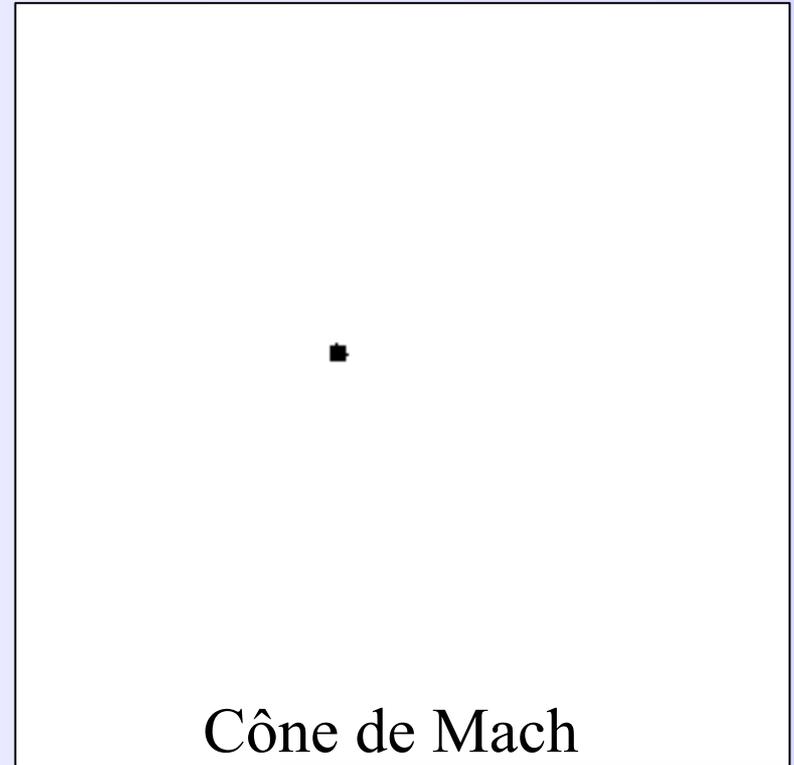
$\Delta$  la vitesse  $U$   
 $\Delta$  la taille  $L$

# Cas d'un avion supersonique

Vitesse du son  $c = \text{Cste}$

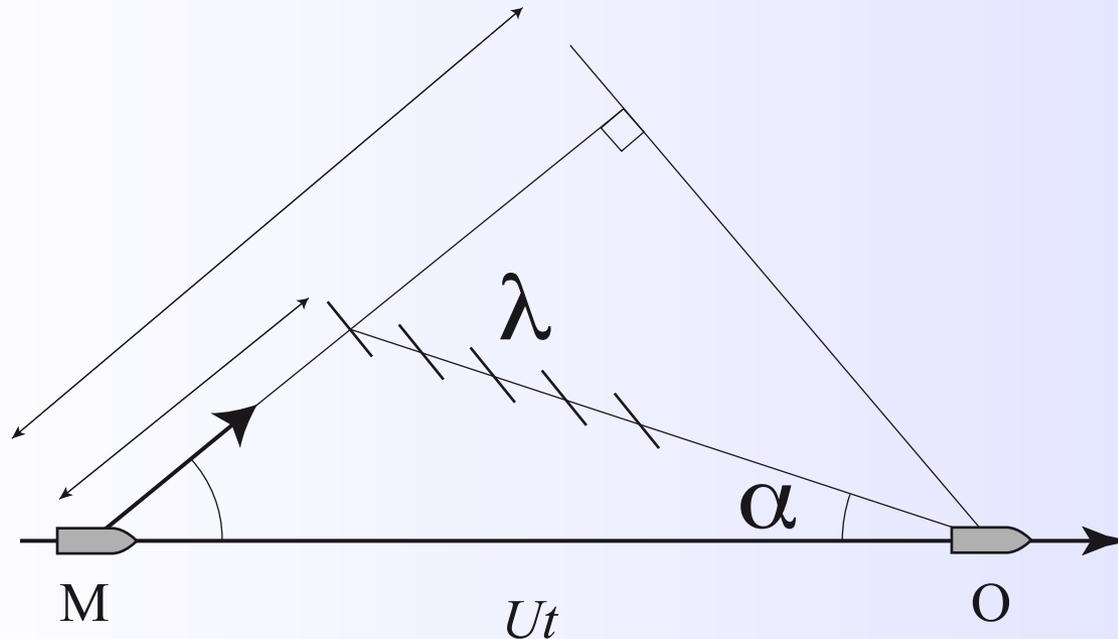


A l'arrêt



En vol supersonique

# Cas des ondes de gravité (eaux profondes)



Milieu dispersif

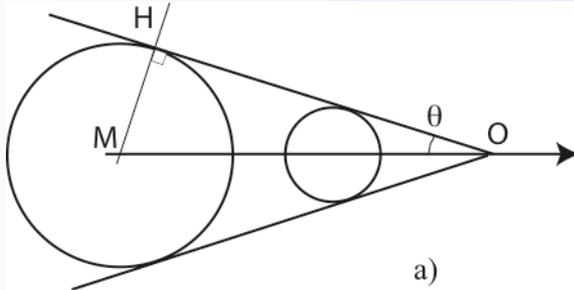
$$\omega^2 = gk$$

$$V_{\varphi}(k) = \frac{\omega}{k} = \sqrt{\frac{g}{k}}$$

$$V_g(k) = \frac{1}{2} V_{\varphi}(k)$$

$\Rightarrow \alpha(\lambda)$  présente un maximum pour  $\alpha_K = 19,47^\circ$

# Mach construction



Mach cone ( $U > c$ )

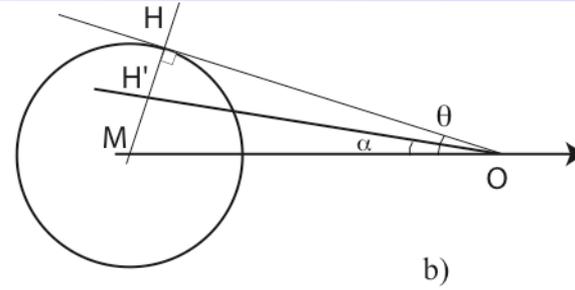
Non dispersive media

$$c = V_g = V_\varphi$$

$$\sin \theta = \frac{c}{U}$$

# Kelvin wedge

For a given  $k$



Kelvin wedge (any  $U$ )

Dispersive media  $V_g = \frac{1}{2} V_\varphi$

$$\tan \alpha = \frac{\tan \theta}{2 + \tan^2 \theta}$$

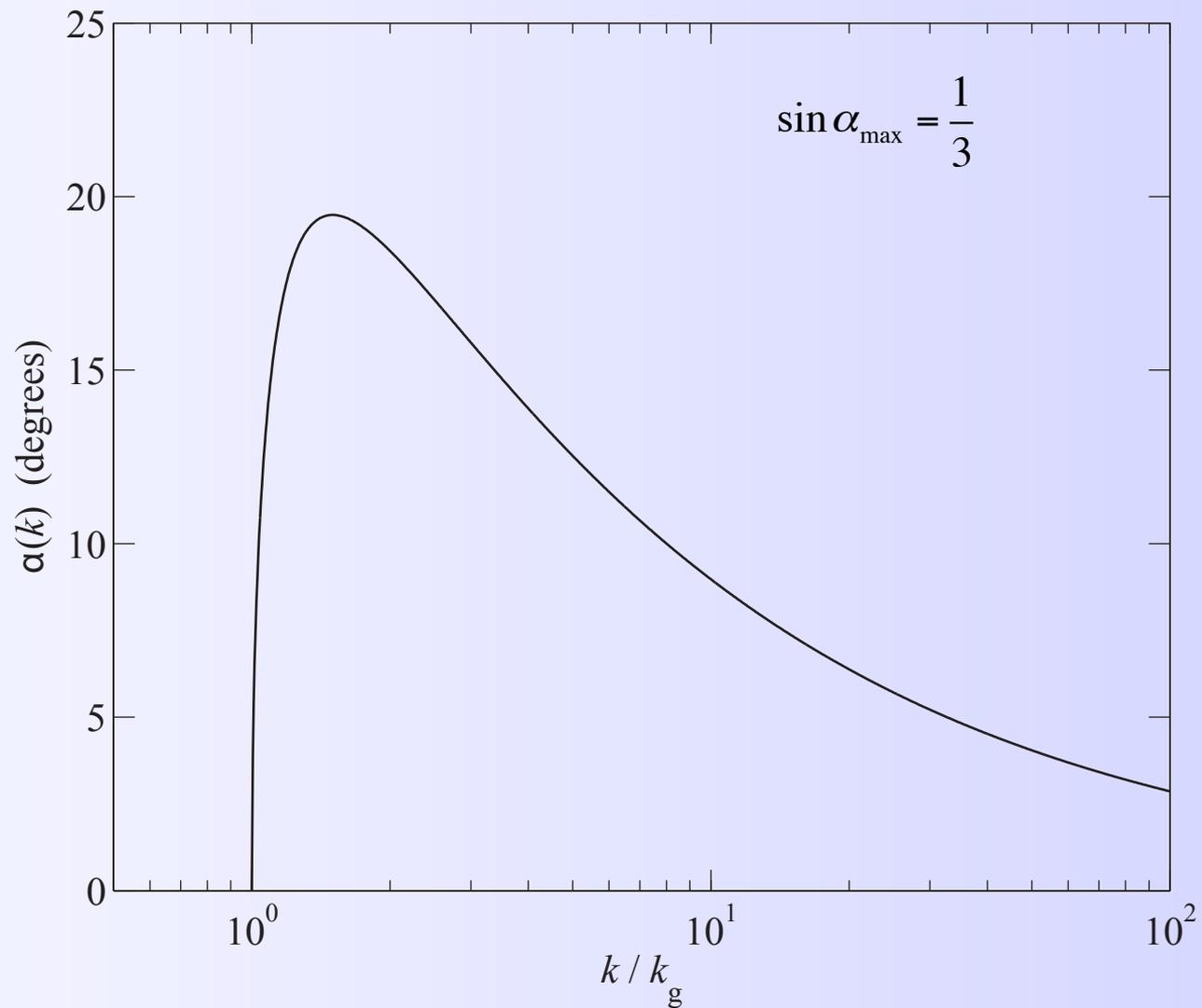
- Crawford, AJP, 1984
- Lamb, Lighthill or Whitham

$$\tan \alpha(\theta) = \frac{\tan \theta}{2 + \tan^2 \theta}$$

$$\tan \alpha(k) = \frac{\sqrt{\frac{k}{k_g} - 1}}{2 \frac{k}{k_g} - 1}$$

avec  $k_g = \frac{g}{U^2}$

$$U \sin \theta(k) = V_\varphi = \sqrt{\frac{g}{k}}$$



# Maximum of deviation

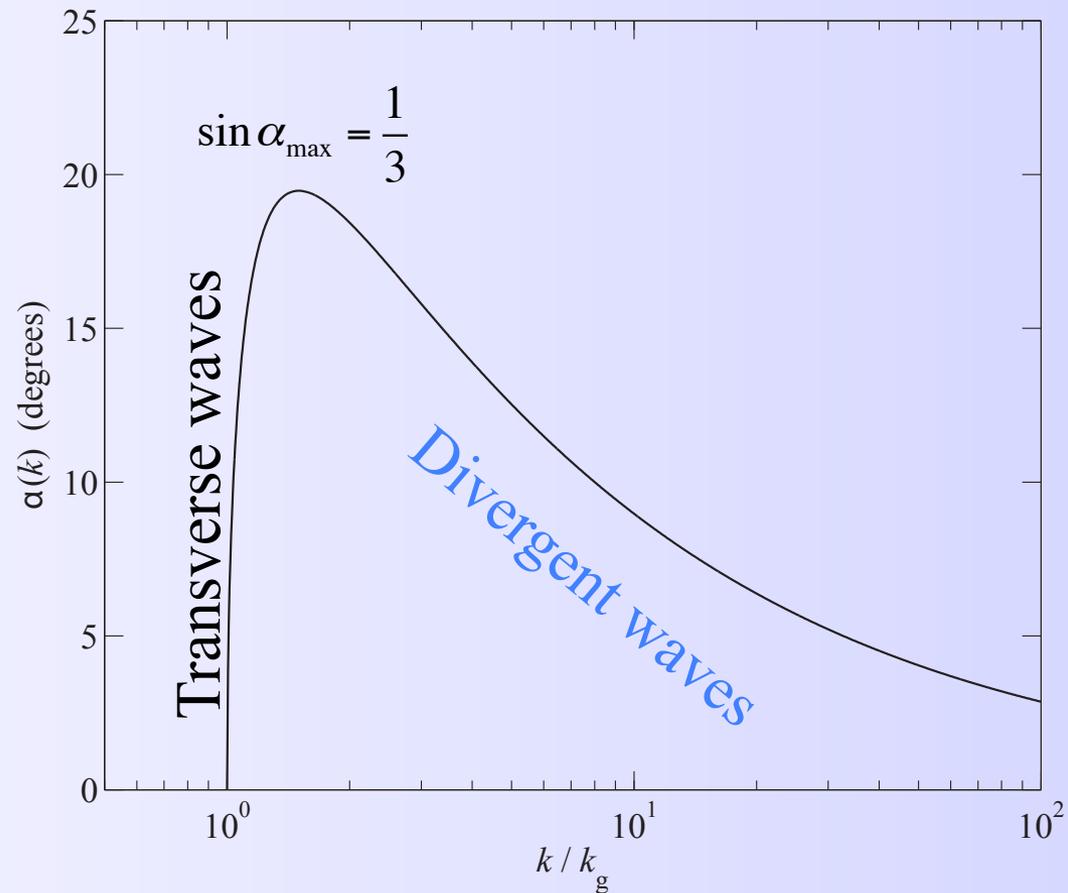
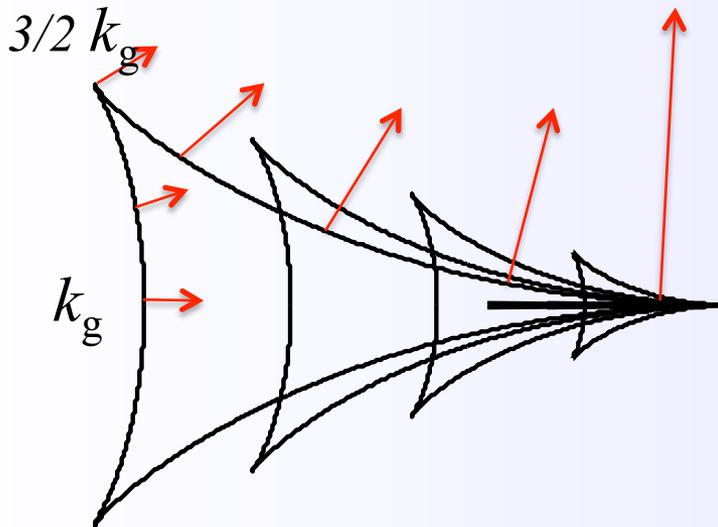
Each wave component  $k$  gives a *radiation angle*  $\alpha(k)$

