Chapitre 2 : des liquides Newtoniens vers les liquides viscoélastiques

1) Les liquides Newtoniens

$\sigma = \eta \dot{\varepsilon}$ $\qquad \eta$ = viscosité
: Pa.s

$\underbrace{\varepsilon = \frac{\partial u}{\partial x}}_{\text{gradient de déplacement}} \leadsto \underbrace{\dot{\varepsilon} = \frac{\partial v}{\partial x}}_{\text{gradient de vitesse}}$ $\qquad u$ = déplacement
v = vitesse

$$\dot{\varepsilon} = \frac{1}{2} \underbrace{\begin{bmatrix} 2\frac{\partial v_x}{\partial x} & \frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y} & \frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z} \\ \frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} & 2\frac{\partial v_y}{\partial y} & \frac{\partial v_z}{\partial y} + \frac{\partial v_y}{\partial z} \\ - & - & 2- \end{bmatrix}}_{\text{tenseur des taux de déformations}}$$

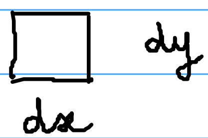
$\sigma = \eta \dot{\varepsilon}$

$P_\eta = \eta \dot{\varepsilon}^2$ $\qquad$ puissance dissipée

$E_\eta = \eta \varepsilon \dot{\varepsilon}$ $\qquad$ énergie dissipée

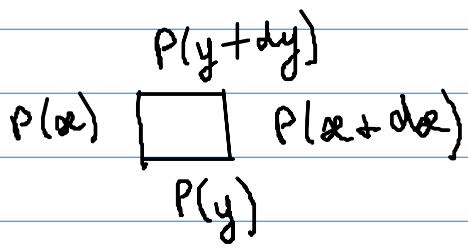
Navier – Stokes

$$\underbrace{\rho \cancel{\frac{D\vec{v}}{Dt}}}_{\text{inertie}} = \rho \vec{g} - \vec{\nabla} P + \eta \vec{\Delta} \vec{v}$$



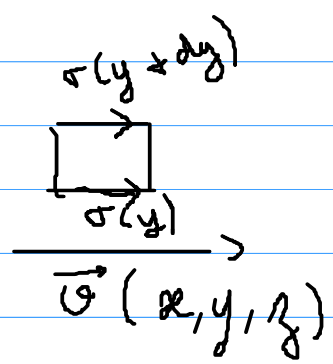
Poids $\rho \, dx \, dy \, dz \, \vec{g}$

$\hookrightarrow \rho \vec{g}$



$$\left(P(x)\, dy\, dz - P(x+dx)\, dy\, dz \right) \vec{e_x}$$
$$+ \left(P(y)\, dx\, dz - P(y+dy)\, dx\, dz \right) \vec{e_y}$$
$$= -\frac{\partial P}{\partial x}\, dx\, dy\, dz\, \vec{e_x}$$
$$-\frac{\partial P}{\partial y}\, dx\, dy\, dz\, \vec{e_y}$$

$\hookrightarrow -\vec{\nabla} P$



$$-\sigma(y)\, dx\, dz + \sigma(y+dy)\, dx\, dz$$
$$= \frac{\partial \sigma}{\partial y}\, dx\, dy\, dz$$

$$\hookrightarrow \frac{\partial \sigma}{\partial y} = \eta \frac{\partial^2 v_x}{\partial y^2} \quad \|$$

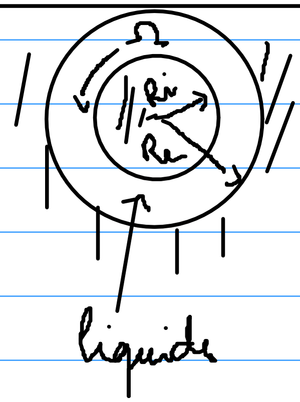
$$\vec{\Delta}\vec{v} = \left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right)\vec{e_x}$$

$$+ \left(\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right)\vec{e_y}$$

$$+ \left(\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right)\vec{e_z}$$

