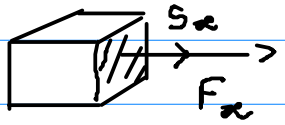
Passage de 1D à 3D

(1D) $\sigma = \frac{F}{S}$ $\quad \varepsilon = \frac{\partial u}{\partial x}$ $\quad \dot{\varepsilon} = \dot{\gamma} = \frac{\partial v}{\partial x}$

(3D) contrainte $\quad \sigma_{ij} = \frac{F_i}{S_j}$

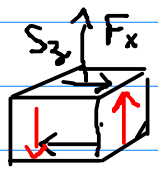


$\sigma_{xx} = \frac{F_x}{S_x}$ contrainte de compression / traction

⚠ $\vec{S_x}$ = vecteurs sortants

$\sigma_{xx} > 0$ traction

$\sigma_{xx} < 0$ compression



$\sigma_{xy} = \frac{F_x}{S_y}$ contrainte de cisaillement

$$\underline{\underline{\sigma}} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$ = tenseur symétrique

$\sigma_{ij} = \sigma_{ji}$

déformation $\quad \varepsilon_{ij} \leftrightarrow \frac{\partial u_i}{\partial x_j}$

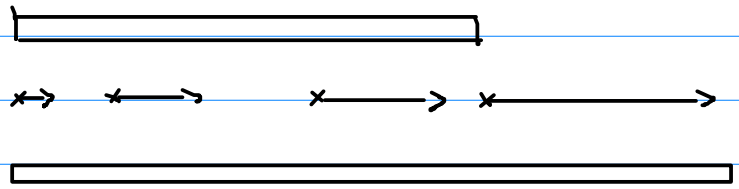
$$\begin{bmatrix} \frac{\partial u_x}{\partial x} & \frac{\partial u_x}{\partial y} & \frac{\partial u_x}{\partial z} \\ \frac{\partial u_y}{\partial x} & \frac{\partial u_y}{\partial y} & \frac{\partial u_y}{\partial z} \\ \partial u_z/\partial x & \partial u_z/\partial y & \partial u_z/\partial z \end{bmatrix}$$

tenseur des gradients de déplacement

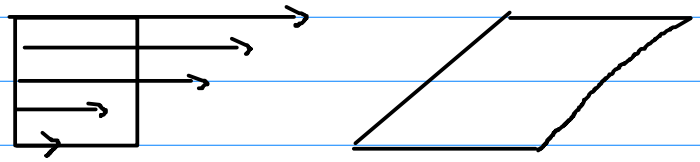
partie symétrique + partie antisymétrique

$$= \underline{\underline{\varepsilon}} + \underline{\underline{\omega}}$$

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$



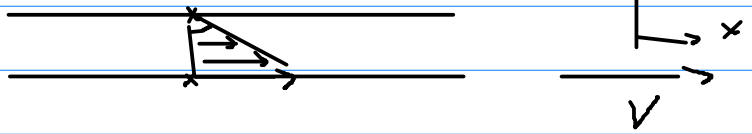
ε_{xx} = dilatation



ε_{xz} = cisaillement

taux de déformation

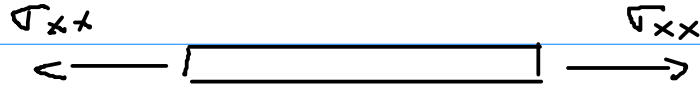
$$\dot{\varepsilon}_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)$$



$$\frac{\partial v_z}{\partial x}$$

$$\dot{\varepsilon}_{xz} = \frac{1}{2}\frac{\partial v_z}{\partial x}$$

Loi de Hook



$$\varepsilon_{xx} = \frac{\sigma_{xx}}{E}$$

$$\varepsilon_{yy} = \varepsilon_{zz} = -\frac{\nu}{E}\sigma_{xx}$$

$$\varepsilon_{xx} = \frac{\sigma_{xx}}{E} - \frac{\nu}{E}\sigma_{yy} - \frac{\nu}{E}\sigma_{zz}$$

$$\varepsilon_{yy} =$$

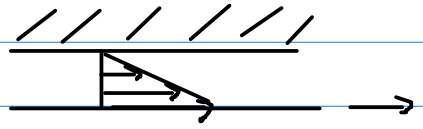
$$\varepsilon_{zz} =$$

$$\varepsilon_{xy} = \frac{\sigma_{xy}}{2\mu} \qquad \varepsilon_{xz} = \frac{\sigma_{xz}}{2\mu} \qquad \varepsilon_{yz} = \frac{\sigma_{yz}}{2\mu}$$

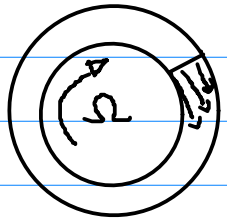
μ = module de cisaillement

Loi des fluides Newtoniens

$$\underline{\underline{\tau}} = 2\eta\,\underline{\underline{\dot{\varepsilon}}}$$



$$\sigma_{xz} = \eta \frac{\partial v_x}{\partial z}$$



$$\sigma_{r\theta} = \eta\left(\frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r}\right)$$