Stationary pattern:

$$U \sin \theta(k) = V_\varphi = \sqrt{\frac{g}{k}}$$

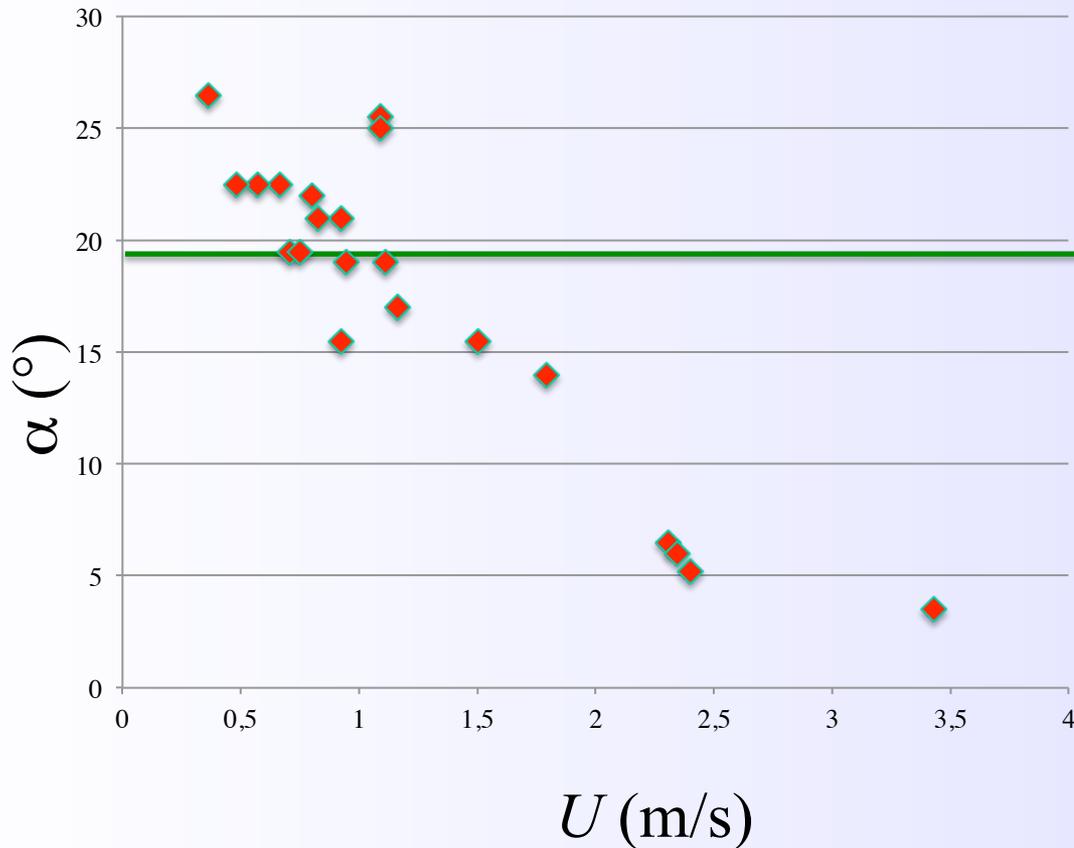
$$\tan \alpha(k) = \frac{\sqrt{k/k_g - 1}}{2k/k_g - 1}$$

$$\text{avec } k_g = \frac{g}{U^2}$$



Et pourtant ...

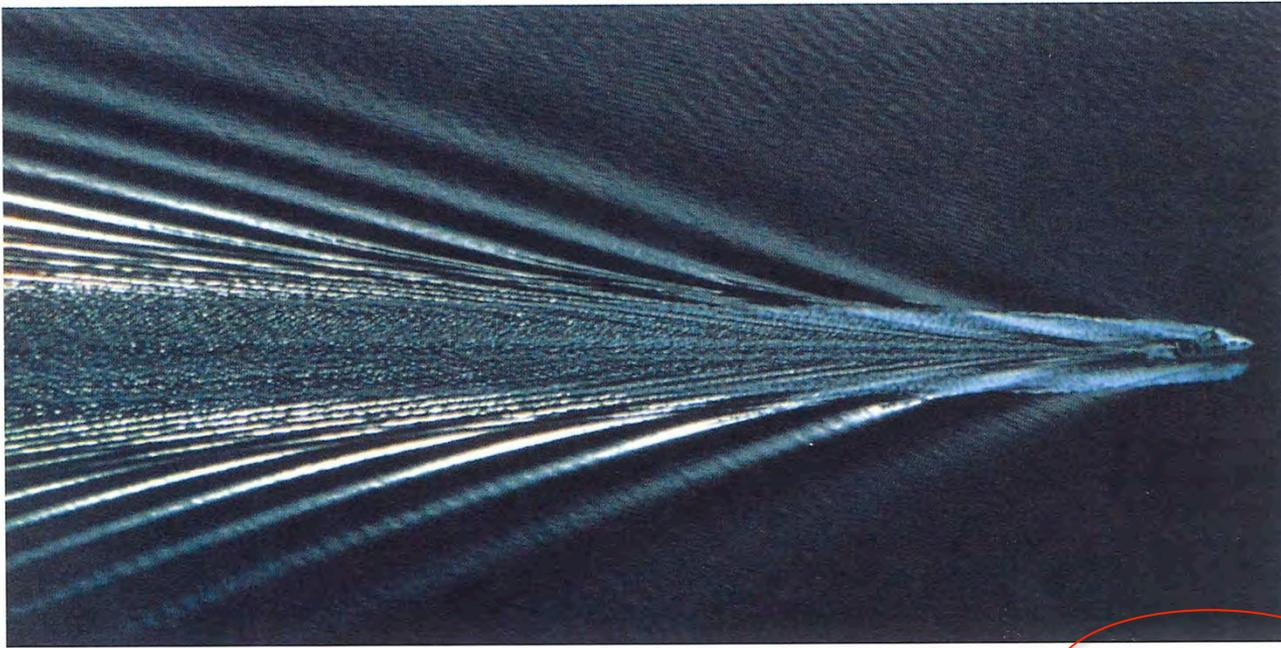
# Et pourtant ...



Kelvin angle

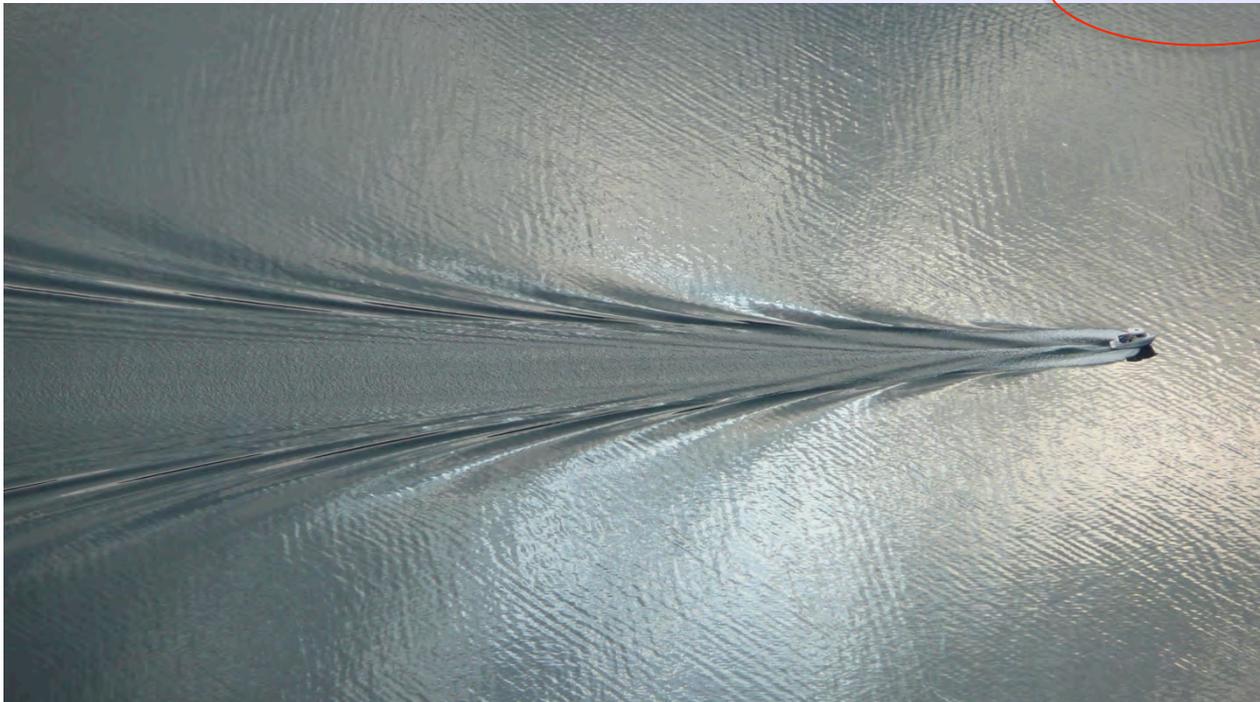
Mesures faites dans  
une piscine de la  
Fondation des Treilles

Et pourtant ...



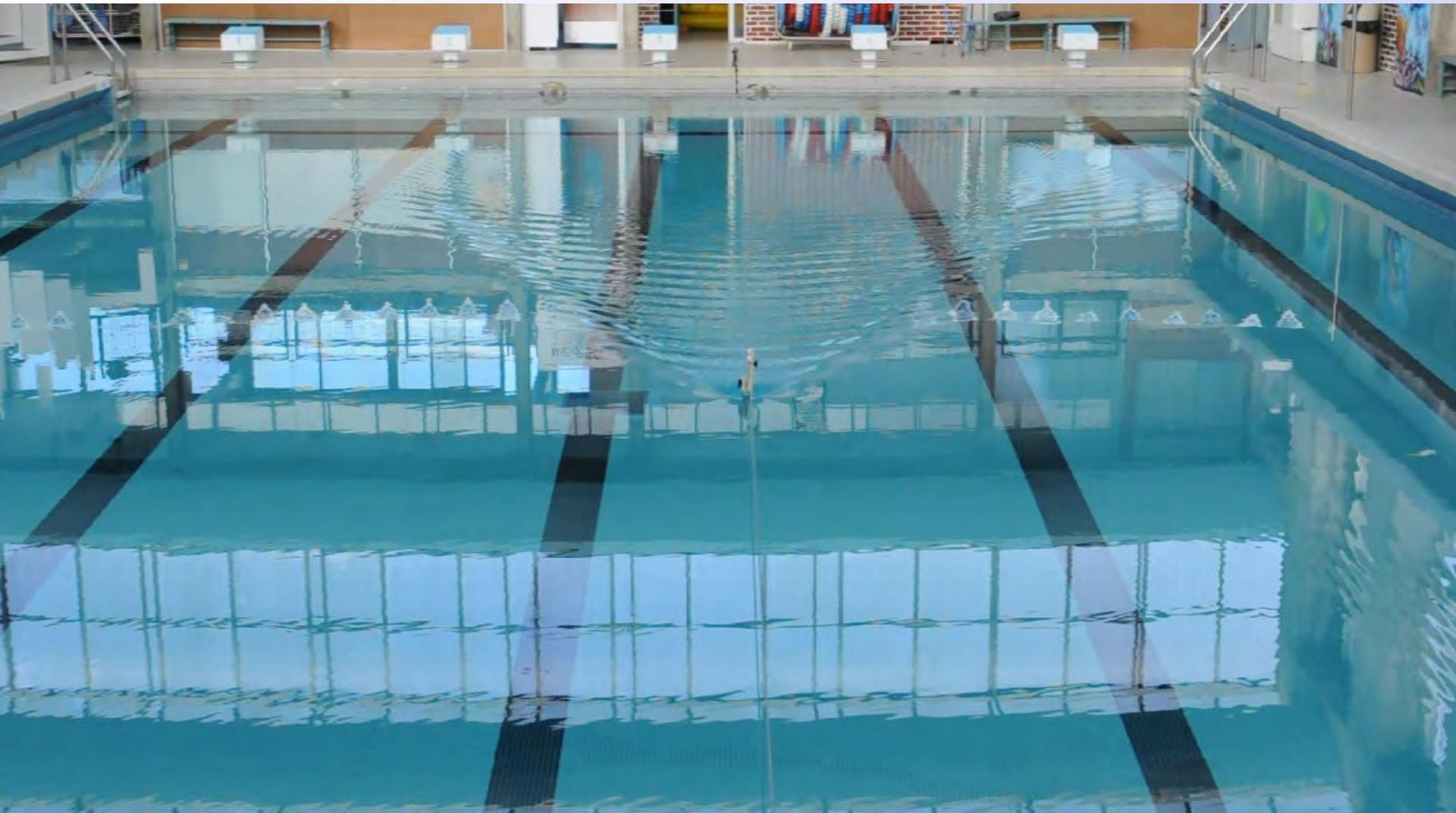
1. VUE AÉRIENNE du sillage d'un bateau à moteur. La figure d'interférence est limitée par un V faisant un angle de 39 degrés.

Jear Walker  
(Pour la Science)



