

# Turbomachinery Aero-Thermodynamics

## Aero-Thermodynamics 2D – Losses

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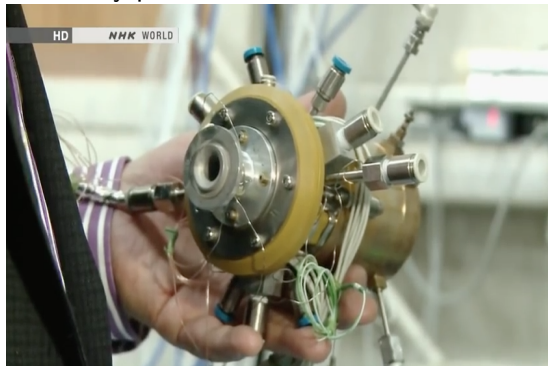
Evaluation for sessions 1 & 2

**QROC I (20 mins)**

# And now what are the stakes and technologies?

Miniaturizing technology

Electricity production – Feed the robots needs



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- Naive derivation
- Rothalpy
- Formal derivation

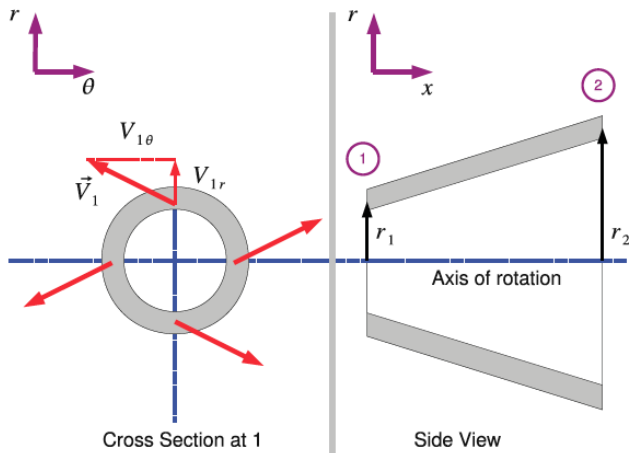
## 2. Velocity triangles

## 3. Losses in axial compressors

- Introduction
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# Euler theorem for turbomachines – Naive derivation

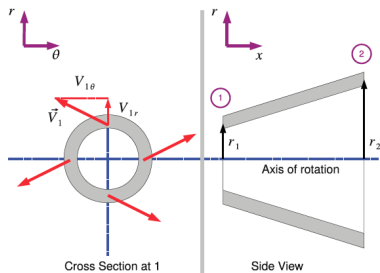
Let's consider the following axial machine



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# Euler theorem for turbomachines – Naive derivation

The torque experienced by a physical system is equal to the temporal change of its angular momentum. i.e.  $C = \frac{dM_{\theta}}{dt}$ (1).



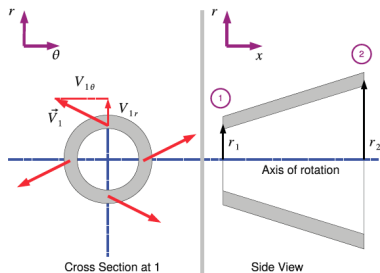
If we apply this relation to the system on the left, assuming velocities are uniform in 1 and 2, we obtain that

$$C = \frac{dM_{\theta_2} - dM_{\theta_1}}{dt}$$

$$C = \frac{dm_2 V_{\theta_2} r_2 - dm_1 V_{\theta_1} r_1}{dt}$$

<sup>1</sup>The angular momentum of a rotating mass is defined as  $M_{\theta} = mV_{\theta}r$

# Euler theorem for turbomachines – Naive derivation



$$\begin{aligned}C &= \dot{m}(V_{\theta 2} r_2 - V_{\theta 1} r_1) \\C\omega &= \dot{m}(V_{\theta 2} r_2 \omega - V_{\theta 1} r_1 \omega) \\P_u &= \dot{m}(V_{\theta 2} U_2 - V_{\theta 1} U_1) \\\Delta w_u &= \Delta(V_{\theta} U)\end{aligned}$$

# Euler theorem for turbomachines – Naive derivation

## Euler theorem for turbomachines

This relation is fundamental. It relates the changes in velocity directions and intensity (aerodynamics) to the effective work (thermodynamics). It applies to all kind of turbomachines (axial, radial, mixed).

$$\Delta w_u = \Delta h_0 = \Delta(V_\theta U)$$

Note! Thanks to the representation of the velocity vectors we can learn about the work exchange.



