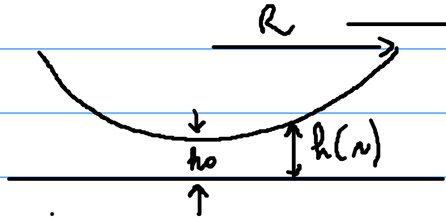
La machine de force

1) 

NS $\rho \frac{\partial \vec{v}}{\partial t} + \vec{v}.\vec{grad}\,\vec{v} = -\vec{\nabla} P + \rho \vec{g} + \vec{\Delta}\vec{v}$

$h_0 \ll R$ $\quad v(z)$

incompressibilité $\quad \mathrm{div}\,\vec{v} = 0$

$$= \underbrace{\frac{\partial v_r}{\partial r}}_{\sim \frac{v_r}{R}} + \underbrace{\frac{\partial v_z}{\partial z}}_{\sim \frac{v_z}{h_0}}$$

$$v_z \sim \frac{h_0}{R} v_r \ll v_r$$

$$\rho \vec{v}.\vec{grad}\,\vec{v} = \rho v_r . \frac{\partial \vec{v_r}}{\partial r} \sim \rho \frac{v_r^2}{R}$$

$$\rho \frac{\partial \vec{v}}{\partial t} \sim \rho \frac{v_r^2}{R}$$

$$\eta \vec{\Delta}\vec{v} \simeq \eta \frac{\partial^2 \vec{v_r}}{\partial z^2} \sim \eta \frac{v_r}{h_0^2}$$

rapport

$$\rho \frac{v_r h_0^2}{\eta R} = \underbrace{\frac{h_0}{R}}_{\ll 1} Re$$

Stokes $\quad \eta \Delta \vec{v} + \rho \vec{g} - \vec{\nabla} P = \vec{0}$

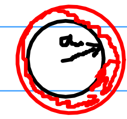
2) C.L. $\quad v_r = 0$ en $z = 0$ et $z = h$

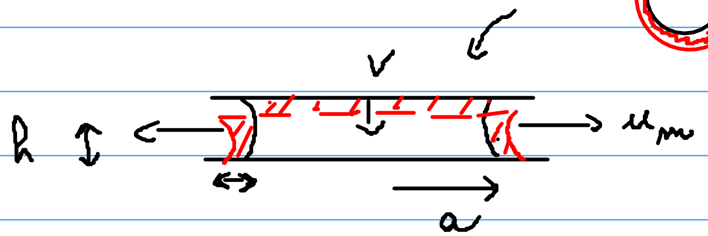
3) $\eta \frac{\partial^2 v_r}{\partial z^2} = \frac{\partial P}{\partial r}$ $\qquad v_r = \frac{1}{2\eta} \frac{\partial P}{\partial r} (z^2 - z h_0)$

$$u_m = \frac{1}{h_0} \int_0^{h_0} \frac{\partial P/\partial r}{2\eta} (z^2 - z h_0)\, dz$$

$$= \frac{1}{2\eta h_0} \frac{\partial P}{\partial r} \left[\frac{h_0^3}{3} - \frac{h_0^2}{2} \right] = -\frac{h_0^2}{12\eta} \frac{\partial P}{\partial r}$$

4) Conservation du debit





$t \to t + dt$

$$\left.\begin{aligned} &\pi r^2 V dt \\ &= 2\pi r h\, u_m dt \end{aligned}\right\} \quad \boxed{u_m = \frac{rV}{2h}}$$

5) $$u_m = \frac{rV}{2h} = -\frac{h^2}{12\eta}\frac{\partial p}{\partial r}$$

$$\int_a^r \frac{\partial p}{\partial r} = -\int_a^r \frac{6\eta r V}{h^3}\, dr$$

$$P(r) = P(a) - 6\eta V \int_a^r \frac{r}{\left(h_0 + \frac{r^2}{2R}\right)^3}\, dr$$

$$= P(a) + \frac{3\eta R V}{\left(h_0 + \frac{r^2}{2R}\right)^2} \qquad \text{pour } a \to +\infty$$

7) $$V = \frac{dh}{dt} \qquad h = h_0 + \delta h\, e^{i\omega t}$$

$$V = i\omega\, \delta h\, e^{i\omega t}$$

$$F^* = \frac{6\pi \eta^* R^2}{h_0}\, \delta h\, i\omega e^{i\omega t}$$

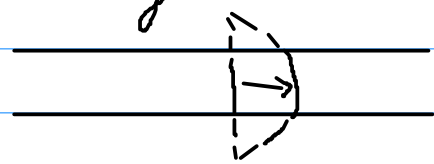
$$\eta^* = \eta' - i\omega\eta''$$

$$F^* = \frac{6\pi R^2 \delta h}{h_0}\left(\eta' i\omega + \eta''\omega^2\right) e^{i\omega t}$$

$$Z_R = \mathrm{Re}\left(\frac{F}{\delta h}\right) = \frac{6\pi R^2 \eta'' \omega^2}{h_0}$$

$$Z_I = \mathrm{Im}\left(\frac{F}{\delta h}\right) = \frac{6\pi R^2 \eta' \omega}{h_0}$$

8) Longueur de glissement $h_0 \to h_0 + b$



$u_N(z) = 0$ en $h = h_0 + b$

$h \to h_0 + b$ dans les expressions précédentes

$1/Z_R \sim h_0 + b$