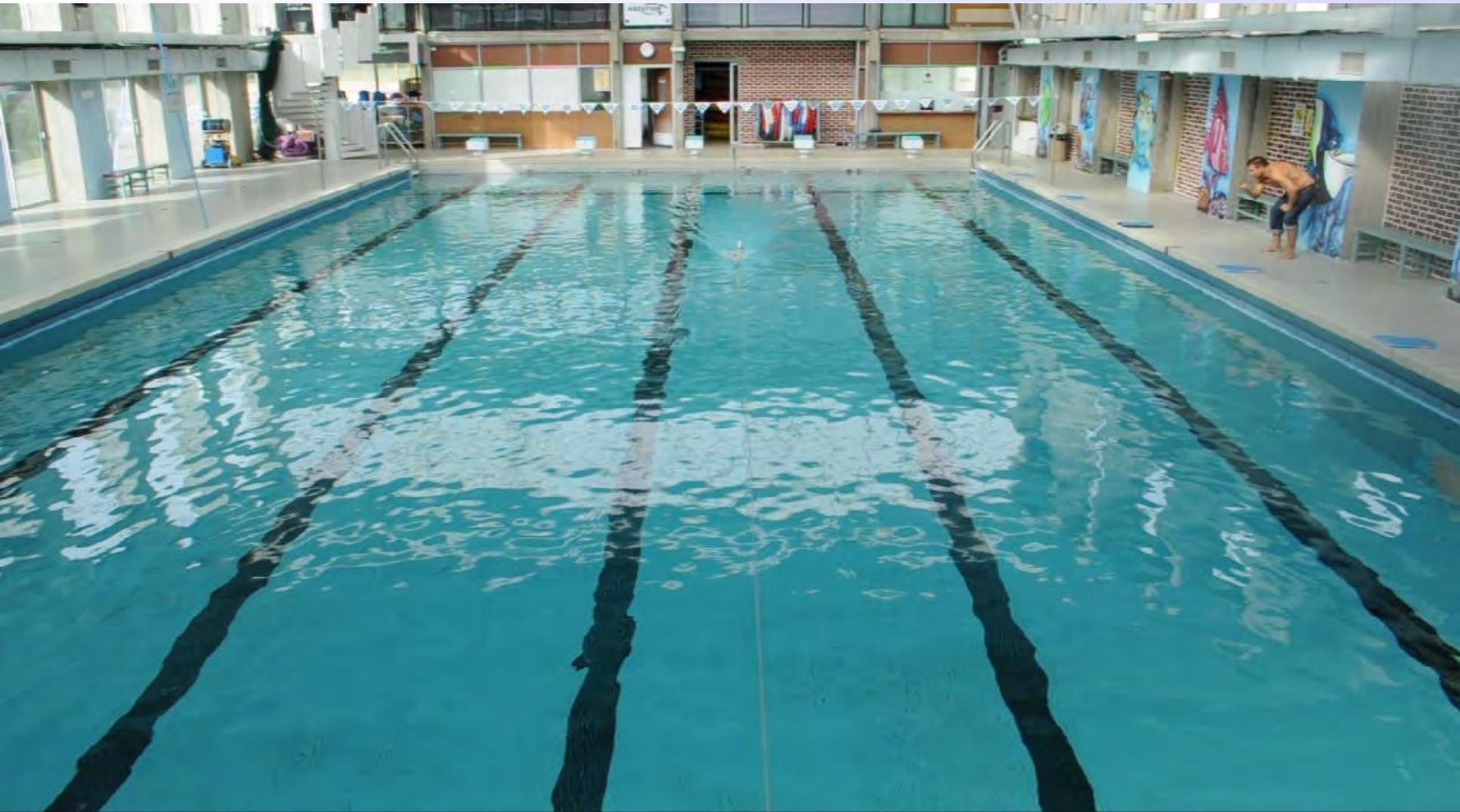
Wikipédia

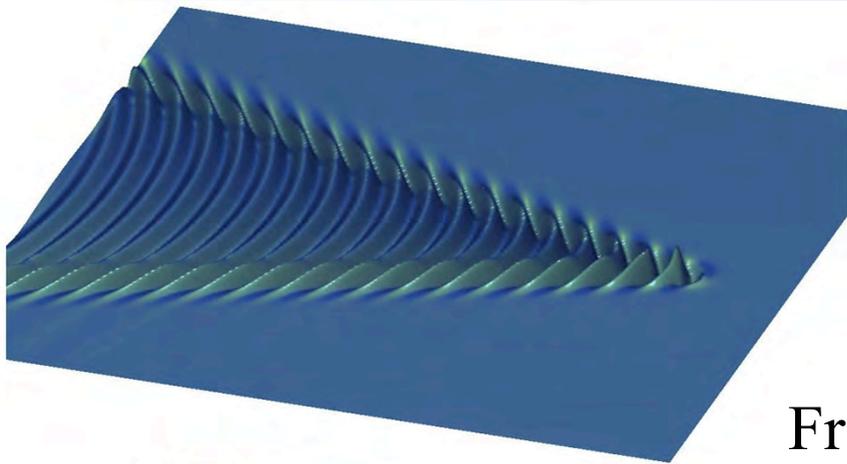
Cylindre de diamètre 3 cm,  $U = 0.75$  m/s



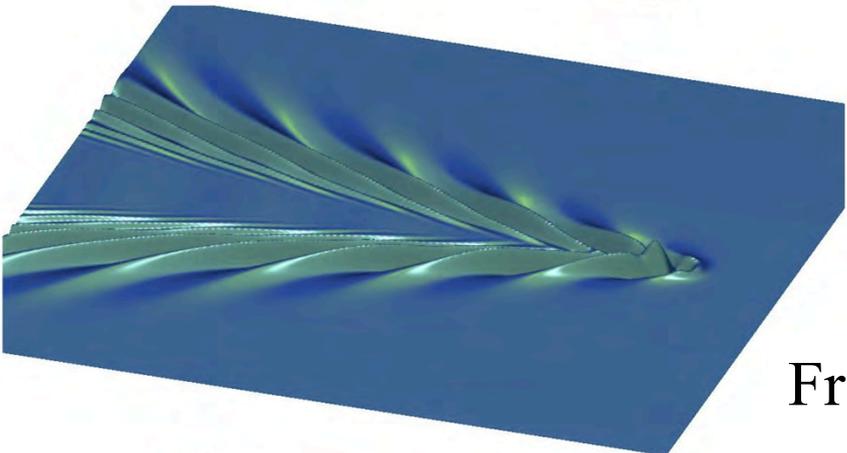
*Piscine Municipale d'Orsay (merci !)*

Cylindre de diamètre 3 cm,  $U = 2.6$  m/s

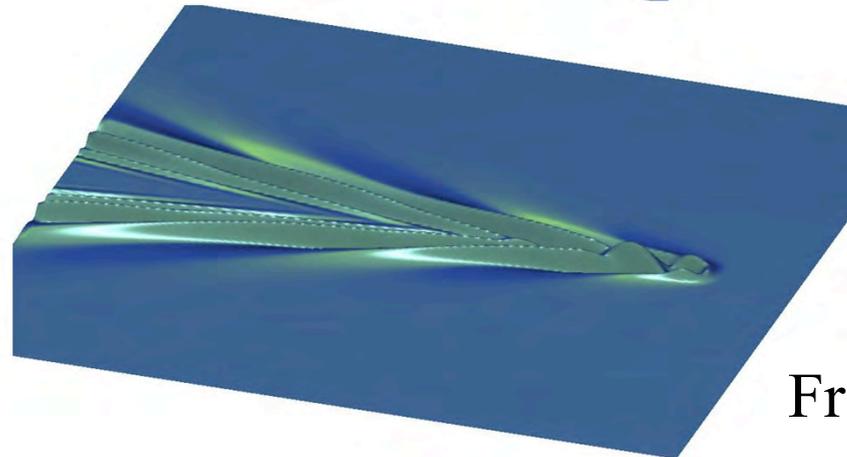




Fr = 0.5



Fr = 1



Fr = 2

Simulation par une  
surpression  
axisymétrique  
de taille  $L$

*(Havelock 1919)*

# Wave-making resistance

Havelock method (1918): Imposed pressure perturbation  $P(x,y)$

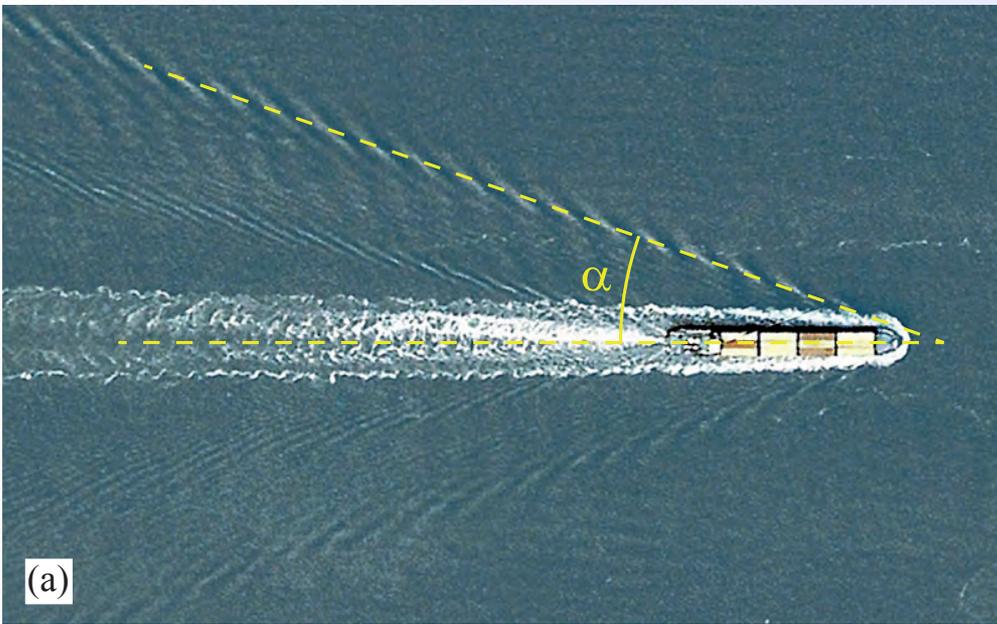
$$R_W = \iint P(x, y) \frac{\partial \zeta}{\partial x} dx dy.$$

Where

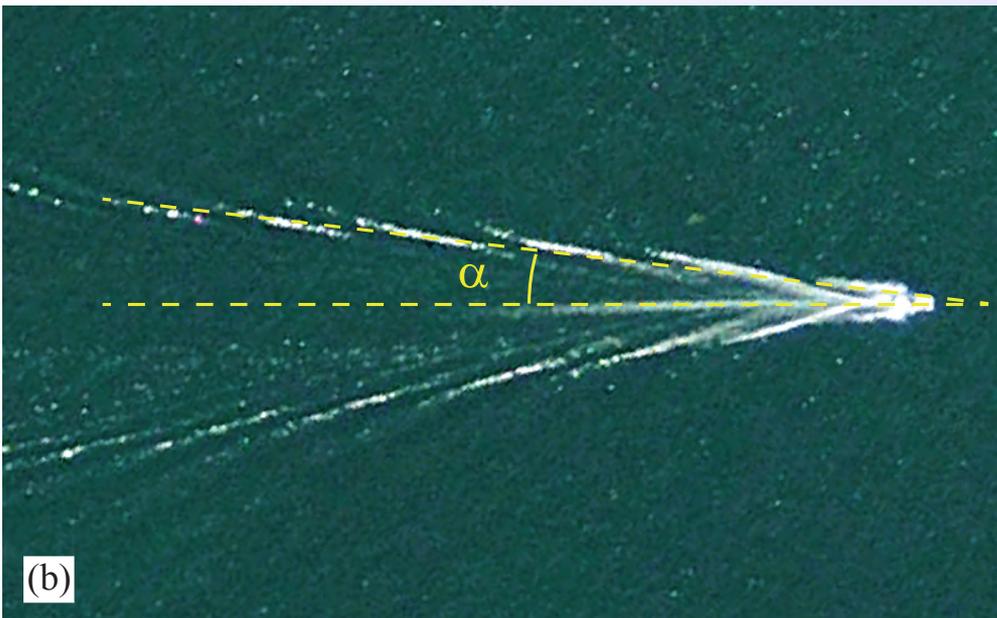
$$\zeta(x, y) = \iint \frac{dk_x}{2\pi} \frac{dk_y}{2\pi} \frac{\hat{P}(k_x, k_y)}{[\gamma(k^2 + k_c^2) - \frac{\rho}{k}(k_x U - i\epsilon)^2]} \exp i[k_x x + k_y y]$$

In the limit  $\epsilon \rightarrow 0$

# Images Google Earth

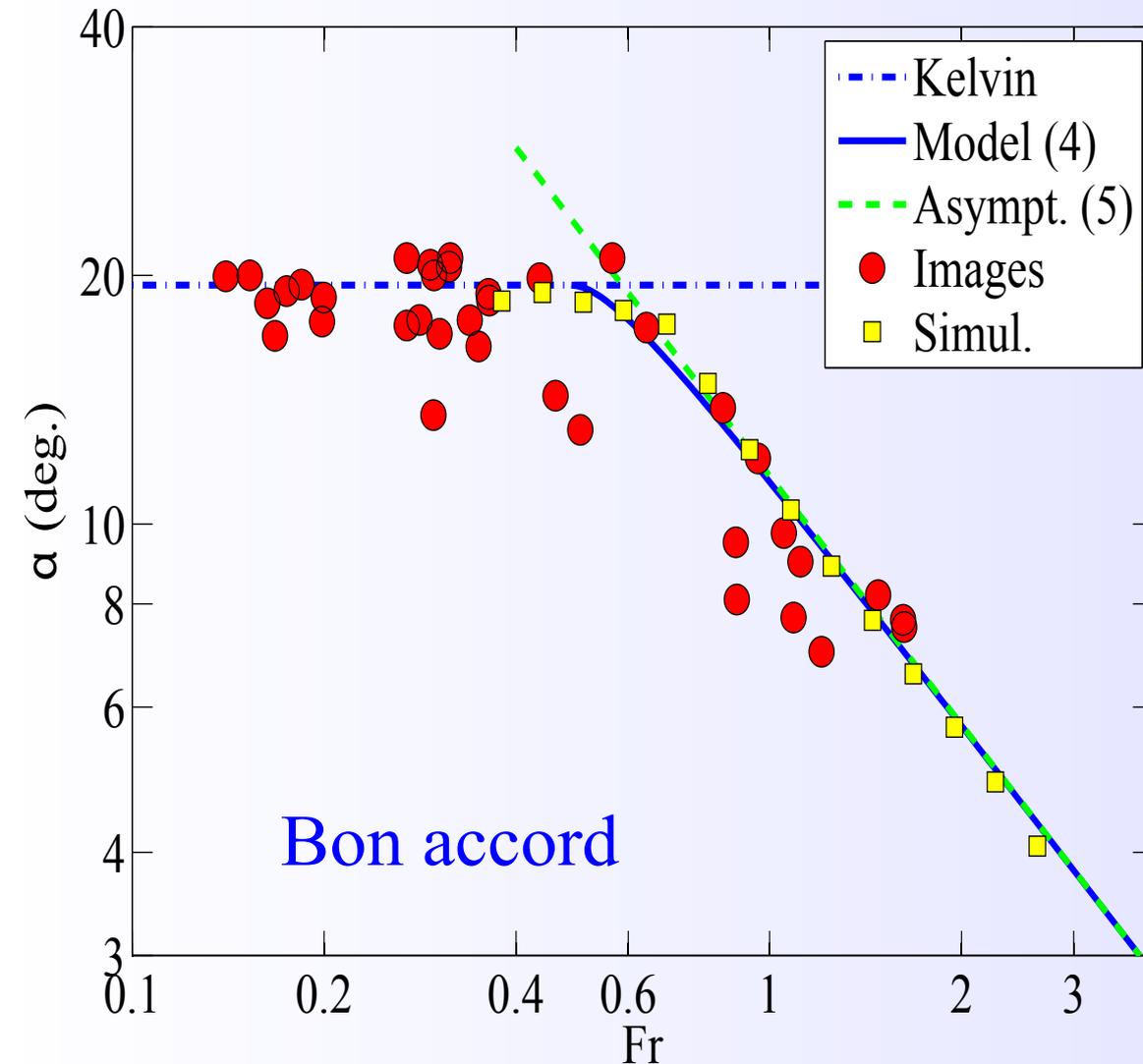


- mesure de l'angle  $\alpha$
- mesure de la longueur  $L$  du bateau
- détermination de sa vitesse  $U$  et donc du Froude :



$$Fr = \frac{U}{\sqrt{gL}}$$

# Evolution de l'angle du sillage



- 2 longueurs  $L$  et  $\lambda_g$ ,
- 1 nombre sans dim.  $Fr$

A grand Froude :

$$\sin(\alpha) \sim \frac{1}{Fr} = \frac{\sqrt{gL}}{U}$$

Comme dans le régime de Mach !

$$\sin(\alpha) = \frac{c}{U}$$

- *Phys. Rev. Letters* 110, 214503 (2013).