2 - Mesure de la viscosité

Rhéomètre de Couette



Calcul du champ de vitesse

$* \ \vec{v}(r, \cancel{\theta}, \cancel{z}) = v(r)\,\vec{u_\theta}$

$* \ 0 = \rho\vec{g} - \vec{\nabla}P + \eta\,\vec{\Delta}\vec{v}$

$$\eta \frac{\partial}{\partial r}\left(\frac{1}{r}\,\frac{\partial r v_\theta}{\partial r}\right) = 0$$

$$\frac{1}{r}\,\frac{\partial r v_\theta}{\partial r} = A$$

$$r v_\theta = A\frac{r^2}{2} + B$$

$$v_\theta(r) = A\frac{r}{2} + \frac{B}{r}$$

__conditions aux limites__

$$v_\theta(R_e) = 0 = A\frac{R_e}{2} + \frac{B}{R_e}$$

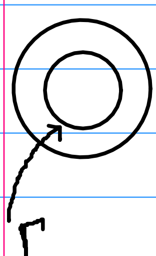
$$v_\theta(R_i) = \Omega R_i = A\frac{R_i}{2} + \frac{B}{R_i}$$

$$B = -A\frac{R_e^2}{2}$$

$$\Omega R_i^2 = \frac{A R_i^2}{2} - A\frac{R_e^2}{2\cancel{R_i}}$$

$$A = \frac{2\Omega R_i^2}{R_i^2 - R_e^2} \qquad B = -\frac{\Omega R_i^2 R_e^2}{R_i^2 - R_e^2}$$

$$v_\theta = -\frac{\Omega R_i^2}{R_e^2 - R_i^2}\, r + \frac{\Omega R_i^2 R_e^2}{R_e^2 - R_i^2}\,\frac{1}{r}$$



$$\Gamma = -2\pi R_i H\, \sigma_{r\theta}(R_i) \times R_i$$

$$\sigma_{r\theta} = \eta\left(\frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r}\right)$$

$$= -\eta\frac{\Omega R_i^2}{R_e^2 - R_i^2} - \eta\frac{\Omega R_i^2 R_e^2}{R_e^2 - R_i^2}\,\frac{1}{r^2}$$

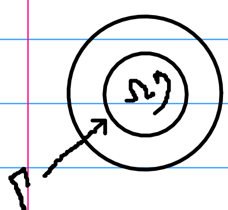
$$+\eta\frac{\Omega R_i^2}{R_e^2 - R_i^2} - \eta\frac{\Omega R_i^2 R_e^2}{R_e^2 - R_i^2}\,\frac{1}{r^2}$$

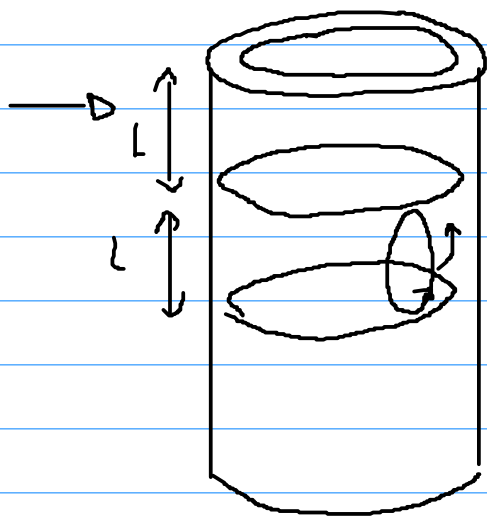
$$\sigma_{r\theta}(R_i) = -\eta\frac{\Omega R_e^2}{R_e^2 - R_i^2}$$

$$\Gamma = 2\pi R_i^2 H \eta \frac{\Omega R_e^2}{R_e^2 - R_i^2} = \eta \times \underbrace{2\pi H \frac{\Omega R_i^2 R_e^2}{R_e^2 - R_i^2}}_{\text{facteur géométrique}}$$

$$\propto \eta$$

<u>Remarque</u> $R_e \sim R_i$ $\Rightarrow$ grands couples



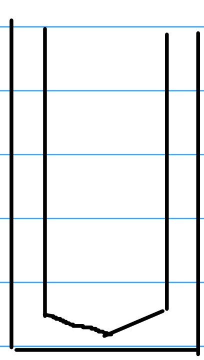


$\rightarrow$ symétrie par translation en z ?

