From Offline to Online pMRI reconstruction

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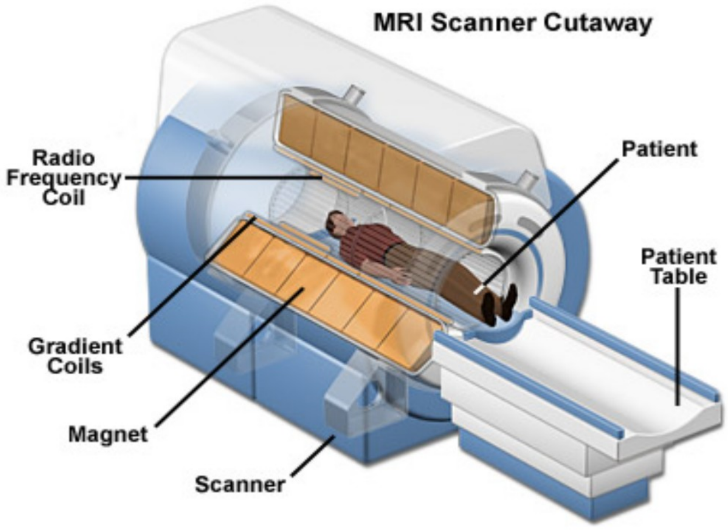
Outline

- Context & Background
- Online Reconstruction
  - Principles
  - Mathematical framework
  - Implementation trick
- Results
- Conclusion and perspectives

Context: MRI in a nutshell

⊕ Non invasive
⊕ high resolution (< $1mm^3$)
⊕ Multi modal

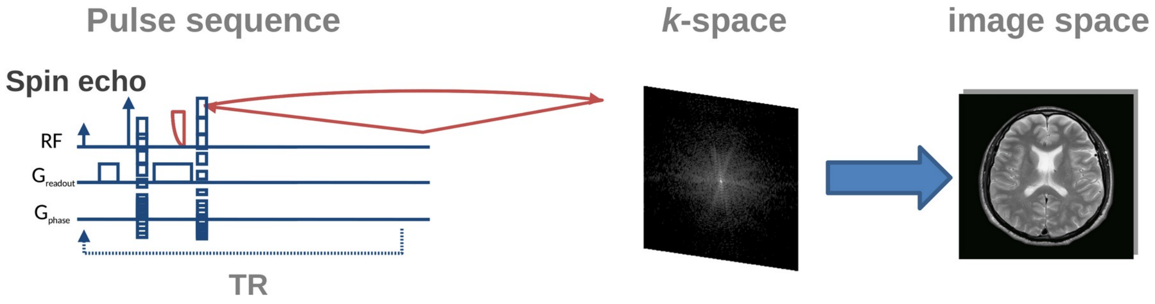
⊖ Expensive
⊖ Complex System



Key principles:
- Excitation and measure relaxation of Hydrogen spin via electromagnetic coils.
- **Data acquired in spatial frequency “k-space”**
- **One pixel ≈ One acquisition**
- **Acquisition + Reconstruction = Image**

TR = time between 2 scans = 50-2000ms
TE = time for a scan = 20-90 ms

High resolution image: **50 min acquisition !**
→ Reducing acquisition time is essential



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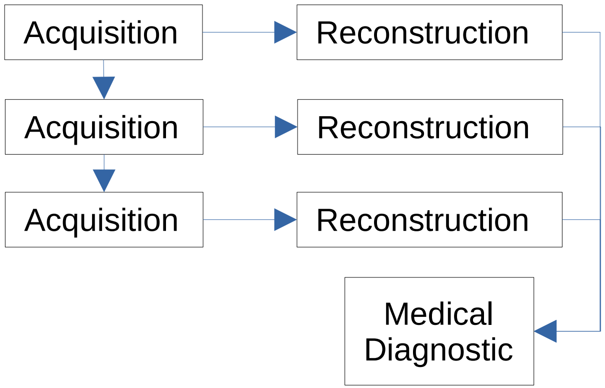
Context: offline vs online reconstruction