- Confirmé depuis analytiquement par Darmon, Benzaquen & Raphaël, *JFM* 2014)

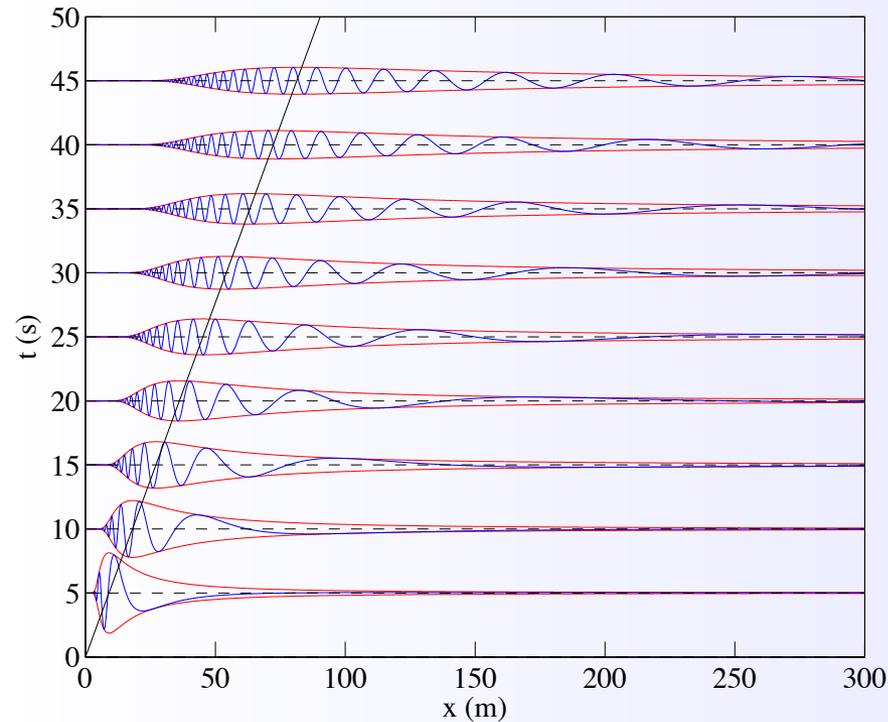
# Kelvin avait-il tort ?

- Comment expliquer que l'angle du sillage décroisse à haute vitesse ?
- Effet de la taille  $L$  de l'obstacle ?

**Jetons un cailloux de taille L dans une mare :**



# The Cauchy-Poisson initial-value problem (1815-1816)



Wave envelope at large distance from the source

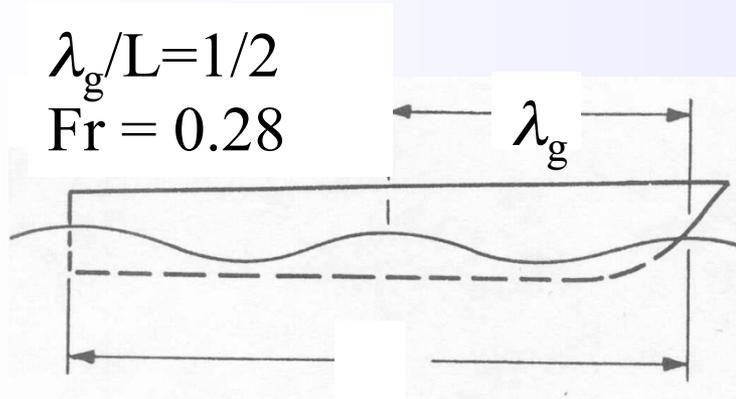
$$\zeta(r, t) \sim \hat{\zeta}(k = k_0(r, t)) \left( r^2 t \left| \frac{\partial c_g}{\partial k}(k = k_0(r, t)) \right| \right)^{-1/2}$$

(Havelock 1908, Lighthill 1978, etc.)

## Key results:

- The wave number of maximum amplitude is  $1/L$
- The wave packet propagates at *constant* velocity  $V_g(1/L)$  ... as in a non-dispersive medium !

# Origin of narrow wake pattern ? Finite size effect !



Largest excited gravity wave:

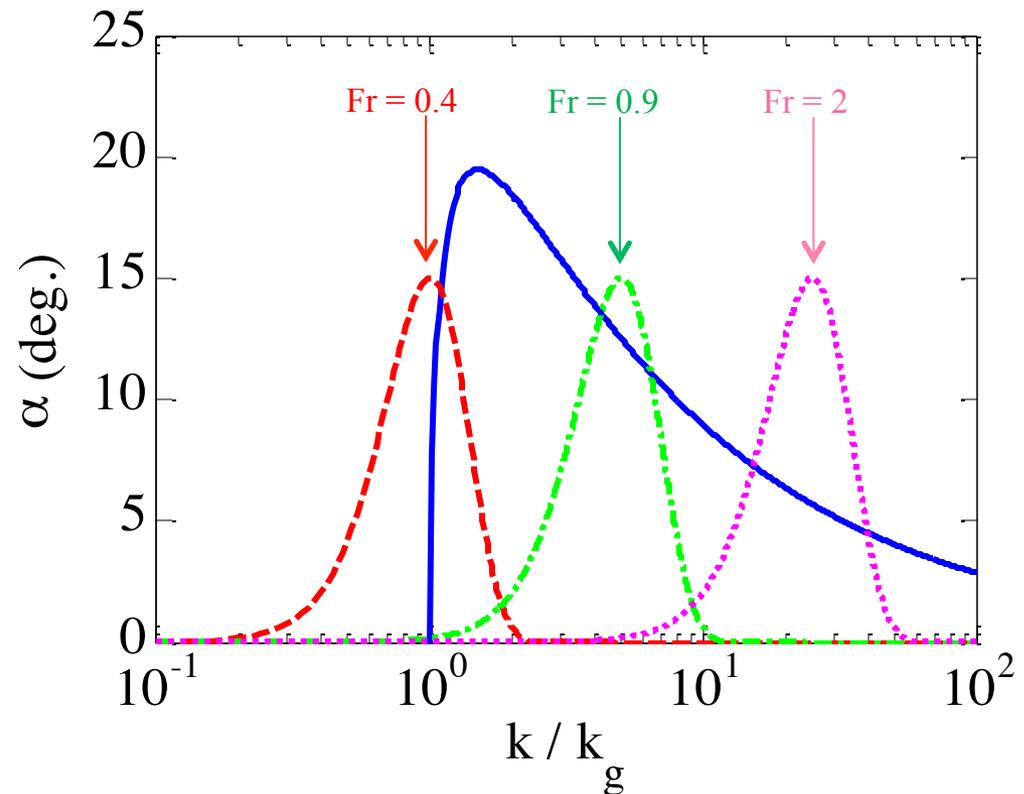
$$\lambda_g = 2\pi/k_g = 2\pi U^2/g$$

$$Fr = \sqrt{\lambda_g/2\pi L}$$

The actual wake pattern  
is an convolution of

$$\tan \alpha(k) = \frac{\sqrt{k/k_g - 1}}{2k/k_g - 1}$$

by the spectrum of the disturbance

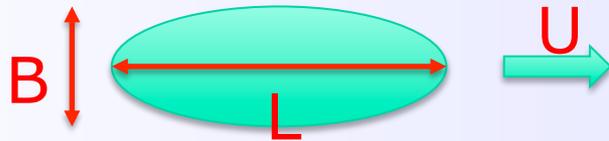


Oui mais les bateaux ne sont pas  
tous axisymétriques !

# Angle de sillage d'une perturbation non axisymétrique

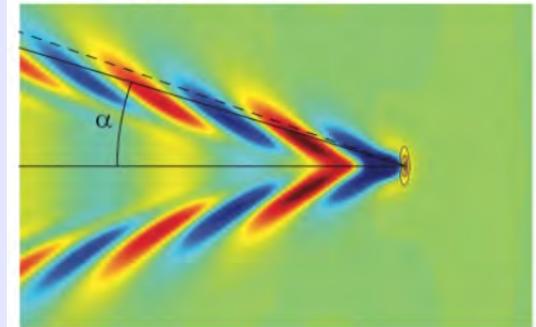
Perturbation de pression gaussienne, de rapport d'aspect  $A = L / B$ :

$$P(\mathbf{r}) = P_0 \exp \left[ -\pi^2 \left( \frac{x^2}{L^2} + \frac{y^2}{B^2} \right) \right]$$

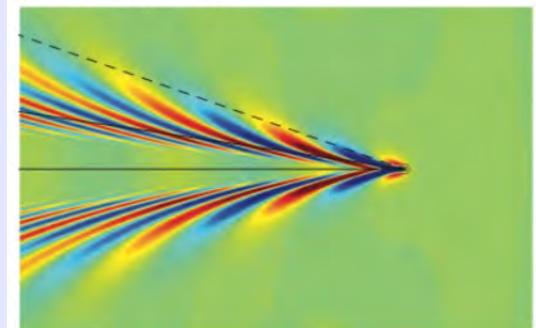


- 3 longueurs  $L$ ,  $B$  et  $\lambda_g$ ,
- 2 nombres sans dim.  $Fr$  et  $A$
- A nombre  $Fr_L$  constant, l'angle décroît lorsque  $A$  augmente.

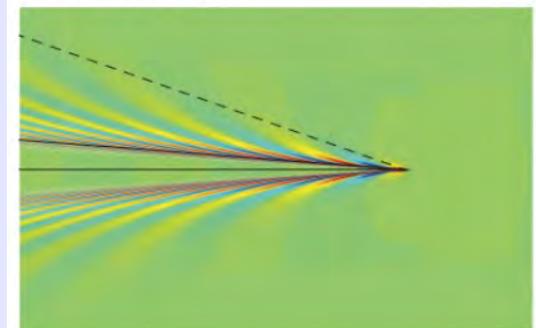
(a)  $A = 0.25$ ,  $\alpha = 18.3^\circ$



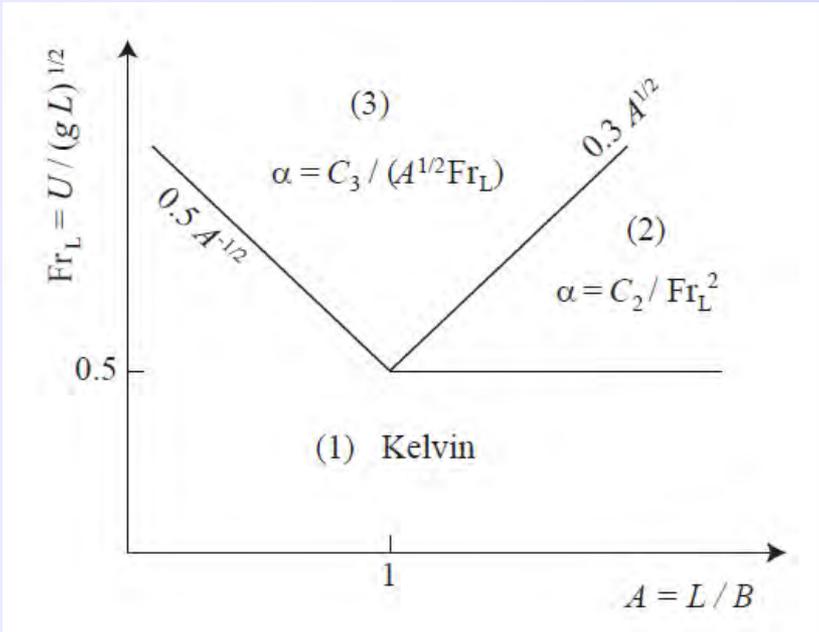
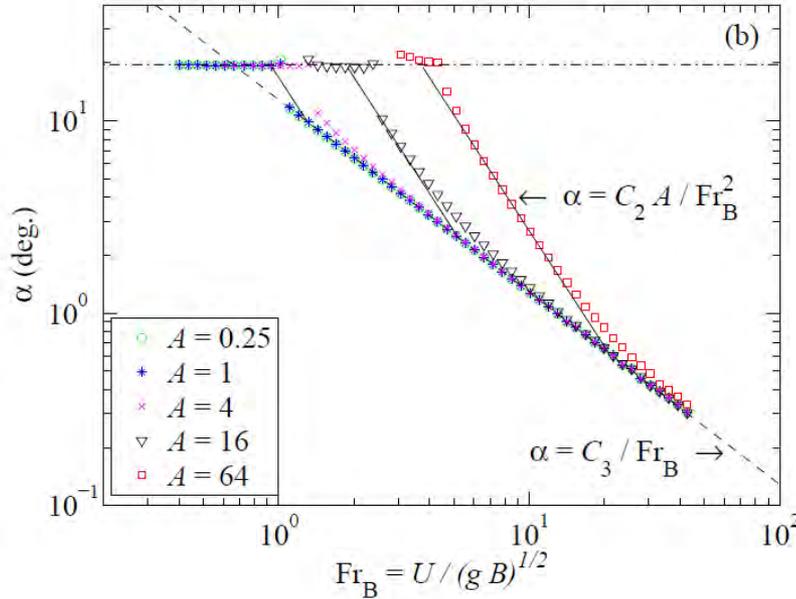
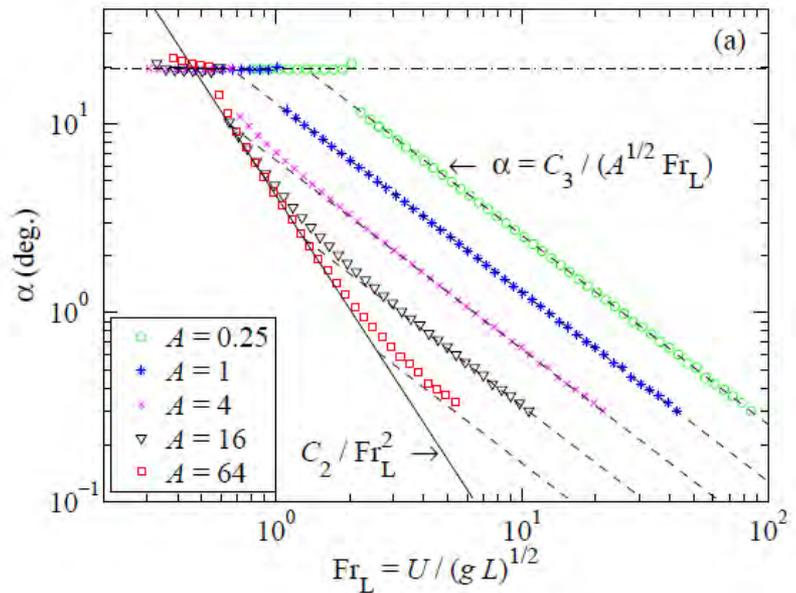
(b)  $A = 1$ ,  $\alpha = 8.6^\circ$



(c)  $A = 4$ ,  $\alpha = 4.5^\circ$



# Angle du sillage d'une perturbation non axisymétrique



- Moisy & Rabaud, PRE 2014
- Benzaquen, Darmon & Raphaël, PoF 2014
- Noblesse et al, Eur. J. Mech B (2014)



*NASA satellite image (MODIS imager on board the Terra satellite) of a wave cloud forming off of Amsterdam Island in the far southern Indian Ocean. Image taken on December 19, 2005.*