$\exists$ instabilité à grande vitesse

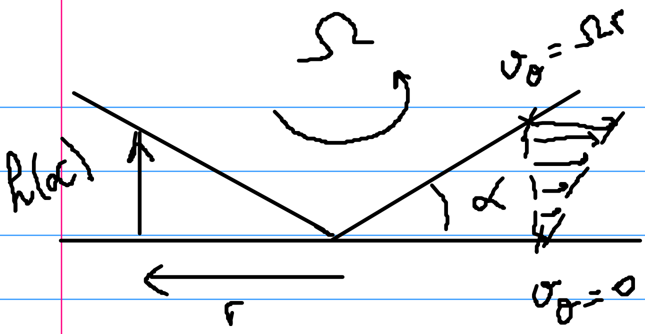
} liquide cisaillé $\Rightarrow$ couple additionnel

<u>Solution</u>



1 - $\dot{\varepsilon} = -2\Omega \dfrac{R_e^2}{R_e^2 - R_i^2}$

2 - $\dot{\varepsilon}$?



$$\tan\alpha = \frac{h}{r} \sim \alpha$$

$$\dot{\varepsilon} \sim \frac{\partial v_\theta}{\partial z} \sim \frac{v_\theta}{h} = \frac{\Omega r}{h}$$

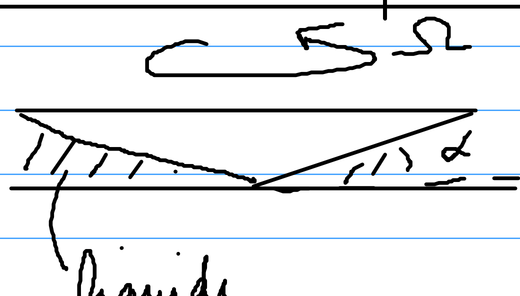
$$\dot{\varepsilon} = \frac{\Omega}{\alpha}$$

⟶ $\dot{\varepsilon}$ indépendant de r

⟶ cisaillement homogène dans le cône

⊕ $\hat{\varepsilon} = \dot{\varepsilon}$ autour du cylindre en fixant α

Rhéomètre cône-plan



$\dot{\varepsilon}$ homogène

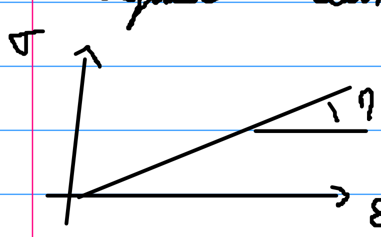
Ω fixée

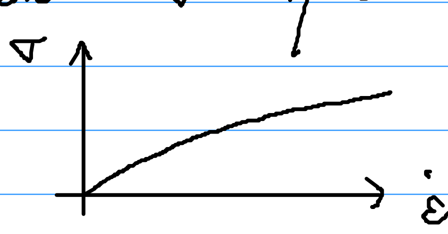
mesure Γ nécessaire pour faire tourner le cône