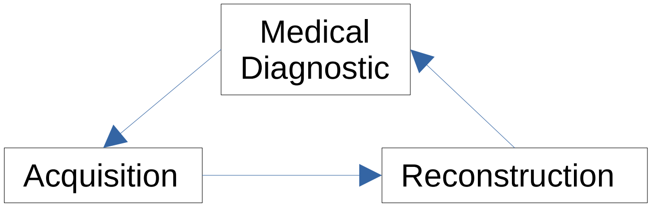
- Typical MRI examination consists of several scans
  - planned ahead of the exams
- The reconstruction step is performed when the acquisition(s) are finished. → waiting time for the results.

Already available accelarations:

- Parallel imaging:
  - Acquisition on multiple coils
- Compressed sensing (CS):
  - Undersampling + sparsity prior



Classical MRI examination
Offline Reconstruction



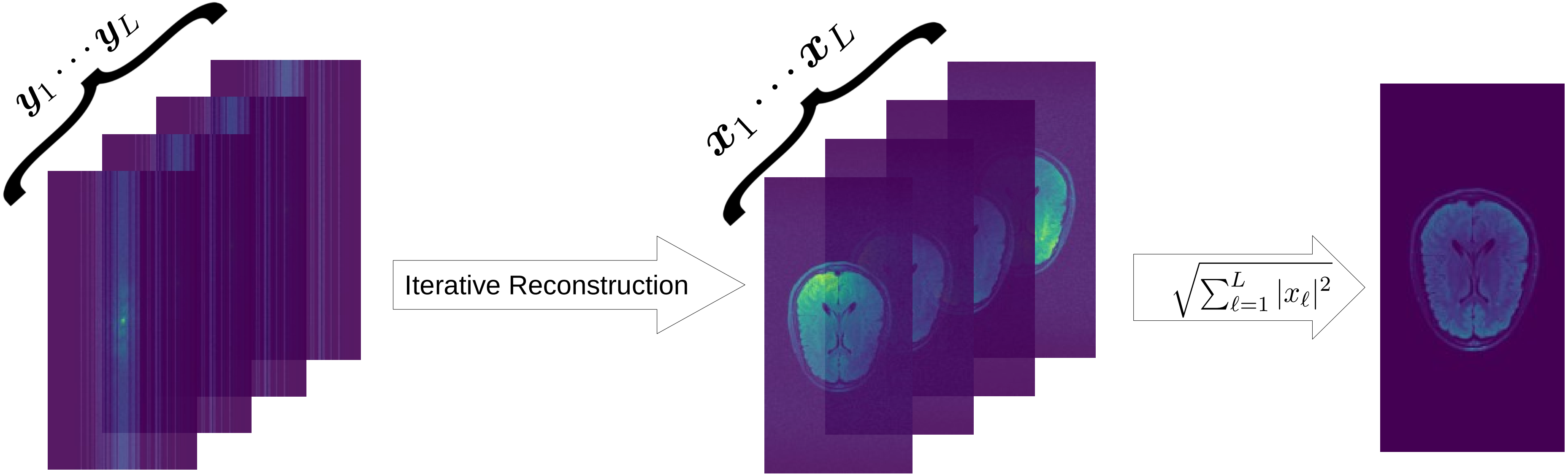
Ideal MRI examination
using online reconstruction

Goals of online reconstruction:

- Reduced the exam time
- “Realtime” Feedback

Offline multicoil CS reconstruction

$$\underline{\hat{\boldsymbol{x}}} = \arg\min_{\underline{\boldsymbol{x}}\in\mathbb{C}^{L\times N}} \underbrace{\frac{1}{2}\sum_{\ell=1}^{L} \|\mathcal{F}_\Omega(\boldsymbol{x}_\ell) - \boldsymbol{y}_\ell\|_2^2}_{f(\underline{\boldsymbol{x}})} + \underbrace{R(\underline{\boldsymbol{\Psi}}\,\underline{\boldsymbol{x}})}_{g(\underline{\boldsymbol{x}})}$$