## 3 – Sillage du canard

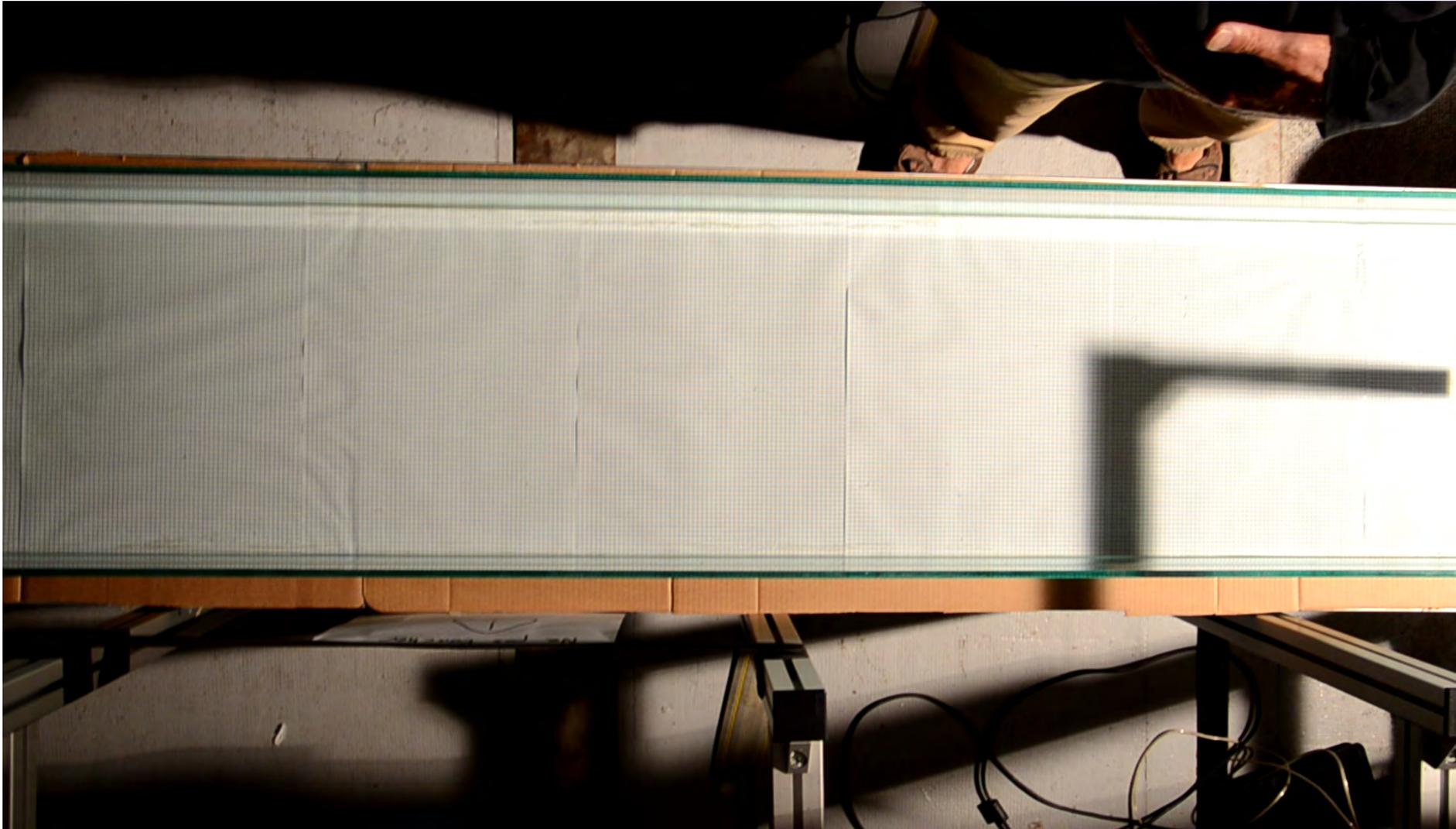
(cas gravito-capillaire)

## Et les canards ? Effets capillaires ?



- 3 longueurs  $L$ ,  $l_{\text{cap}}$  et  $\lambda_g$
- 2 nombres sans dim.  $Fr_L$  et  $Bo = L/l_{\text{cap}}$

Cylindre de diamètre 1.5 mm,  $U = 0.6$  m/s ( $Fr = 4.9$ )



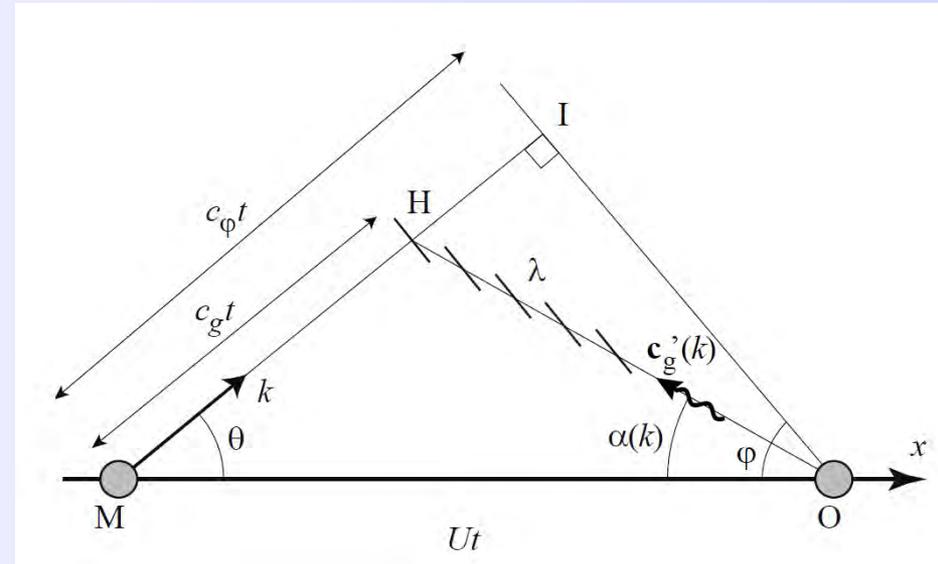
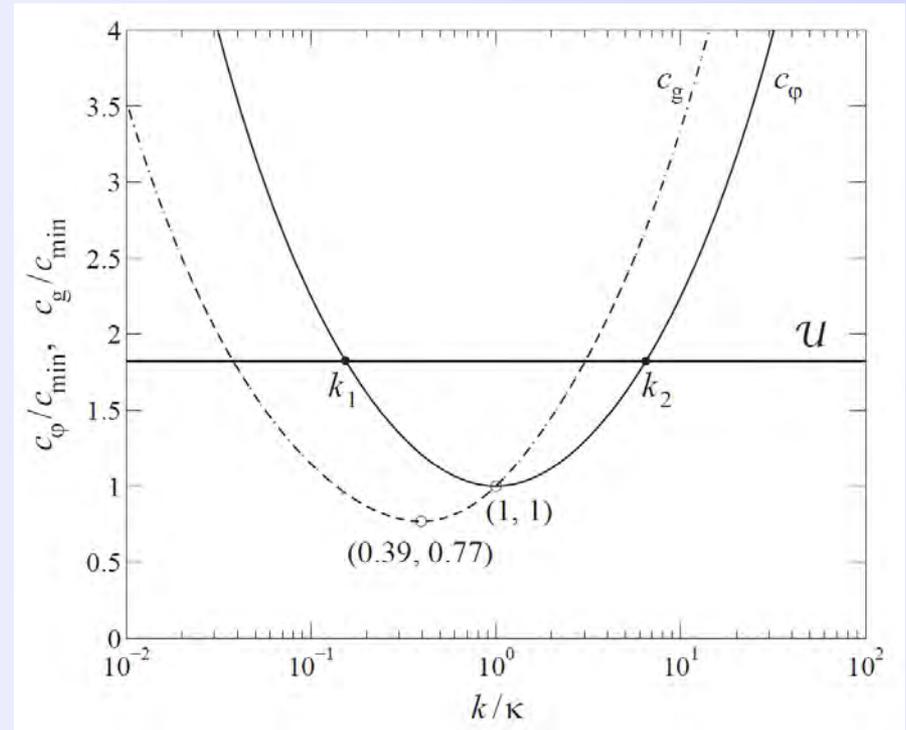
## With capillary-gravity waves

Each wave component  $k$  propagates according to its radiation angle

$$\omega(k) = \sqrt{gk + \frac{\gamma}{\rho} k^3}$$

$$U \cos \theta = c_\varphi = \frac{\omega}{k}$$

$$\tan \alpha(k) = \frac{c_g(k) \sqrt{U^2 - c_\varphi^2(k)}}{U^2 - c_g(k) c_\varphi(k)}$$



## With capillary waves

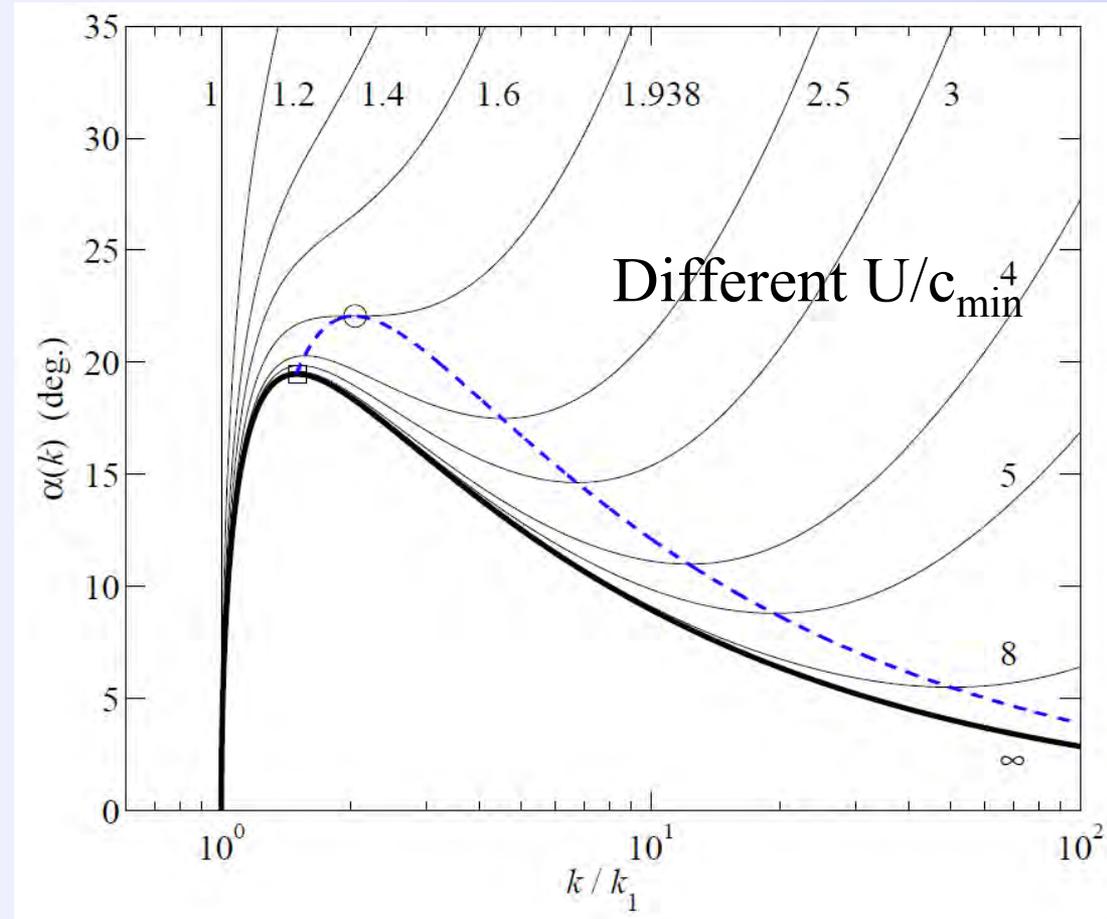
$$\frac{U}{c_{\min}} = Fr \sqrt{\pi B \sigma}$$

$$\omega(k) = \sqrt{gk + \frac{\gamma}{\rho} k^3}$$

$$U \sin \theta = c_{\varphi} = \frac{\omega}{k}$$

$$\tan \alpha(k) = \frac{c_g(k) \sqrt{U^2 - c_{\varphi}^2(k)}}{U^2 - c_g(k) c_{\varphi}(k)}$$

$$\text{avec } k_g = \frac{g}{U^2}$$

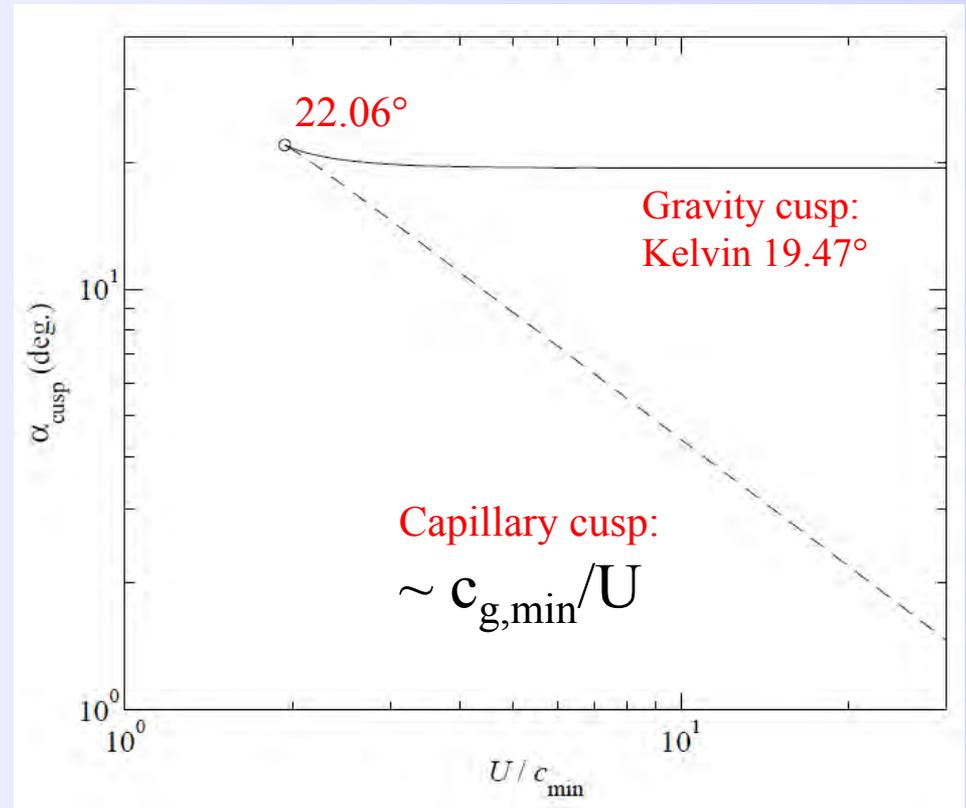
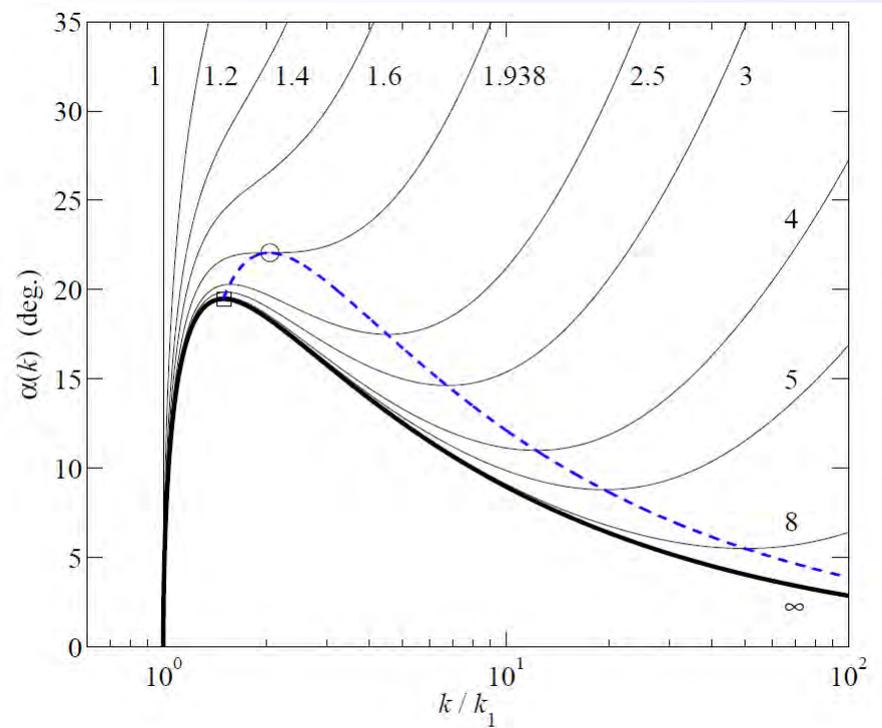