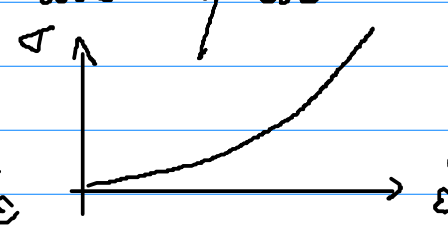
↳ η

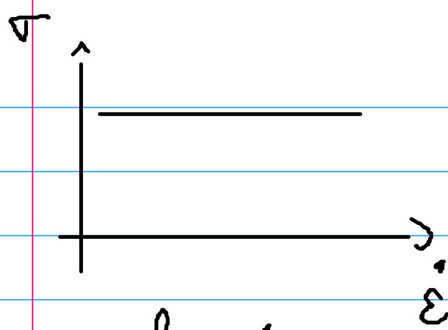
2_ Les liquides non Newtoniens

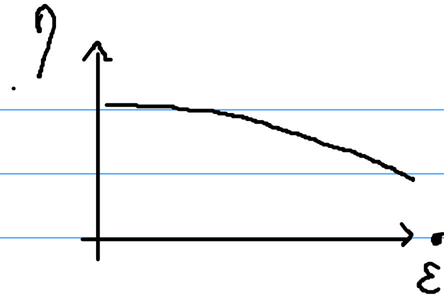
liquide newtonien $\sigma = \eta\,\dot{\varepsilon}$ avec η cst

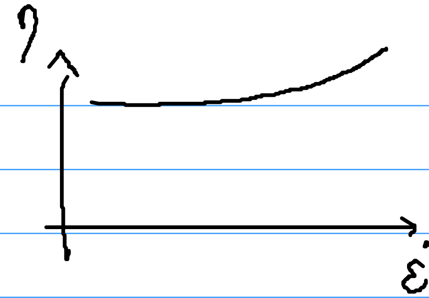








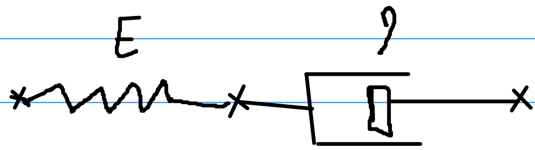




liquide newtonien — rhéofluidifiant — rhéoépaississant

3 – Les liquides viscoélastiques

a) Le modèle de Maxwell



$$\dot{\sigma} + \frac{E}{\eta}\sigma = E\dot{\varepsilon}$$

$$\tau = \frac{\eta}{E}$$

b) Temps caractéristique et nombre de Deborah

$$\tau = \frac{\eta}{E} \longleftrightarrow \tau_S \text{ temps de sollicitation}$$

$$De = \frac{\tau}{\tau_S} \quad \text{Nombre de Deborah} \rightarrow$$

$De \gg 1$ ⤳ élastique
$De \ll 1$ ⤳ visqueuse

eau: $\eta \simeq 10^{-3}$ Pa.s, $E \simeq 10^{9}$ Pa, $\tau \sim 10^{-12}$ s

huiles silicones: $\eta \simeq 2500$ Pa.s, $E \simeq 5000$ Pa, $\tau \sim 0{,}5$ s

<u>Glace</u> $\eta \sim 10^{13}$ Pa-s $\quad \tau \sim 10^4$ s
$E \simeq 10^9$ Pa

<u>Métaux - Minéraux</u> $\eta = 10^{21}$ Pa-s $\quad \tau \sim 10^{11}$ s
$E = 10^{11}$ Pa

c) <u>Mesure de viscoélasticité - Rhéologie</u>

liquide Newtonien $\qquad \sigma = \eta \dot{\varepsilon}$

liquide { complexe / viscoélastique $\qquad \sigma^* = \eta^* \dot{\varepsilon}^*$

$$\eta^* = \eta - i\frac{E}{\omega}$$

$$\varepsilon^* = \varepsilon_0 e^{i\omega t}$$

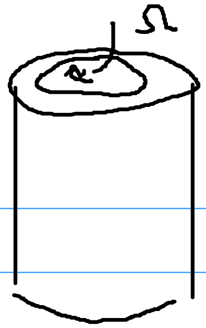
$$\sigma^* = \left(\eta - i\frac{E}{\omega}\right)\varepsilon_0 e^{i\omega t} \times i\omega$$