# Euler theorem for turbomachines – A few comments

This fundamental equation brings a few important comments

- Note!  $\Delta w_u > 0$  for a compressor and  $\Delta w_u < 0$  for a turbine. In some textbooks dedicated to turbines, the relation is multiplied by -1 to have positive quantities...Be careful!
- Note!  $M_{theta} = f(\rho, V_\theta, r)$ . Modifying the radius of a stage between 1 and 2 will therefore lead to potentially more power delivered (turbine) or a higher compression rate (compressors). **This effect is the main reason for the development of centrifugal compressors and turbines.**

# Rothalpy

The rothalpy is defined as

$$I = h_0 - UV_\theta$$

This definition comes in handy because of the Euler equation for turbomachines which states that through the rotor

$$\Delta h_0 = \Delta(UV_\theta)$$

We therefore have

$$\Delta(h_0 - UV_\theta) = \Delta I = 0$$

## Rothalpy

The rothalpy  $I = h_0 - UV_\theta$  is conserved through a turbomachinery stage.<sup>a</sup>

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<sup>a</sup>Note however that the rothalpy is not a thermodynamic variable per se, it depends of the frame of reference.

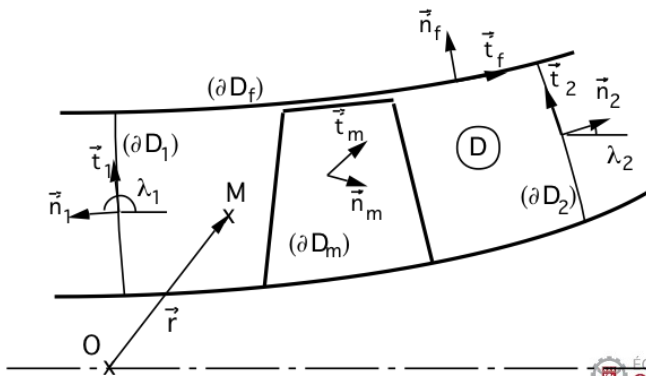
# Euler theorem for turbomachines – Formal derivation

Let's start from the conservation equation for the angular momentum.

$$\underbrace{\frac{\partial(\vec{r} \wedge \rho \vec{V})}{\partial t} + \text{div}((\vec{r} \wedge \rho \vec{V}) \otimes \vec{V})}_{\rho \frac{D(\vec{r} \wedge \vec{V})}{Dt}} = \vec{r} \wedge \rho \vec{g} + \text{div}(\vec{r} \wedge \vec{\sigma})$$

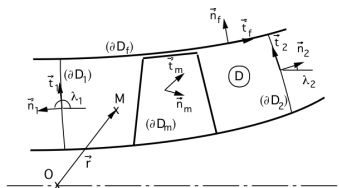
# Euler theorem for turbomachines – Formal derivation

Let's now consider the integration volume  $D$  represented by its meridional view below



# Euler theorem for turbomachines – Formal derivation

The integration volume is only composed of fluid particles. The integration volume is enclosed in its external surface  $\partial D$  which is composed of



- $\partial D_1$  Inlet surface. fixed. Azimuthal symmetry
- $\partial D_2$  Outlet surface. fixed. Azimuthal symmetry
- $\partial D_m$  Solid rotating surface (blades and hub)
- $\partial D_f$  Solid fixed surface (shroud). Azimuthal symmetry

