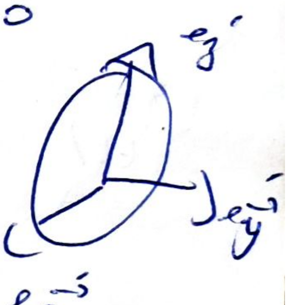
1] Solide en rotation (~~selon~~ autour d'un axe ~~preferencielle~~ préférentiel)

1) Solide en rotation ~~est~~ et angles d'Euler

Soit un solide indéformable S soumis à aucune force avec $I_S(O) = \begin{pmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{pmatrix}$

$$\left.\frac{d\vec{L}^*}{dt}\right|_R = \left.\frac{d\vec{L}^*}{dt}\right|_{R'} + \vec{\Omega}_{S/R} \times \vec{L}^* = \vec{0}$$



$$\begin{cases} I_x \dot{\omega}_x + (I_z - I_y)\,\omega_y \omega_z = 0 \\ I_y \dot{\omega}_y + (I_x - I_z)\,\omega_x \omega_z = 0 \end{cases}$$

Sym axiale selon $\vec{e_z'}$ $\qquad \omega_0 = \omega_z \dfrac{I_z - I_{xy}}{I_{xy}}$

$\omega_{z'} = \text{cst}$ $\qquad \dot{\omega}_{x'} = -\omega_0\, \omega_{y'}$

$\qquad \dot{\omega}_{y'} = +\omega_0\, \omega_{x'}$

2) $\omega_{S/R'} = \dot{\theta} u + \dot{\psi} e_z + \dot{\varphi} e_{z'}$

θ

ψ

φ

2) Approximation gyroscopique

○ $\overrightarrow{\omega_{S/R'}} \simeq \omega \overrightarrow{e_z'}$

○ $\dot{\varphi} \gg \dot{\theta}$ et $\dot{\varphi} \gg \dot{\psi}$

3) Effet gyroscopique

(a) $\frac{d\vec{L_o}}{dt} = \overrightarrow{OC} \times m\vec{g} = \vec{L_o} \times \frac{m\vec{g}c}{L_o}$

$= \vec{\omega_p} \times \vec{L}$ $\quad \vec{\omega_p} \approx \frac{mgc}{I_3\omega} \vec{e_z}$

(1) $L_o \cdot (a) \rightarrow \underline{L_o = cst}$

(2) $e_z \cdot (a) \rightarrow \underline{L_{oz} = cst}$

○ Couple gyroscopique

$\left(\frac{d\vec{L_c}}{dt}\right)_R = \vec{\Gamma}_{c\, ex \to gyro}$

$\left(\frac{d\vec{L_c}}{dt}\right)_R = \left(\frac{d\vec{L_c}}{dt}\right)_\Delta + \overrightarrow{\omega_{\Delta/R}} \times \vec{L_c} = -\Gamma_{c\, gyro \to ex}$

$M_{C, g \to sx} = L_C \times \omega_{A/R}$

2) Application : du gyroscope

a) Précession des équinoxes

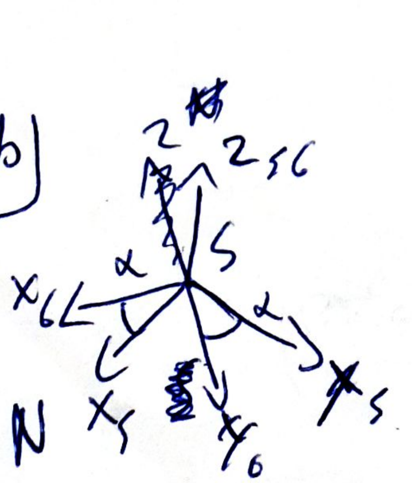
$M_T = -A \sin(2\theta_0) \vec{e_\psi} \sin(\theta_0)$ $\quad \dot{\phi} = 7{,}3 \cdot 10^{-5} \gg \omega_p$

$$\dot{\psi} = \frac{M_{yC}}{L_0} = \frac{-A \sin(2\theta_0) / \sin(\theta_0)}{I_3 \dot{\phi}}$$

$T_a^2 / r_S^3 = 4\pi^2 / G M_S$ $\quad \dot{\phi} = \frac{2\pi}{T_j}$

$T_a \simeq 25\,900$ années

b)



δ angle latitude

α alignement avec le nord sud

ψ rotation gyro

$$\frac{d}{dt}\left(\frac{d\mathcal{L}_1}{d\dot{\alpha}}\right) = \frac{\partial \mathcal{L}_1}{\partial \alpha}$$

$$I_2 \ddot{\alpha} = L_x \Omega \sin(\delta) \sin(\alpha) + \frac{1}{2} I_2 \Omega^2 \sin^2(\delta) \sin(2\alpha)$$

Pôle $\sin \delta = 0$

$$L_x = I_1 \dot{\varphi} = \text{cst}$$

$$I_2 \ddot{\alpha} = 0$$

sur une latitude constante δ

$\sin \delta \neq 0 \qquad \alpha \simeq 0$

$$L_x \simeq I_1 (\dot{\varphi} - \Omega \sin \delta)$$

$$I_2 \ddot{\alpha} \simeq (L_x \Omega \sin(\delta) + I_2 \Omega^2 \sin^2(\delta)) \alpha$$

$L_x < 0 \qquad$ gyro $|\dot{\varphi}| \gg \Omega$

$$L_x \simeq -I_1 |\dot{\varphi}| = \text{cst}$$

$$I_2 \ddot{\alpha} \simeq -I_1 |\dot{\varphi}| \Omega \sin(\delta) \alpha$$

oscillation $\qquad \hat{\omega} = \left(\frac{I_1 |\dot{\varphi}| \Omega \sin(\delta)}{I_2} \right)^{1/2}$

c) $\vec{\mu} = i S \vec{n} = \frac{\delta q}{dt} \pi r^2 \vec{n} = \frac{-e v r}{2} \vec{n}$

$L_o = r \times m_e v$

soit $\vec{\mu} = \gamma \vec{L_c}$

Système $\{e^-\}$ moment du en champ B

$$\left(\frac{d L_o}{dt}\right)_R = \gamma \vec{L_o} \times \vec{B} = \omega_L \times L_o$$

$\omega_L = -\gamma B$ pulsation de

Larmor

$1T \rightarrow 10^{11}\ rad.s^{-1}$ $\frac{\Omega}{\omega_0}$ faible

gyro ok