$$= (E + i\omega\eta)\,\varepsilon_0 e^{i\omega t} = (\underbrace{G'}_{\text{module de stockage}} + i\underbrace{G''}_{\text{module de perte}})\,\varepsilon^*$$

$$G' = E \qquad G'' = \eta\omega$$

$$\eta^* = \eta - i\frac{E}{\omega} = \eta' - i\eta'' \qquad \begin{cases} \eta' = \eta \\ \eta'' = E/\omega \end{cases}$$

$$\sigma = \underbrace{E\varepsilon_0 \cos\omega t}_{\text{en phase}} - \underbrace{\omega\eta\varepsilon_0 \sin\omega t}_{\text{en opposition de phase}}$$



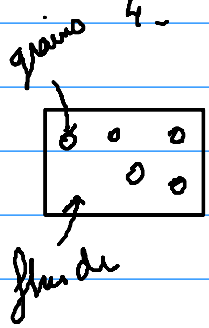
$$\Gamma = 2\pi R_i^2 H \sigma$$
$$= \Gamma' \cos \omega t + \Gamma'' \sin \omega t$$

$$\sigma = \frac{2R_e^2}{R_e^2 - R_i^2} \Omega \eta$$

$$\Gamma'' = 4\pi H \frac{R_i^2 R_e^2}{R_e^2 - R_i^2} \Omega \eta'' \qquad \eta'' = \frac{E}{\omega}$$

$$\Gamma' = 4\pi H \frac{R_i^2 R_e^2}{R_e^2 - R_i^2} \Omega \eta' \qquad \eta' = \eta$$

4. L'exemple des suspensions



$$\phi = \frac{V_{grains}}{V_{total}}$$

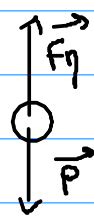
a) Le nombre de Peclet

$$Pe = \frac{\text{effets hydro}}{\text{effets de diffusion}} = \frac{UL}{D}$$

exemple convection par sedimentation vs diffusion

Coefficient de diffusion $D = \dfrac{k_B T}{6\pi\eta R}$

Sedimentation



$$F_\eta = 6\pi\eta R U$$
$$P = \rho \frac{4}{3}\pi R^3 g$$

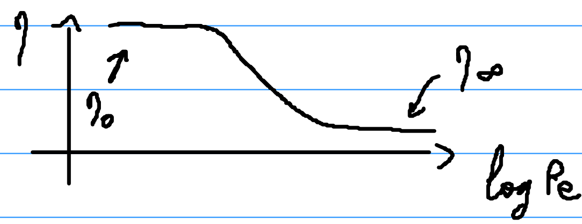
$$F_\eta = P \Rightarrow U = \frac{2}{9}\rho R^2 g / \eta$$

$$Pe = \frac{U \times 2R}{D} = \frac{2}{9}\frac{\rho R^2 g}{\eta} \times 2R \times \frac{6\pi\eta R}{k_B T} = \frac{8\pi}{3}\frac{\rho g R^4}{k_B T}$$

$$\Rightarrow \quad Pe < 1 \qquad R < \left(\frac{3k_BT}{8\pi\rho g}\right)^{1/4} = 200 \text{ nm}$$

$R < 200$ nm $\rightarrow$ particules browniennes
$\rightarrow$ suspensions stables

b) viscosité des suspensions



Pe petit $\rightarrow$ vitesse faible η_0
Pe grand $\rightarrow$ gde vitesse η_∞

} caractère rhéofluidifiant = alignement des particules à grande vitesse.