# Euler theorem for turbomachines – Formal derivation

Let's address the first term,

$$\int_D \rho \frac{Df}{Dt} dv = \int_D \left[ \underbrace{\frac{\partial \rho f}{\partial t}}_{\text{Fixed frame}} + \text{div}(\rho f \vec{V}) \right] dv,$$
$$\int_D \rho \frac{Df}{Dt} dv = \int_D \left[ \underbrace{\frac{\delta \rho f}{\delta t}}_{\text{Rotating frame}} + \text{div}(\rho f \vec{W}) \right] dv,$$

The relative velocity  $\vec{W}$  is defined as  $\vec{W} = \vec{V} - \vec{U}$ .  $\vec{W}$  is zero on the rotating solid surfaces.

The absolute velocity  $\vec{V}$  is zero on the fixed solid surfaces.

# Euler theorem for turbomachines – Formal derivation

Now let's integrate the equation for the angular momentum over D

$$\int_D \frac{\delta \vec{r} \wedge \rho \vec{V}}{\delta t} dv + \int_{\delta D} (\vec{r} \wedge \rho \vec{V})(\vec{V} - \vec{U}) \cdot \vec{n} ds =$$
$$\int_D (\vec{r} \wedge \rho \vec{g}) dv + \int_{\delta D} (\vec{r} \wedge \vec{\sigma} \vec{n}) ds$$

The flow is assumed to be steady in the rotating frame so that

$$\int_{\delta D} (\vec{r} \wedge \rho \vec{V})(\vec{V} - \vec{U}) \cdot \vec{n} ds = \int_D (\vec{r} \wedge \rho \vec{g}) dv + \int_{\delta D} (\vec{r} \wedge \vec{\sigma} \vec{n}) ds$$

# Euler theorem for turbomachines – Formal derivation

- $\vec{W} = 0$  on  $\partial D_m$
- $\vec{V} = 0$  on  $\partial D_f$
- $\int_D (\vec{r} \wedge \rho \vec{g}) dv = 0$  because of the axisymmetry

$$\int_{\partial D_1 \cup \partial D_2} (\vec{r} \wedge \rho \vec{V})(\vec{V} - \vec{U}) \cdot \vec{n} ds = \int_{\delta D} (\vec{r} \wedge \bar{\tau} \vec{n}) ds - \int_{\delta D} (\rho \vec{r} \wedge \vec{n}) ds$$

- $\vec{U} \cdot \vec{n} = 0$  on  $\partial D_1 \cup \partial D_2$

$$\int_{\partial D_1 \cup \partial D_2} (\vec{r} \wedge \rho \vec{V}) \vec{V} \cdot \vec{n} ds = \int_{\delta D} (\vec{r} \wedge \bar{\tau} \vec{n}) ds - \int_{\delta D} (\rho \vec{r} \wedge \vec{n}) ds$$



# Euler theorem for turbomachines – Formal derivation

Balance of angular momentum along the  $\vec{j}$  axis

$$\int_{\partial D_1 \cup \partial D_2} r V_{\theta} \rho \vec{V} \cdot \vec{n} ds = \int_{\delta D} (\vec{r} \wedge \bar{\vec{\tau}} \vec{n}) \cdot \vec{j} ds - \int_{\delta D} (p \vec{r} \wedge \vec{n}) \cdot \vec{j} ds$$

The axial torque imposed by the mobile solid boundaries on the fluid is

$$C = \int_{\delta D_m} (\vec{r} \wedge \bar{\vec{\tau}} \vec{n}) \cdot \vec{j} ds - \int_{\delta D_m} (p \vec{r} \wedge \vec{n}) \cdot \vec{j} ds$$

- $\bar{\vec{\tau}} \approx 0$  on  $\partial D_1 \cup \partial D_2$
- $(p \vec{r} \wedge \vec{n}) \cdot \vec{j} = 0$  on  $\partial D_1 \cup \partial D_2 \cup D_f$  because  $\vec{n} \cdot \vec{i}_{\theta} = 0$

$$\int_{\partial D_1 \cup \partial D_2} r V_{\theta} \rho \vec{V} \cdot \vec{n} ds = C + \underbrace{\int_{\delta D_f} (\vec{r} \wedge (\bar{\vec{\tau}} \vec{n})_{i_{\theta}}) \cdot \vec{j} ds}_{C_{\text{friction}} = \text{Shroud axial friction torque}}$$

# Euler theorem for turbomachines – Formal derivation

Let's now use a streamtube as our integration volume.