Final image

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Online Reconstruction

- Principles
- Mathematical framework
- Implementation tricks

Online reconstruction

Assumptions:

- The acquisition can be partitioned in several “shots”
- We access the same data as in the offline case

Mask of sequential sampling in kspace

Goals of Online Reconstruction:

- Quality **as good as offline** reconstruction
- **Earlier reconstruction results**
- Feedback to clinician during the exam

This Work:

- Theoretical analysis and preliminary **convergence results**.
- **Faster** Cartesian reconstruction
- Retrospective analysis on FastMRI dataset (cartesian)
- *First non Cartesian online (SPARKLING) tests (WIP)*

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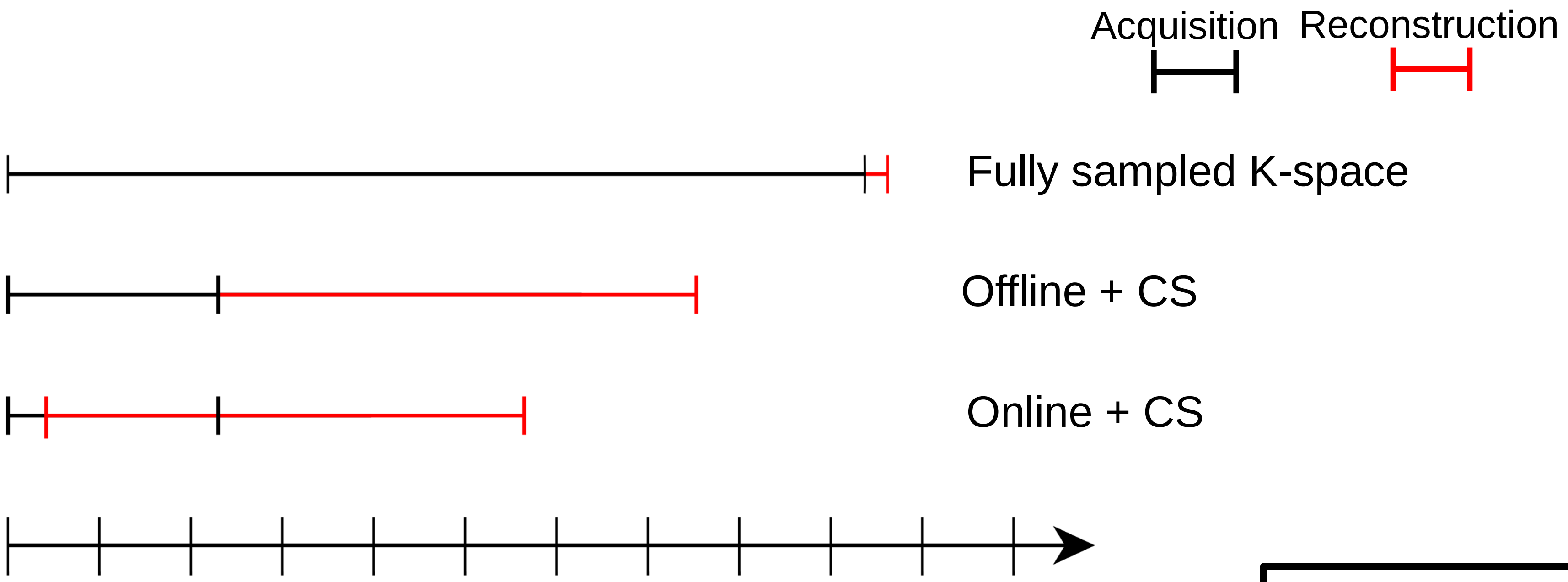
Online reconstruction principle



First Work on Online Reconstruction in *Loubna Elgueddari's PhD Thesis.*

Online reconstruction:

- Reduced scan and reconstruction time
- "Real-time" feedback to physician

[1] Gueddari et al, 2019 -- Online MR image reconstruction for compressed sensing acquisition in T2* imaging *SPIE Conference - Wavelets and Sparsity XVIII*