c) Evolution de la viscosité avec ϕ

Loi d'Einstein ϕ petit $\rightarrow$ comportement linéaire

$$\eta = \eta_s(1 + 2{,}5\,\phi)$$

Modèle d'Arrhenius

Solution = suspension à ϕ
$\hookrightarrow$ suspension à $\phi + d\phi$ ↙ Einstein

$$\eta(\phi + d\phi) = \eta(\phi)\,(1 + 2{,}5\,d\phi)$$

$$d\eta = 2{,}5\,\eta(\phi)\,d\phi$$

volume exclu

$$d\eta = 2{,}5\,\eta(\phi)\,\frac{d\phi}{1 - K\phi}$$

$$\boxed{\eta = \eta_s \left(1 - K\phi\right)^{-\frac{2,5}{K}}}$$

<u>ex</u> : les emulsions appoloniennes

$\begin{cases} \text{emulsions huile dans eau} \\ \text{particules suspendues = gouttes d'huile} \end{cases}$

d) <u>Modéliser la rhéologie d'une suspension</u>

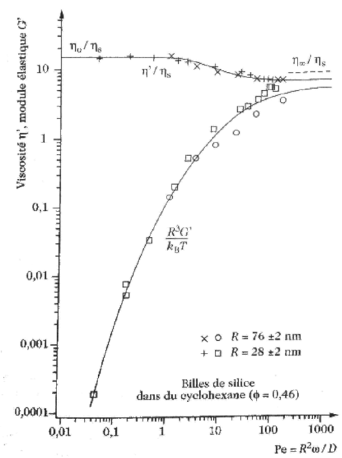
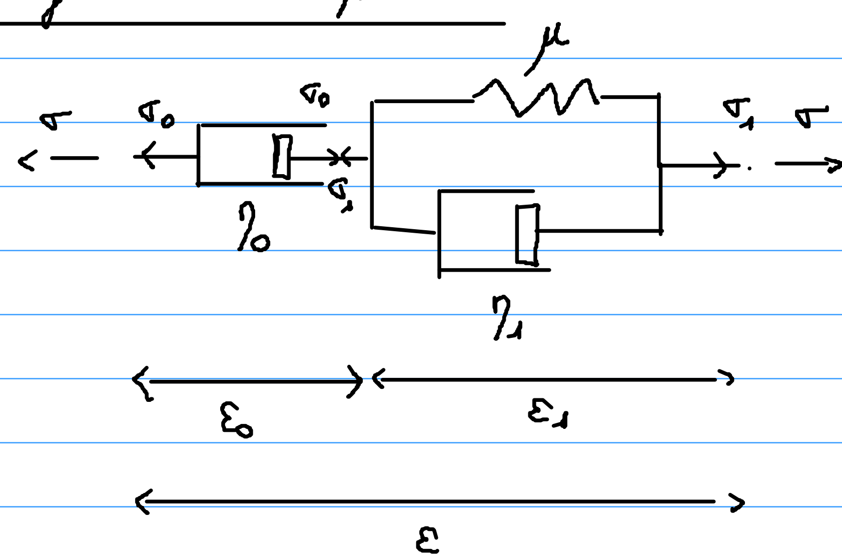


Figure 7.31. Viscosité η' et module élastique G' en fonction du nombre de Péclet [Mellema et al. 1987].



$$\varepsilon = \varepsilon_0 + \varepsilon_1 \qquad \sigma = \eta_0 \dot{\varepsilon}_0 = \mu_1 \varepsilon_1 + \eta_1 \dot{\varepsilon}_1$$

$$\dot{\varepsilon} = \frac{\sigma}{\eta_0} + \frac{\sigma}{\eta_1} - \frac{\mu_1}{\eta_1}\varepsilon_1$$

$$= \left(\frac{1}{\eta_0} + \frac{1}{\eta_1}\right)\sigma - \frac{\mu_1}{\eta_1}\left(\varepsilon - \varepsilon_0\right)$$

$$\ddot{\varepsilon} = \left(\frac{1}{\eta_0} + \frac{1}{\eta_1}\right)\dot{\sigma} - \frac{\mu_1}{\eta_1}\left(\dot{\varepsilon} - \frac{\sigma}{\eta_0}\right)$$

$$\varepsilon = \varepsilon_0 \cos\omega t \qquad \varepsilon^* = \varepsilon_0 e^{i\omega t} \qquad \sigma^* = \sigma_0 e^{i\omega t}$$