- $r = \text{constant}$  and  $V_\theta = \text{constant}$  on  $\partial D_1 \cup \partial D_2$
- $dm = \int_{\partial D_2} \rho \vec{V} \cdot \vec{n} ds = \text{constant}$
- No fixed surface  $\rightarrow$  no friction torque

$$\begin{aligned} dm \Delta(rV_\theta) &= dC \\ dm \Delta(r\omega V_\theta) &= dC\omega = dP \\ \Delta(UV_\theta) &= \Delta w_u \end{aligned}$$

Since the transformation is assumed to be adiabatic ( $\Delta q = 0$ ), we have

$$\Delta h_0 = \Delta(UV_\theta)$$

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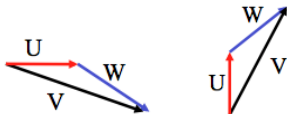
- Introduction
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# Velocity triangle

In a turbomachines, the velocity composition is considered on the cascade surface and leads to the following vectorial expression:

$$\vec{V} = \vec{W} + \vec{U}$$

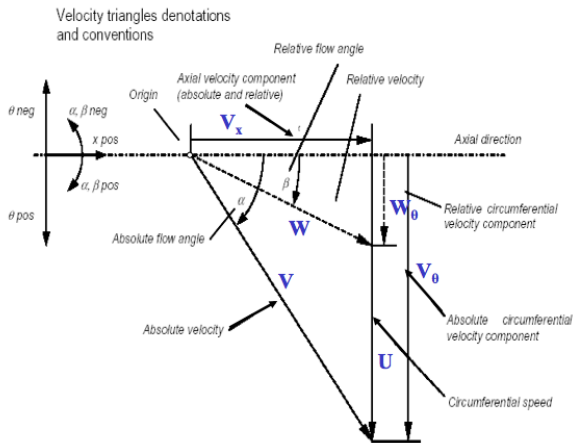
It is represented as follows



Both pictures represent the same velocity triangle. The only difference between the two pictures is the convention used to place  $\vec{U}$ .

# Velocity triangle

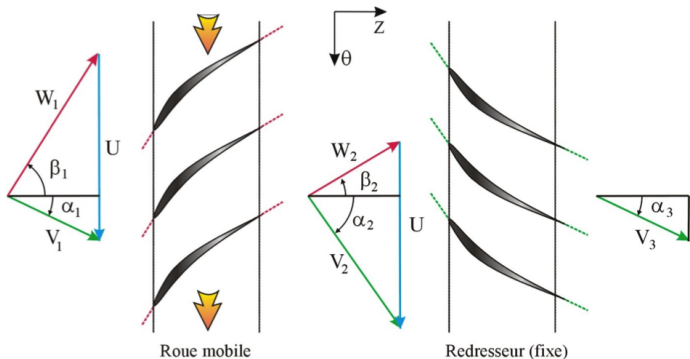
The following picture presents with more details the velocities and angles involved with the velocity triangle representation.



The angles  $\beta$  (relative angle) and  $\alpha$  (absolute angle) are **positive** in the direction of  $\vec{U}$

# Velocity triangles: Compressor

The following picture presents velocity triangles together with the blades in the cascade plane for an axial compressor.



