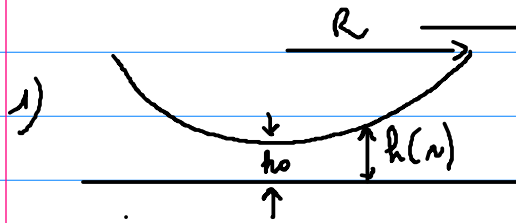


La machine de force



$$\frac{N.S}{h_0 \ll R} \quad \rho \frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \text{grad} \vec{v} = -\vec{\nabla} P + \rho \vec{g} + \Delta \vec{v}$$

$v(r)$

incompressibilité $\text{div} \vec{v} = 0$

$$= \frac{\partial v_r}{\partial r} + \frac{\partial v_z}{\partial z}$$

$$\sim \frac{v_r}{R} \quad \sim \frac{v_z}{h_0}$$

$$v_z \sim \frac{h_0}{R} v_r \ll v_r$$

$$\left. \begin{aligned} \rho \vec{v} \cdot \text{grad} \vec{v} &= \rho v_r \cdot \frac{\partial \vec{v}}{\partial r} \sim \rho \frac{v_r^2}{R} \\ \rho \frac{\partial \vec{v}}{\partial t} &\sim \rho \frac{v_r^2}{R} \end{aligned} \right\}$$

$$\eta \Delta \vec{v} \sim \eta \frac{\partial^2 v_r}{\partial z^2} \sim \eta \frac{v_r}{h_0^2}$$

rapport

$$\rho \frac{v_r h_0^2}{\eta R} = \frac{h_0}{R} Re \ll 1$$

Stokes $\eta \Delta \vec{v} + \rho \vec{g} - \vec{\nabla} P = \vec{0}$

2) C.L. $v_r = 0$ en $z=0$ et $z=h$

3) $\eta \frac{\partial^2 v_r}{\partial z^2} = \frac{\partial P}{\partial r}$

$$v_r = \frac{1}{2\eta} \frac{\partial P}{\partial r} \left(z^2 - z h_0 \right)$$

$$u_m = \frac{1}{h_0} \int_0^{h_0} \frac{\partial P}{2\eta} (z^2 - z h_0) dz$$

$$= \frac{1}{2\eta h_0} \frac{\partial P}{\partial r} \left[\frac{h_0^3}{3} - \frac{h_0^3}{2} \right] = -\frac{h_0^2}{12\eta} \frac{\partial P}{\partial r}$$

4) Conservation du débit

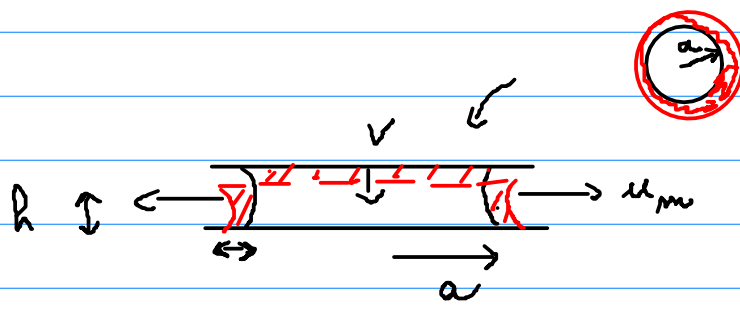


Diagram illustrating a fluid element of length a and thickness h moving with velocity v . The element is shown at two positions, t and $t+dt$, with a change in length da . The volume of the element is $V = a \cdot h \cdot 2\pi R$. The mass of the element is $m = \rho \cdot V$. The force on the element is $F = \mu_m \cdot 2\pi R \cdot h \cdot da$. The equation of motion is $m \frac{dv}{dt} = F$. This leads to the relationship $\mu_m = \frac{R \cdot V}{2h}$.

$$5) \quad \mu_m = \frac{rV}{2h} = - \frac{h^3}{12\eta} \frac{\partial p}{\partial r}$$

$$\int_a^r \frac{\partial p}{\partial r} = - \int_a^r \frac{6\eta r V}{h^3} dr$$

$$P(r) = P(a) - 6\eta V \int_a^r \frac{r}{\left(h_0 + \frac{r^2}{2R}\right)^3} dr$$

$$= P(a) + \frac{3\eta R V}{\left(h_0 + \frac{r^2}{2R}\right)^2} \quad \text{pour } a \rightarrow +\infty$$

$$7) \quad V = \frac{dh}{dt} \quad h = h_0 + \delta h e^{i\omega t}$$

$$V = i\omega \delta h e^{i\omega t}$$

$$F^* = \frac{6\pi \eta^* R^2}{h_0} \delta h i\omega e^{i\omega t}$$

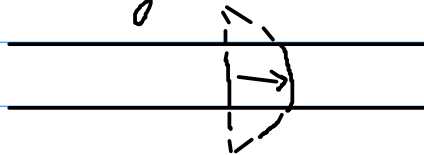
$$\eta^* = \eta' - i\omega \eta''$$

$$F^* = \frac{6\pi R^2 \delta h}{h_0} (\eta' i\omega + \eta'' \omega^2) e^{i\omega t}$$

$$Z_R = \operatorname{Re} \left(\frac{F}{8h} \right) = \frac{6\pi R^2 \eta'' \omega}{h_0}$$

$$Z_I = \operatorname{Im} \left(\frac{F}{8h} \right) = \frac{6\pi R^2 \eta' \omega}{h_0}$$

8) longueur de glissement $h_0 \rightarrow h_0 + b$



$u_v(y) = 0$ en $h = h_0 + b$

$h \rightarrow h_0 + b$ dans les expressions précédentes

$$1/Z_R \sim h_0 + b$$