$$-\omega^2 \varepsilon^* = \left(\frac{1}{\eta_0} + \frac{1}{\eta_1}\right) i\omega\sigma^* - \frac{\mu_1}{\eta_1}\left(i\omega\varepsilon^* - \frac{\sigma^*}{\eta_0}\right)$$

$$\left(-\omega^2 + \frac{\mu_1}{\eta_1} i\omega\right)\varepsilon^* = \left(\left(\frac{1}{\eta_0} + \frac{1}{\eta_1}\right) i\omega + \frac{\mu_1}{\eta_0\eta_1}\right)\sigma^*$$

$$\sigma^* = \frac{-\omega^2 + i\omega \frac{\mu_1}{\eta_1}}{\left(\frac{1}{\eta_0} + \frac{1}{\eta_1}\right) i\omega + \frac{\mu_1}{\eta_0 \eta_1}} \varepsilon^*$$

$$= \frac{\left(-\omega^2 + i\omega \frac{\mu_1}{\eta_1}\right)\left(\frac{\mu_1}{\eta_0\eta_1} - i\omega\left(\frac{1}{\eta_0} + \frac{1}{\eta_1}\right)\right)}{\left(\frac{\mu_1}{\eta_0\eta_1}\right)^2 + \left(\frac{1}{\eta_0} + \frac{1}{\eta_1}\right)^2 \omega^2} \varepsilon^*$$

$$\frac{\sigma^*}{\varepsilon^*} = \frac{-\omega^2 \cancel{\mu_1}/\eta_0\eta_1 + \omega^2 \frac{\mu_1}{\eta_1}\left(\cancel{\frac{1}{\eta_0}} + \frac{1}{\eta_1}\right)}{\left(\frac{\mu_1}{\eta_0\eta_1}\right)^2 + \left(\frac{1}{\eta_0} + \frac{1}{\eta_1}\right)^2 \omega^2} + i\omega \frac{\frac{\mu_1^2}{\eta_0 \eta_1^2} + \omega^2\left(\frac{1}{\eta_0} + \frac{1}{\eta_1}\right)}{\left(\frac{\mu_1}{\eta_1\eta_0}\right)^2 + \left(\frac{1}{\eta_0} + \frac{1}{\eta_1}\right)^2 \omega^2}$$

$$= \underbrace{\frac{\mu_1 \omega^2 \eta_0^2}{\mu_1^2 + (\eta_1 + \eta_0)^2 \omega^2}}_{G'} + i\omega\, \eta_0 \underbrace{\frac{\mu_1^2 + \eta_1(\eta_0 + \eta_1)\omega^2}{\mu_1^2 + (\eta_1 + \eta_0)^2 \omega^2}}_{G'' = \eta\omega}$$

$$\eta_\infty = \frac{\eta_1 \eta_0}{\eta_1 + \eta_0} \qquad \tau_0 = \frac{\eta_1}{\mu_1} \qquad \tau_1 = \frac{\eta_1 + \eta_0}{\mu_1}$$

$$\tau_3 = \frac{\eta_0}{\mu_1}$$

$$\frac{\eta_1}{\eta_\infty} = \frac{\eta_1 + \eta_0}{\eta_0}$$

$$G' = \frac{\eta_0^2 \mu_1 \omega^2}{\mu_1^2 + (\eta_1 + \eta_0)^2 \omega^2} = \frac{\mu_1 \omega^2}{\frac{1}{\tau_3^2} + \left(\frac{\eta_1}{\eta_\infty}\right)^2 \omega^2}$$

$$\xrightarrow[\omega \to 0]{} \quad G' \to 0 \qquad G' \sim \tau_3^2 \mu_1 \omega^2$$

$$\xrightarrow[\omega \to +\infty]{} \quad \left(\frac{\eta_\infty}{\eta_1}\right)^2 \mu_1$$

$$\eta(\omega) = \eta_0 \frac{\mu_1^2 + \eta_1(\eta_1+\eta_0)\omega^2}{\mu_1^2 + (\eta_1+\eta_0)^2\omega^2}$$