# Velocity triangles: Turbine

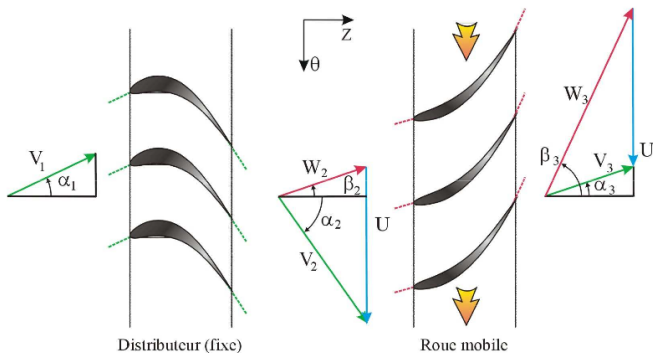
## Exercice

Draw the velocity triangles for a full periodic turbine stage. You will also draw the blade profiles for the rotor and stator.

The absolute inlet flux angle is arbitrarily chosen between  $-45^\circ$  and  $+45^\circ$

# Velocity triangles: Turbine

The following picture presents velocity triangles together with the blades in the cascade plane for an axial turbine.





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# Link between loading and flow coefficients in an axial compressor

Euler equation in an axial machine where  $U_1 = U_2$

$$\frac{\Delta h_0}{U^2} = \frac{V_{\theta 2}}{U} - \frac{V_{\theta 1}}{U}$$
$$\frac{\Delta h_0}{U^2} = 1 - \frac{V_z}{U} \tan \beta_2 - \frac{V_z}{U} \tan \alpha_1$$
$$\Psi = 1 - \Phi(\tan \beta_2 + \tan \alpha_1)$$

## Special case

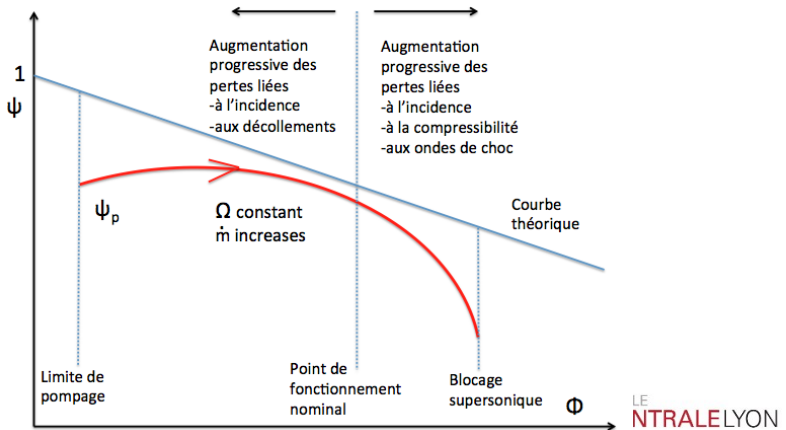
Can be simplified if

- no flux relative angle at inlet ( $\alpha_1 = 0$ )
- no flux relative angle at outlet ( $\beta_2 = 0$ )

In this case,  $\Psi = 1 - \Phi$ . For a given rotation speed, the effective work linearly decreases with the volume flow rate.

# The general picture

Let's represent the theoretical evolution of the load coefficient along with typical experimental curves.



# The general picture I

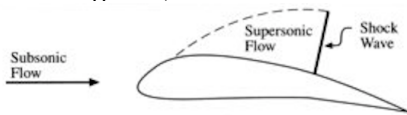
## Theoretical Vs Experimental load coefficient

The pressure load coefficient is always smaller than its theoretical value.  
This is due to the different types of losses

# The general picture II

To understand this phenomenon, let's first increase the flow coefficient.

- $|\beta_1|$  decreases so that the incidence angle becomes more and more negative  $\rightarrow$  more losses
- the relative Mach number  $M_R = \frac{W_1}{c}$  increases  $\rightarrow$  more losses
- when  $M_R \approx 1$ , shock waves form  $\rightarrow$  more losses



- if  $M_R = 1$  at  $A = A^*$ ,  $\dot{m} = \dot{m}_{max}$ . The mass flow rate cannot be made higher without having a supersonic inflow velocity in the absolute frame.<sup>2</sup>

<sup>2</sup>remember that  $A^*$  is the smallest section

# The general picture III

Let's now decrease the flow coefficient. The following phenomena occur

- $|\beta_1|$  increases so that the incidence angle becomes positive.
- At first this is beneficial, the pressure loading coefficient will increase.
- Yet if the flow coefficient is further decreased, the incidence angle reaches a critical value at which the turbulent boundary layer detaches at the suction side. Losses increase dramatically and load coefficient plunges.