But damping in  $\exp(-2\nu k^2 t)$

For  $U/c_{\min} \approx 1.938$  : saddle-node apparition of two extrema,  $\partial\alpha/\partial k = 0$

# Two cusps angles

$$\tan \alpha(k) = \frac{v_g(k) \sqrt{U^2 - v_\phi^2(k)}}{U^2 - v_g(k)v_\phi(k)}$$



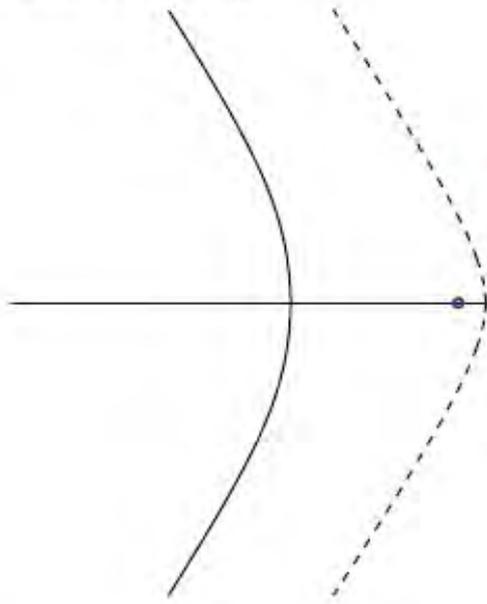
For  $U \gg c_{\min}$ ,  $\alpha = c_{g,\min}/U \approx A c_{\min}/U$

with

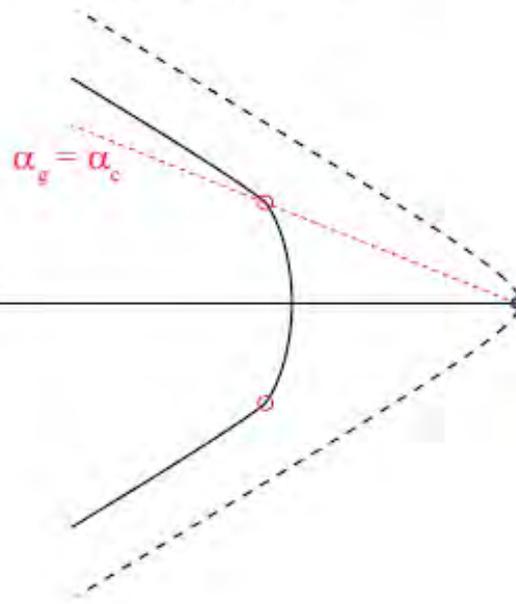
$$A = \frac{1}{2}(\sqrt{3} - 1) \left( \frac{3}{2/\sqrt{3} - 1} \right)^{1/4} \simeq 0.768$$

# Formes des crêtes stationnaires

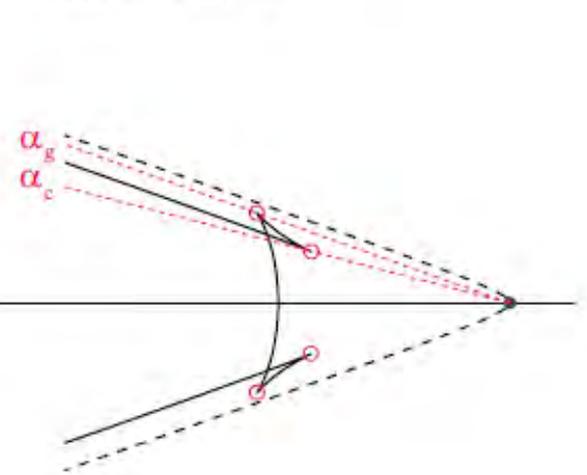
(a)  $U/c_{\min} = 1.2$



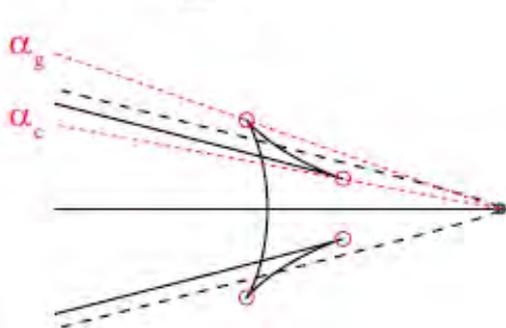
(b)  $U/c_{\min} = 1.938$



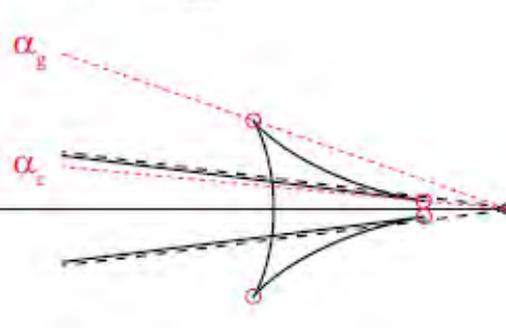
(c)  $U/c_{\min} = 3$



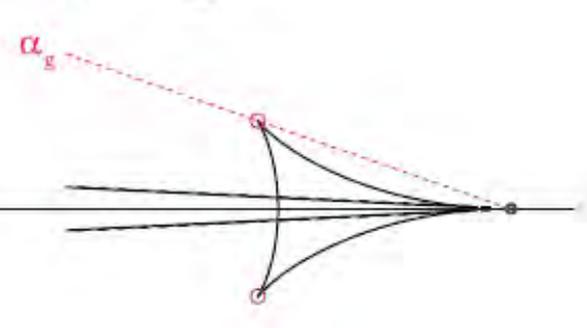
(d)  $U/c_{\min} = 4$



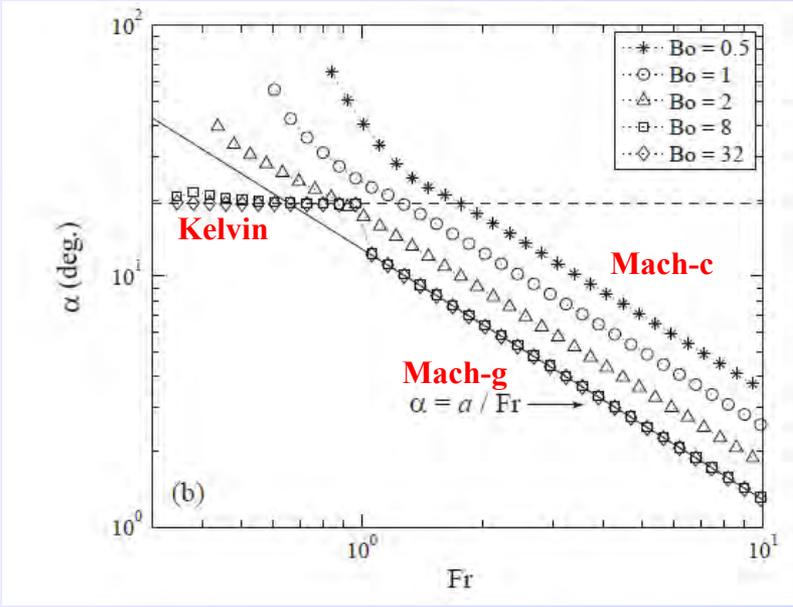
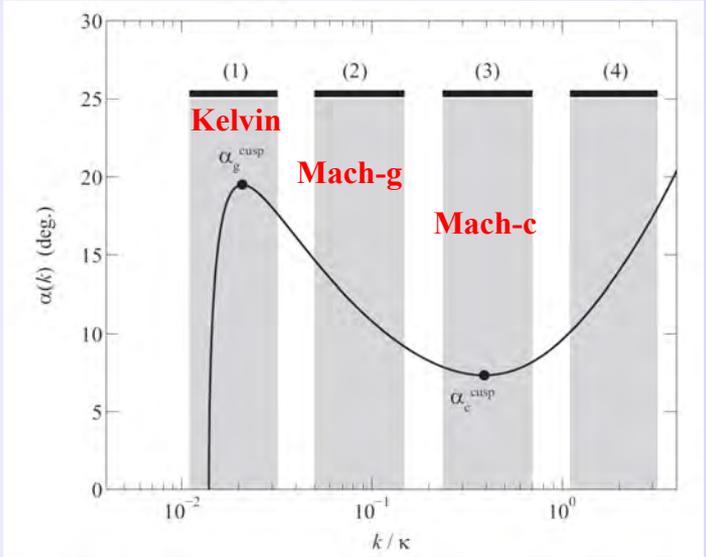
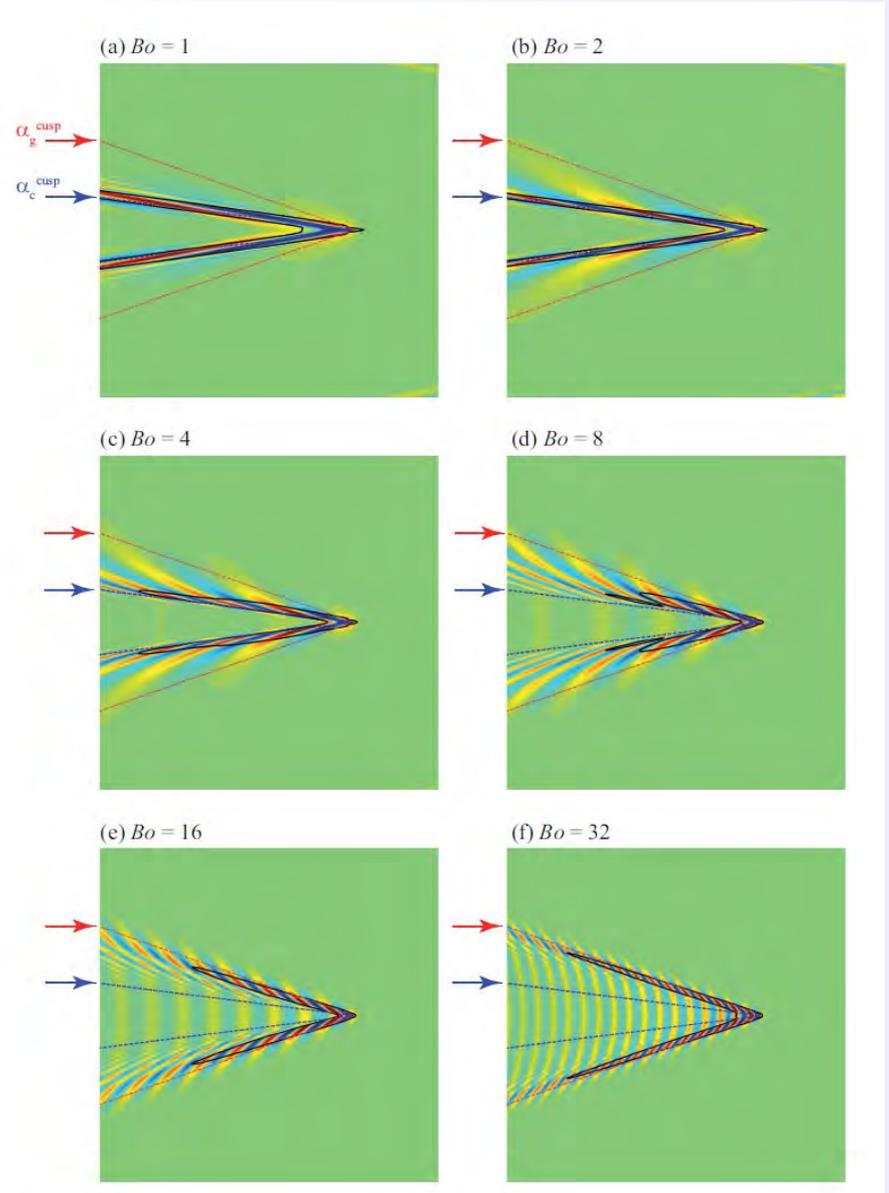
(e)  $U/c_{\min} = 8$



(f)  $U/c_{\min} = 20$



# Capillary-gravity wake simulation

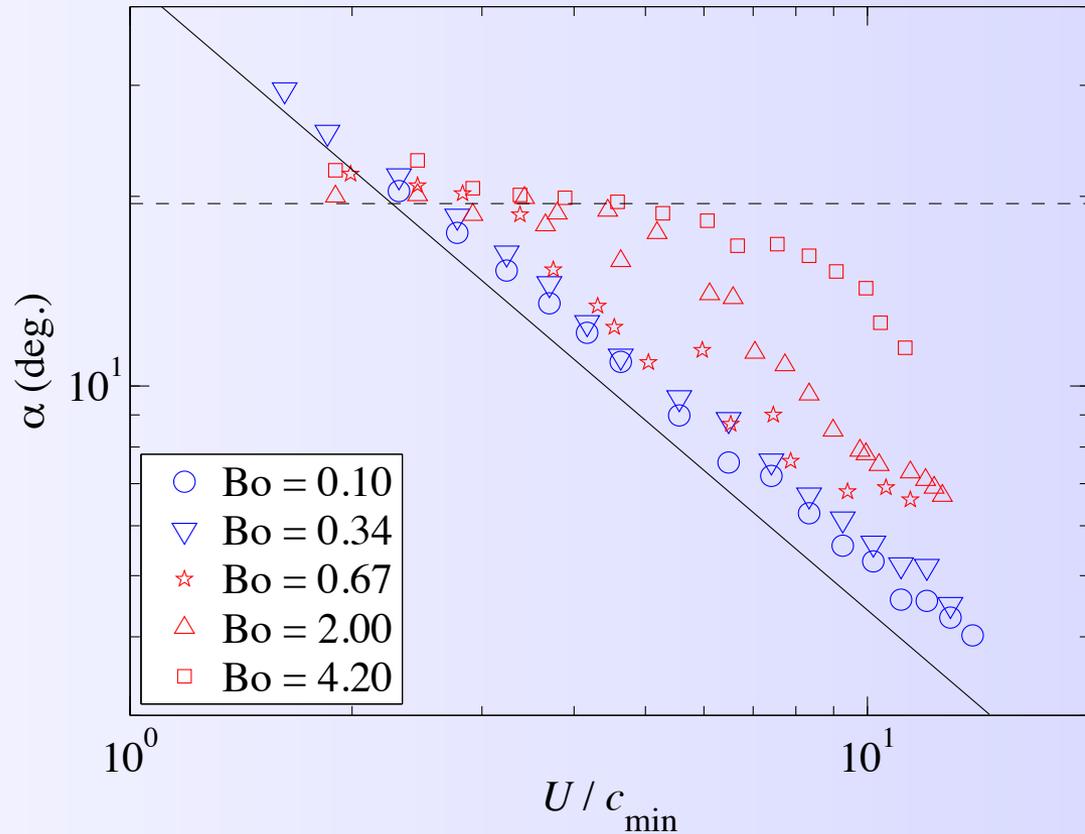


With capillary waves

$$D \in [1.5, 3, 5, 10, 20, 62] \text{ mm}$$

Bond number:

$$Bo = \frac{D}{\lambda_c}$$



# Conclusions sur les sillages

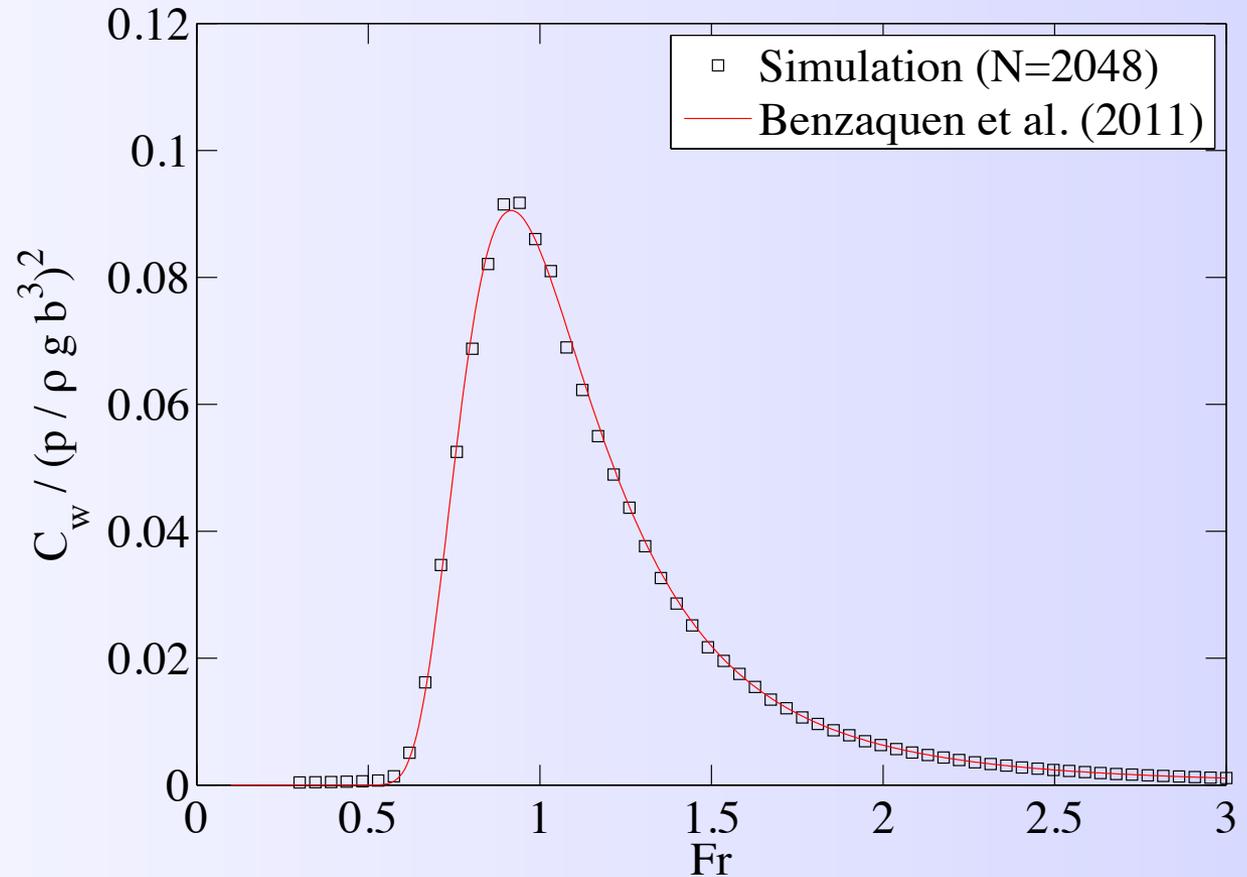
- Les sillages plus étroits que la solution de Kelvin existent bien et ils n'étaient pas expliqués.
- Tout n'est pas compris dans les vrais sillages (coques rigides, non-linéarités, présence de vagues ...)
- Le lien entre coque planante, diminution de l'angle du sillage et diminution de la traînée de vague reste à faire à mon avis.

# Wave-making resistance

A decrease of the wave drag is found.



Planing hull



Analytical result of Benzaquen, Chevy, Raphaël for an imposed gaussian pressure field.

$$C_W = \left(\frac{D}{L^3}\right)^2 \frac{1}{Fr^8} \int_0^{\pi/2} \frac{d\theta}{\cos^5 \theta \exp\left[\left(\sqrt{2\pi} Fr \cos \theta\right)^{-4}\right]}$$

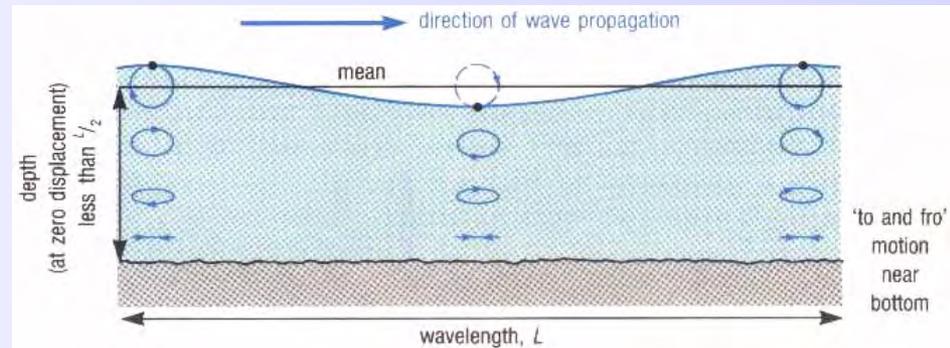
## Compléments sur les vagues

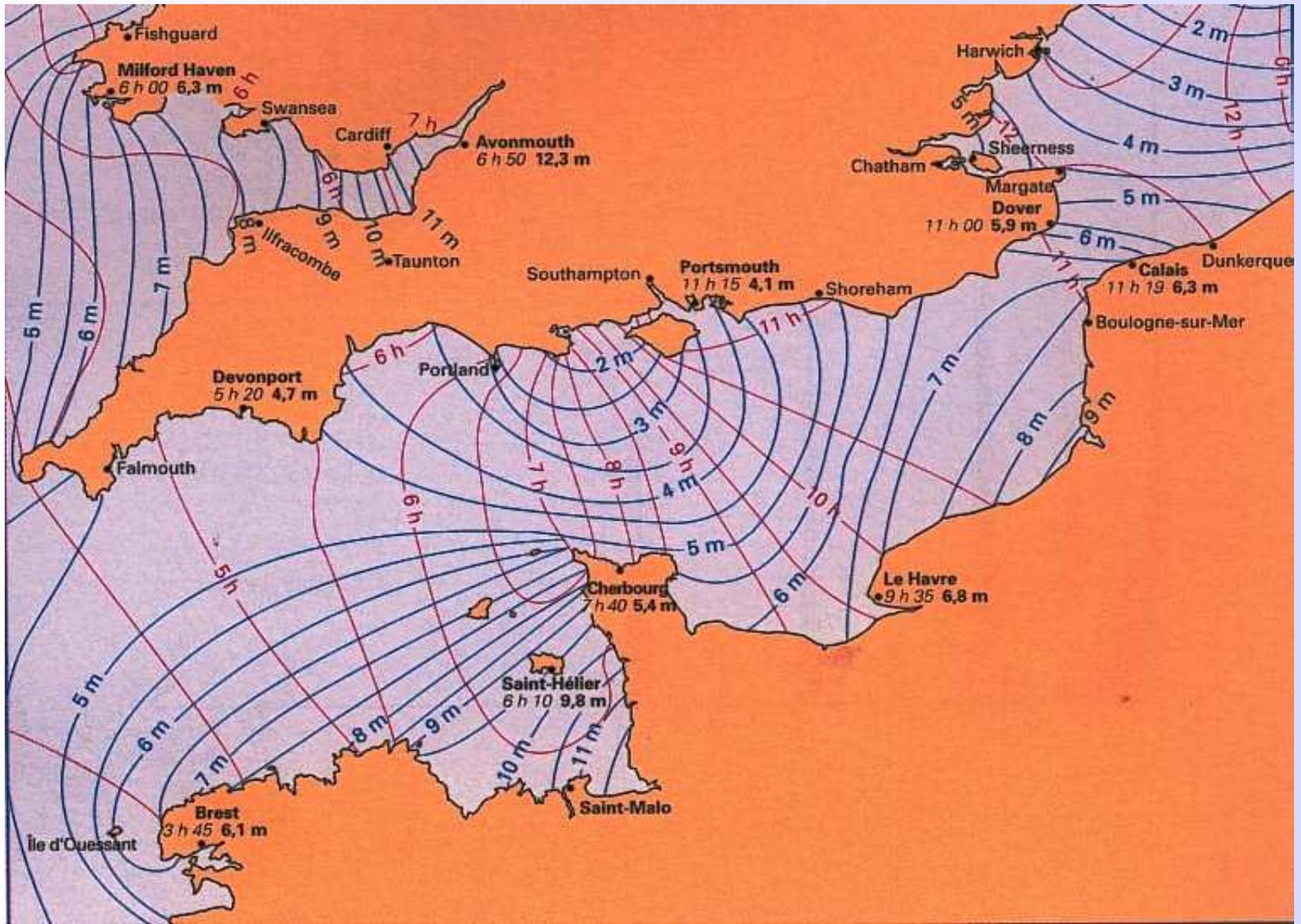
Les ondes de grandes longueurs d'onde ou en eau peu profonde

$$\lambda > 4 \text{ ou } 5 H$$
$$v_{\varphi} = v_g = \sqrt{gH}$$

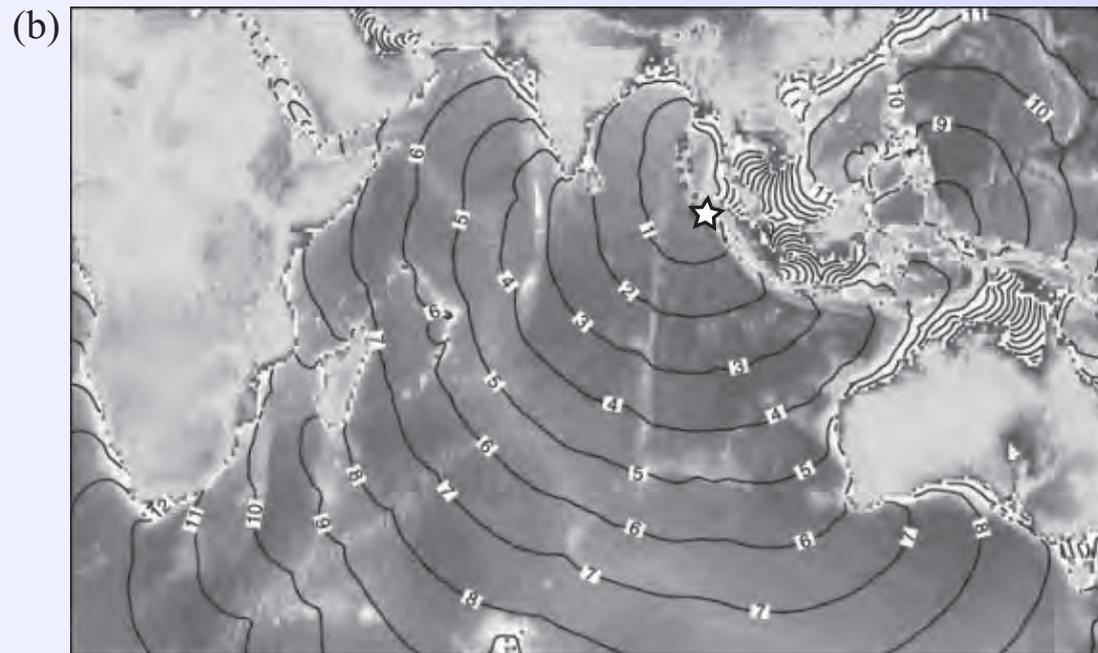
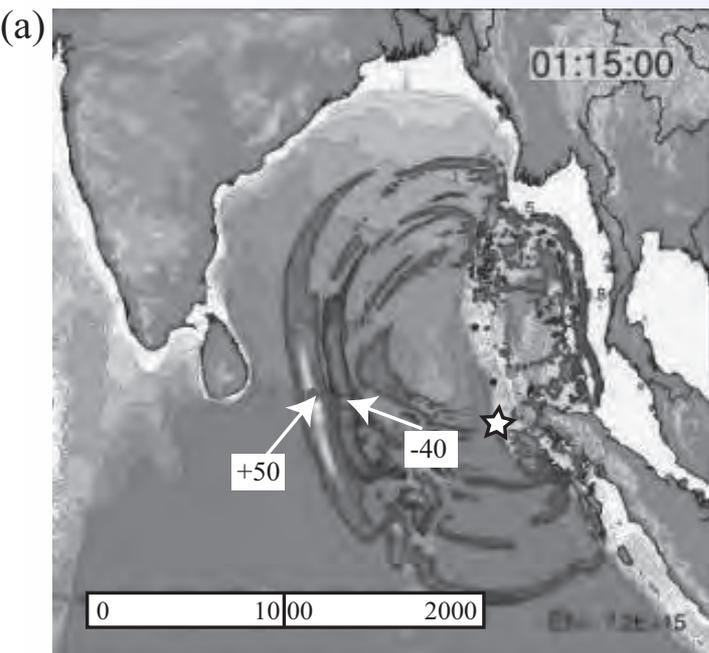
(onde de marée, tsunami, ressaut hydraulique, soliton, seiche, mascaret)