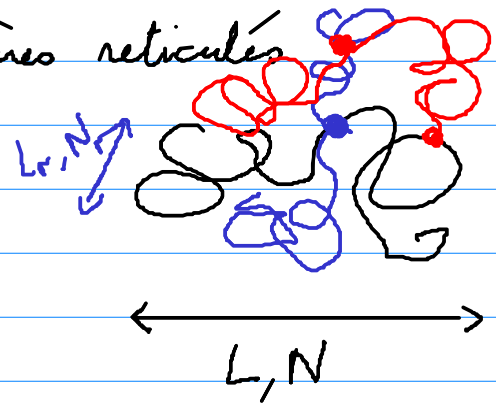
$$= \eta_0 \frac{1 + \tau_0\tau_1\omega^2}{1+\tau_1^2\omega^2}$$

$$\xrightarrow[\omega \to 0]{} \eta_0$$

$$\xrightarrow[\omega \to +\infty]{} \eta_\infty = \frac{\eta_1\eta_0}{\eta_1+\eta_0}$$

5) L'exemple des solutions de polymères

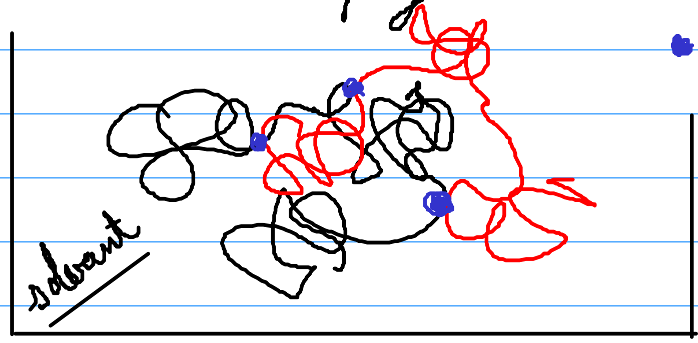
polymères reticulés



$L = \sqrt{N}\, a$

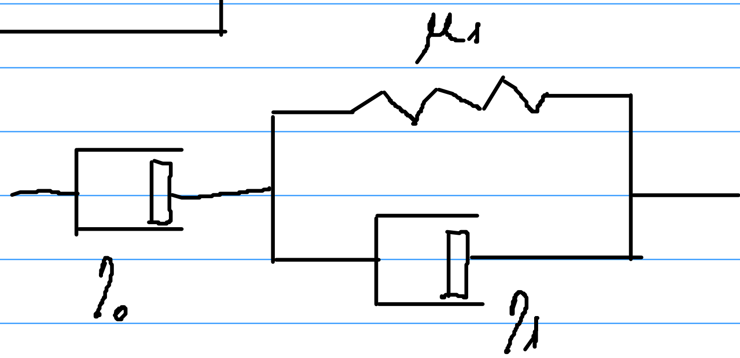
$L_r = \sqrt{N_r}\, a$

ici = solution de polymère



• noeuds physiques

un modèle mécanique



$$G' = \mu_1 \frac{\eta_0^2 \omega^2}{\mu_1^2 + (\eta_1+\eta_0)^2 \omega^2} \quad \xrightarrow{\omega \to 0} \frac{\eta_0^2}{\mu_1}\omega^2 \qquad \xrightarrow{\omega \to +\infty} \mu_1 \frac{\eta_0^2}{(\eta_1+\eta_0)^2}$$

$$G'' = \eta_0 \omega \frac{\mu_1^2 + \eta_1(\eta_0+\eta_1)\omega^2}{\mu_1^2 + (\eta_0+\eta_1)^2 \omega^2} \quad \xrightarrow{\omega \to 0} \eta_0 \omega \qquad \xrightarrow{\omega \to +\infty} \frac{\eta_0 \eta_1}{\eta_0+\eta_1}\omega$$