# Profile losses

Profile loss are already in Euler equation  $\rightarrow$  the velocity triangle is not influenced.

**Profile losses limit the pressure loading coefficient**

## profile losses

Profile loss are due to the viscous friction on the blades.

$$\bar{\omega} = \frac{\Delta p_{0R}}{\rho_1 W_1^2 / 2}$$

# Profile losses

One can link the profile loss coefficient to the momentum thickness ( $\theta^*$ ) and shape ( $H$ ) of the boundary layer<sup>3</sup>.

## profile losses

$$\bar{\omega} = 2 \left( \frac{\theta^*}{c} \right)_2 \frac{\sigma}{\cos\beta_2} \left( \frac{\cos\beta_1}{\cos\beta_2} \right)^2 \underbrace{\frac{\frac{2H_2}{3H_2-1}}{\left[ 1 - \left( \frac{\theta^*}{c} \right)_2 \frac{\sigma H_2}{\cos\beta_2} \right]}}_{\text{close to unity}}$$

where  $c$  is the chord of the blade

<sup>3</sup> $H$  is the ratio between the displacement and the momentum thickness  $H \approx 1.1$



# Profile losses

Two problems arise with the previous relation

- there is no easy way to 'a-priori' know the evolution of the non dimensional boundary layer  $\frac{\theta^*}{c}$ ,
- there is a strong need to be able to predict the separation of the boundary layer as losses largely increase if it occurs.

To gain some insight into the problem, let's write the Von-Karman equation for the incompressible momentum thickness.

$$\frac{d\theta^*}{dx} = \frac{\tau_w}{\rho W_l^2} - (H + 2) \frac{\theta^*}{W_l} \frac{dW_l}{dx}$$

where  $W_l$  is the streamwise velocity.

## Profile losses

$$\begin{aligned}(H + 2) \frac{\theta^*}{W_l} \left| \frac{dW_l}{dx} \right| \frac{\rho W_l^2}{\mu} &\geq \frac{dW_l}{dz} \\ \frac{\theta^*}{W_l} \left| \frac{dW_l}{dx} \right| \frac{\rho W_l^2}{\mu} &\geq A \frac{W_l}{\theta^*} \\ \frac{\theta^*}{W_l} \left| \frac{dW_l}{dx} \right| Re_\theta &\geq A\end{aligned}$$

In 1931, prior to Von Karman derivation, Bury proposed the following diffusion criterion

$$D_{Bury} = \frac{\theta^*}{W_l} \left| \frac{dW_l}{dx} \right| Re_\theta^n \geq K$$

In both cases if the criterion relation is met, the boundary layer will separate.

# Profile losses

The problem with those criterion is that they rely on local quantities. When one designs a turbomachine, there is no knowledge of local quantities...

During the 50's, Lieblein lead an experimental research effort to express the diffusion coefficient in terms of inlet and outlet quantities. If one assumes that the velocity linearly decreases from  $W_{max}$  (reached at a% of the chord  $c$ ) to  $W_2$  at the trailing edge, Bury's diffusion criterion can be written as

$$D_{Bury} \approx \frac{\theta_m^*}{W_m} \frac{W_{max} - W_2}{(1 - a)c}$$

where  $\theta_m^*$  and  $W_m$  are the mean values of the boundary layer thickness and velocity.