# Trajectoire de particule en eau peu profonde

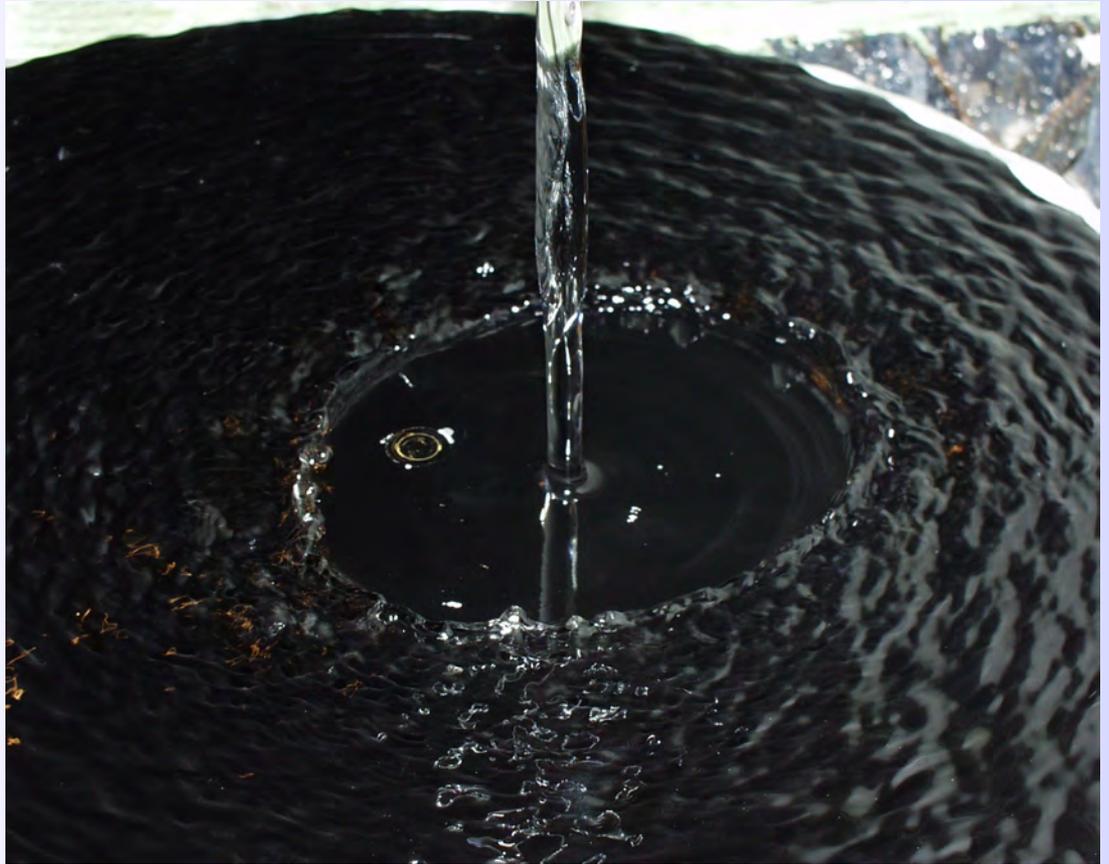




L'onde de marée dans la Manche

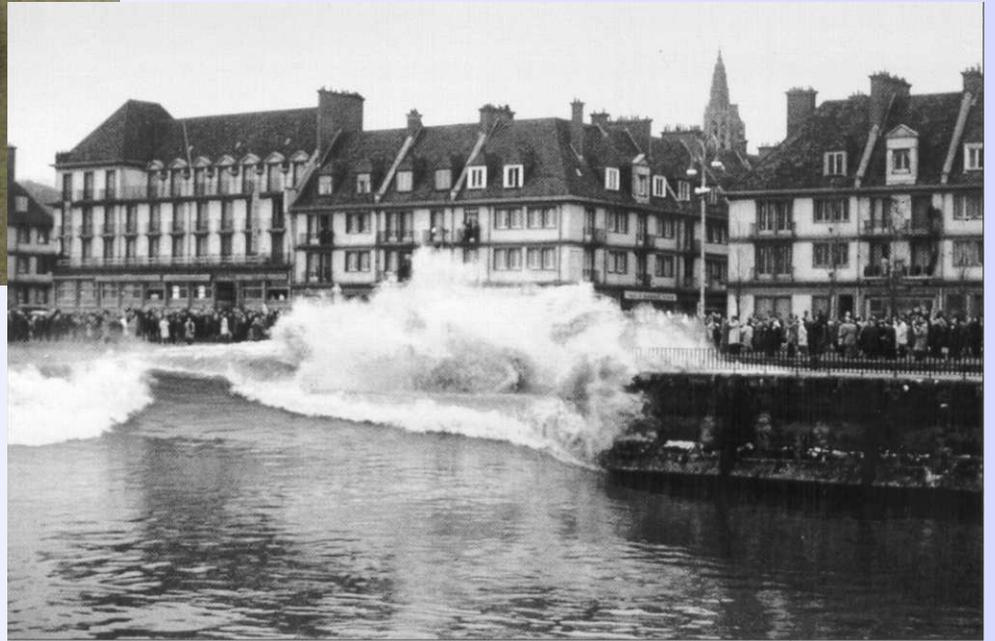
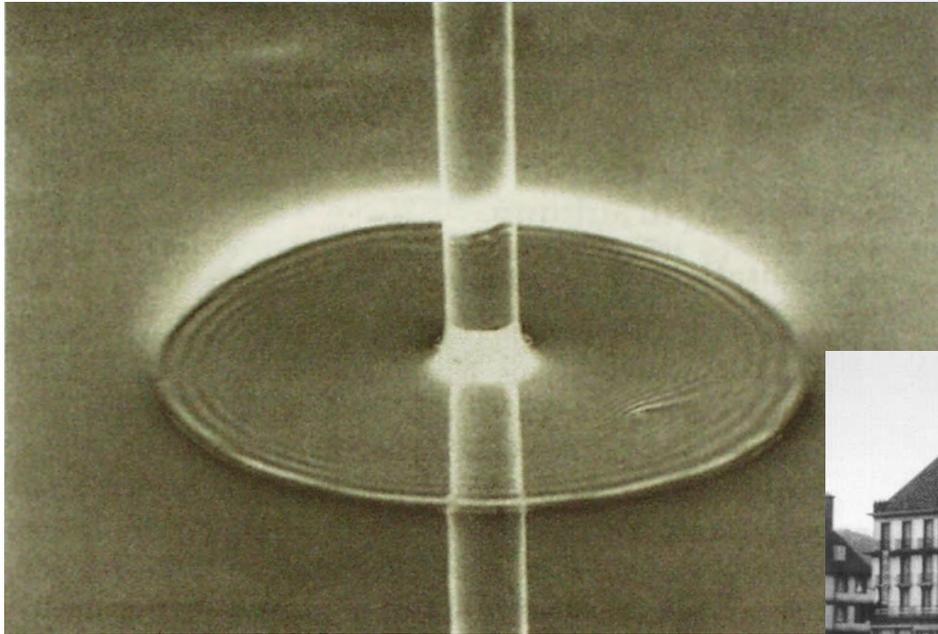


Propagation dans l'Océan Indien du tsunami suite au séisme (★) du 26 décembre 2004



Ressaut hydraulique

# Ressaut hydraulique et mascaret



Caudebec (1960)

Mascaret de St-Pardon (Gironde)

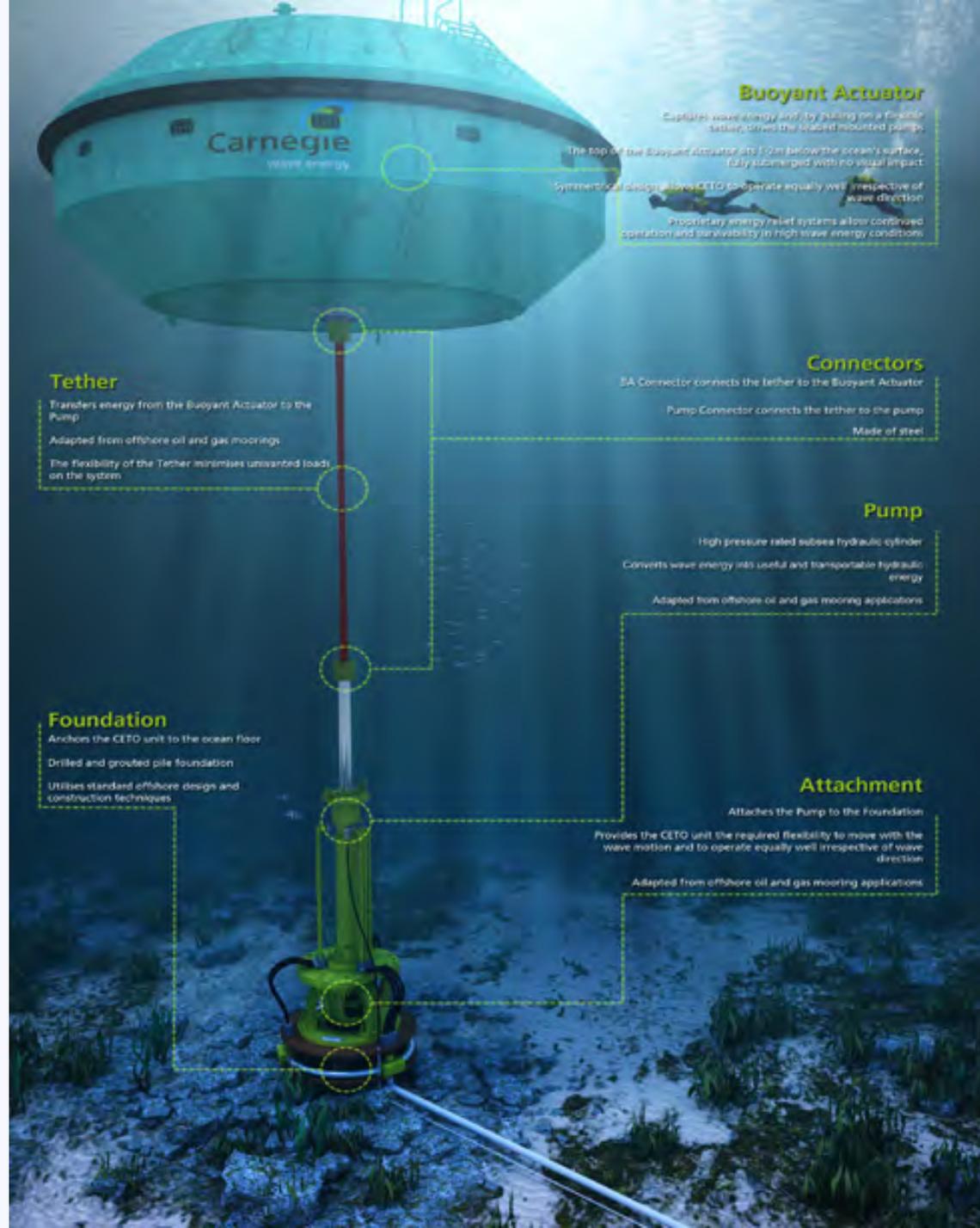
Mascaret de l'Amazone (record)

## 7 – Utilisation de l'énergie des vagues

$$\frac{E_g}{S} = \frac{1}{2} \rho A_0^2 g$$

L'énergie se propage à la vitesse de groupe  $v_g = \sqrt{\frac{g}{k}} \propto \sqrt{\lambda}$





### Buoyant Actuator

Captures wave energy and by pulling on a flexible tether, drives the seabed mounted pumps.

The top of the Buoyant Actuator sits 1.2m below the ocean's surface, fully submerged with no visual impact.

Symmetrical design allows CETO to operate equally well irrespective of wave direction.

Proprietary energy relief systems allow continued operation and survivability in high wave energy conditions.

### Connectors

EA Connector connects the tether to the Buoyant Actuator.

Pump Connector connects the tether to the pump.

Made of steel.

### Pump

High pressure rated subsea hydraulic cylinder.

Converts wave energy into useful and transportable hydraulic energy.

Adapted from offshore oil and gas mooring applications.

### Attachment

Attaches the Pump to the Foundation.

Provides the CETO unit the required flexibility to move with the wave motion and to operate equally well irrespective of wave direction.

Adapted from offshore oil and gas mooring applications.

### Tether

Transfers energy from the Buoyant Actuator to the Pump.

Adapted from offshore oil and gas moorings.

The flexibility of the Tether minimises unwanted loads on the system.

### Foundation

Anchor the CETO unit to the ocean floor.

Drilled and grouted pile foundation.

Utilises standard offshore design and construction techniques.

# « Wave Glider »

(Liquid Robotic Inc., Californie)

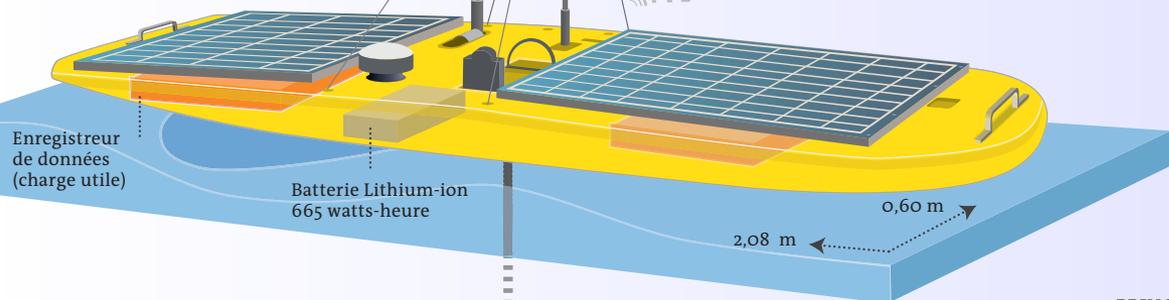


# Le Wave Glider, un robot semi-submersible 100% autonome

La société américaine Liquid Robotics a conçu, en 2008, un semi-submersible capable d'effectuer des mesures océanographiques de façon autonome, grâce à l'énergie solaire et à la force de la houle.

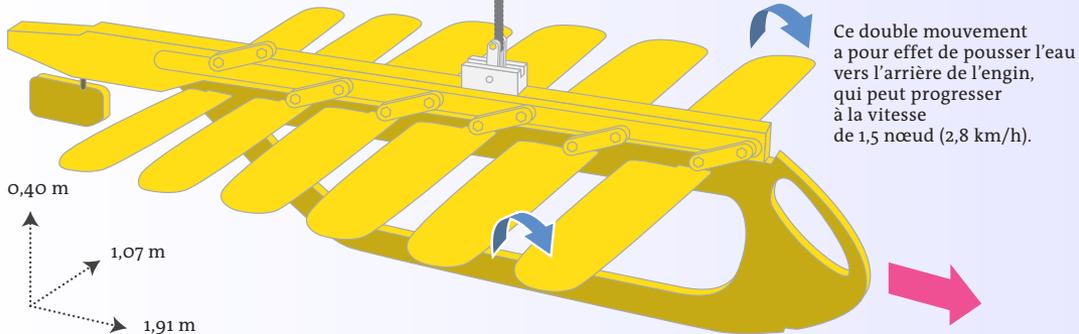
Le Wave Glider est destiné à des missions de défense, de surveillance de l'environnement et d'assistance à l'industrie de la pêche.

Le Wave Glider, doté de panneaux solaires, d'antennes et de GPS sur sa partie émergée, peut respecter un plan de route préétabli ou être dirigé vers un point particulier, tout en transmettant des données météorologiques ou océanographiques.



La force motrice est apportée par la partie sous-marine du Wave Glider, reliée au flotteur en surface par une barre de 6 mètres.

Cette structure comporte des pales horizontales mobiles, qui s'inclinent vers le bas quand le flotteur, soumis à la houle, est soulevé, et s'élevent quand il redescend.



## À L'ASSAUT DU PACIFIQUE

Le 17 novembre 2011, quatre Wave Glider ont quitté le port de San Francisco pour traverser le Pacifique.

La flotte se séparera à Hawaï, deux des robots devant rejoindre le Japon, et les deux autres l'Australie. Ce voyage transocéanique, qui doit durer 300 jours, est destiné à prouver les capacités de ces engins, qui doivent collecter quelque 2,25 millions de données sur la salinité, la température de l'eau, le taux d'oxygène, etc.

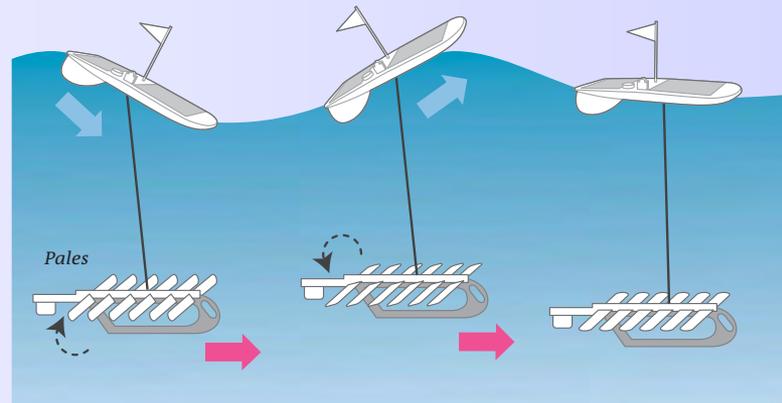
Cette odyssee peut être suivie sur le site <http://pacxdata.liquidr.com/>



Position approximative début 2012

## PRINCIPE DE PROPULSION

Transformation du mouvement vertical des vagues en poussée horizontale



Wave Glider (Liquid Robotics)



**Systeme qui désalinise l'eau de mer grâce à l'énergie des vagues.** ECOH20

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*Film :*

- Waves in Fluids, A.E. Bryson, National Committee For Fluid Mechanics Films, <http://web.mit.edu/hml/ncfmf.html>