# Profile losses

One still has to model  $W_{max}$  as a function of known parameters. To do so Lieblein writes that

$$W_{max} = W_1 + \Delta W_f + \Delta W_d$$

- $\Delta W_f$  is the acceleration at the suction side related to the circulation created by the incidence and camber. We have  $\Delta W_f = b_f \Delta W_\theta / \sigma$  where  $\sigma = c/g$  is the solidity of the blade
- $\Delta W_d$  is the acceleration at the suction side related to blocking effect of the blade thickness. We have  $\Delta W_d = b_d e_{max} / c$

# Profile losses

By assuming that  $b_d \approx 0$ ,  $b_f \approx 0.5$ ,  $W_m \approx W_1$ ,  $\frac{\theta_m^*}{(1-a)_c} \approx \text{constant}$ , one obtains the diffusion coefficient of Lieblein

## Lieblein's diffusion coefficient

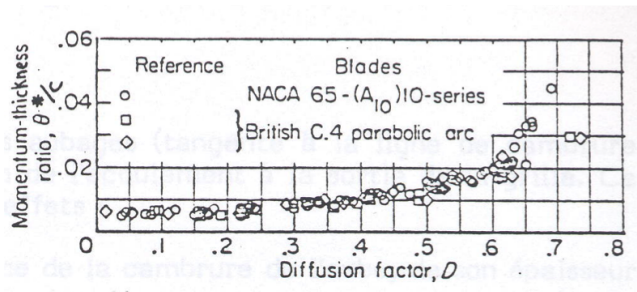
Lieblein's diffusion coefficient relates the inlet and outlet velocities to the relative boundary layer thickness.

Lieblein's diffusion coefficient provides the critical value beyond which the boundary layer separates. It writes:

$$D_{\text{Lieblein}} = 1 - \frac{W_2}{W_1} + \frac{|W_{\theta 2} - W_{\theta 1}|}{2\sigma W_1}$$

# Profile losses

Lieblein made extensive experiments to show the correlation of his coefficient with the relative boundary layer thickness.



## Lieblein's diffusion coefficient

The limit beyond which a separation can occur and largely increase losses is arbitrarily fixed to 0.6.

The curve can be approximated by  $\frac{\theta^*}{c} = 0.0804D^2 - 0.0272D + 0.0071$

$$D_{Lieblein} \geq 0.6 \iff \text{Turbulent boundary layer separation}$$

## Effect of the incidence angle

The previous diffusion criterion are only valid at the nominal working point. It should be modified to include the effect of the incidence of the blade on the turbulent boundary layers separation. This improvement is the equivalent diffusion coefficient which writes

$$D_{eq} = \frac{\cos \beta_2}{\cos \beta_1} \left[ 1.12 + a|i - i^*|^{1.43} + 0.61 \frac{\cos^2 \beta_1}{\sigma} |\tan \beta_2 - \tan \beta_1| \right]$$

$$D_{eq} \leq 2.0$$

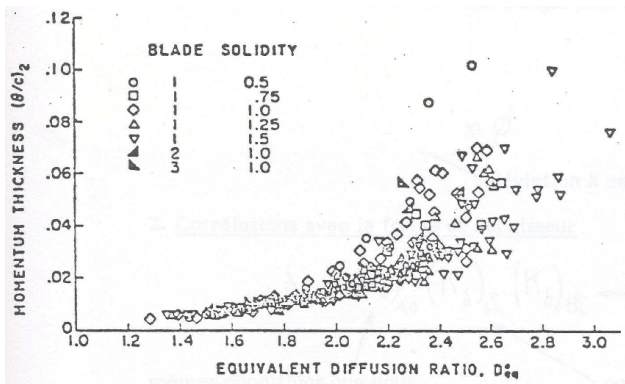
where  $a = 0.0117$ ,  $i$  is the incidence angle and  $i^*$  is the optimum incidence angle.<sup>4</sup>

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<sup>4</sup>The optimum incidence angle is the one for which the glide ratio ( $C_L/C_D$ ) is maximum.

# Effect of the incidence angle

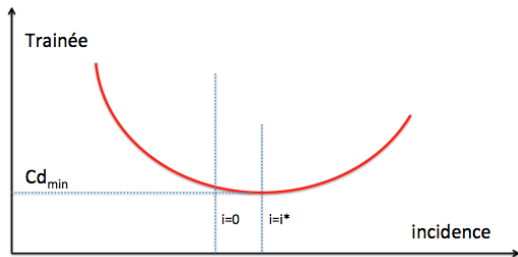
The criterion diverges as its value passes 2.0 regardless of the type of blade profile and blade solidity.





# Effect of the incidence angle

As the profile is set with incidence angle, the drag coefficient evolves along a curve depending of the type of the profile but looking most probably to the one presented below.



## Effect of the incidence angle

The optimal angle of incidence is not necessarily zero (it is only the case for symmetrical profiles). To compute losses in incidence, one uses the following link between total pressure losses and the drag coefficient. We have

$$\bar{\omega} = C_D \frac{\sigma}{\cos \beta_m}$$

where  $\beta_m$  is the average angle defined by the relation  $\tan \beta_m = \frac{1}{2}(\tan \beta_1 + \tan \beta_2)$ .

## Other types of losses I

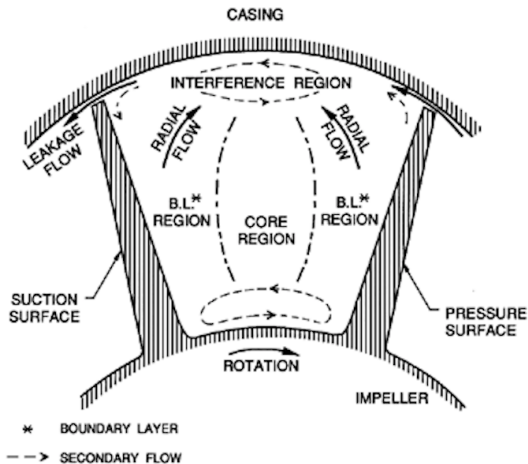
There are other losses that limit the efficiency of a compressor stage.

- Friction losses due to the disks on which the compressor blades are fixed.



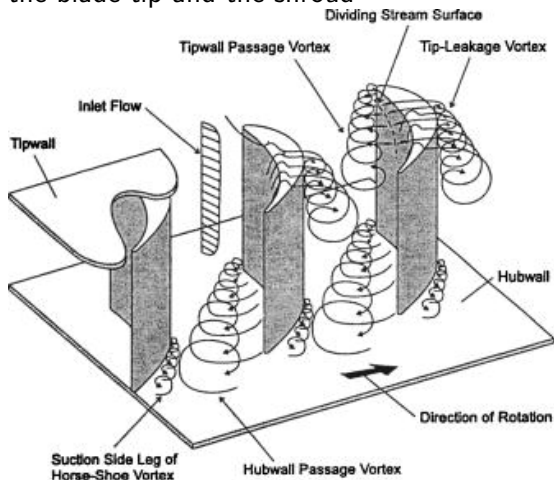
# Other types of losses II

- Friction losses at the hub and shroud



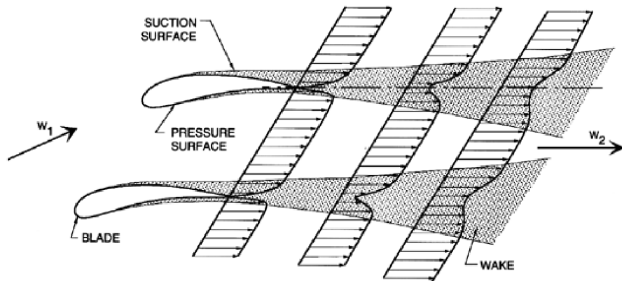
## Other types of losses III

- Losses due to the secondary flow passing through the gap between the blade tip and the shroud



## Other types of losses IV

- Losses due to the wake of the rotating of fixed blades



- Losses due to the remaining kinetic energy present in the outlet flow (not all kinetic energy was transformed in internal one (pressure))

## Other types of losses

All these losses phenomena are the topic of active research at LMFA and elsewhere. One tries to

- understand them into details
- understand what are the main physical parameters that control those losses
- propose innovations to limit or even suppress them