Online formulation of CS reconstruction

- Single coil Cartesian *example* → straightforward extension to multicoil acquisition
- *K* shot acquired

offline

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{x}\in\mathbb{C}^N} \underbrace{\frac{1}{2}\|\boldsymbol{\Omega}\boldsymbol{\mathcal{F}}\boldsymbol{x} - \boldsymbol{y}\|_2^2}_{f(\boldsymbol{x})} + \underbrace{R(\boldsymbol{\Psi}\,\boldsymbol{x})}_{g(\boldsymbol{x})}$$

$$\boldsymbol{y} = \sum_{k=1}^{K} \delta\boldsymbol{\Omega}_k \boldsymbol{y} = \sum_{k=1}^{K} \delta\boldsymbol{y}_k$$

online

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{x}\in\mathbb{C}^N} \sum_{k=1}^{K} \underbrace{\frac{1}{2}\|\delta\boldsymbol{\Omega}_k\boldsymbol{\mathcal{F}}\boldsymbol{x} - \delta\boldsymbol{y}_k\|_2^2}_{f_k(\boldsymbol{x})} + \underbrace{R(\boldsymbol{\Psi}\,\boldsymbol{x})}_{g(\boldsymbol{x})}$$

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Theoretical Result

- f and g are proper continous convex functions
- gradient of f is 1-Lipschitz.
- η is the descent step: “learning rate”

Offline Forward Backward

$$\boldsymbol{T} = \mathrm{prox}_{\eta g}\left(Id - \eta \boldsymbol{\nabla} f\right)$$

$$\boldsymbol{x}^{(n+1)} = \boldsymbol{T}\boldsymbol{x}^{(n)}$$

- Convergence to a fixed point of *T*

$$\mathrm{Fix}\,\boldsymbol{T} = \arg\min f + g$$

- Acceleration methods
  - FISTA, POGM, Barista, etc
- If g = 0 FB algorithm ⇔ gradient descent.

Online Forward Backward

$$\boldsymbol{T}_k = \mathrm{prox}_{\eta g_k}\left(Id - \eta \boldsymbol{\nabla} f_k\right)$$
$$\boldsymbol{x}^{(n,k+1)} = \boldsymbol{T}_k \boldsymbol{x}^{(n,k)}$$

$$\boldsymbol{\mathcal{T}} = \boldsymbol{T}_K \circ \cdots \circ \boldsymbol{T}_1$$
$$\boldsymbol{x}^{(n+1)} = \boldsymbol{x}^{(n,K+1)} = \boldsymbol{\mathcal{T}}\boldsymbol{x}^{(n,1)}$$

- Choice of the T_k:
  - From the available data and mask
  - Condition on convergence to offline:

$$\cap_{k=1}^{K} \mathrm{Zer}\left\{\boldsymbol{\nabla} f_k + \boldsymbol{\partial} g_k\right\} \neq \varnothing \implies \mathrm{Fix}\,\boldsymbol{\mathcal{T}} \subset \mathrm{Fix}\,\boldsymbol{T}$$

[1] Fixed Point Strategies in Data Science, *Combette and Pesquet, 2021*

Online type I and type II

- Online Type I:
  - Data term:

$$F_k(\boldsymbol{x}) = \frac{1}{2}\sum_{j=1}^{k} \|\delta\boldsymbol{\Omega}_j\left(\boldsymbol{\mathcal{F}x} - \boldsymbol{y}\right)\|_2^2$$

  - Iteration:

$$\hat{\boldsymbol{x}}_k = \mathrm{prox}_{\eta g}(Id - \boldsymbol{\nabla} F_k(\boldsymbol{x}_{k-1}))$$

  - Observed data is **accumulated**
  - Robust and easy to study
  - Incremental Memory cost
    - Max to Offline Reconstruction
  - equivalent to Offline

- Online Type II:
  - Data term and regularisation term:

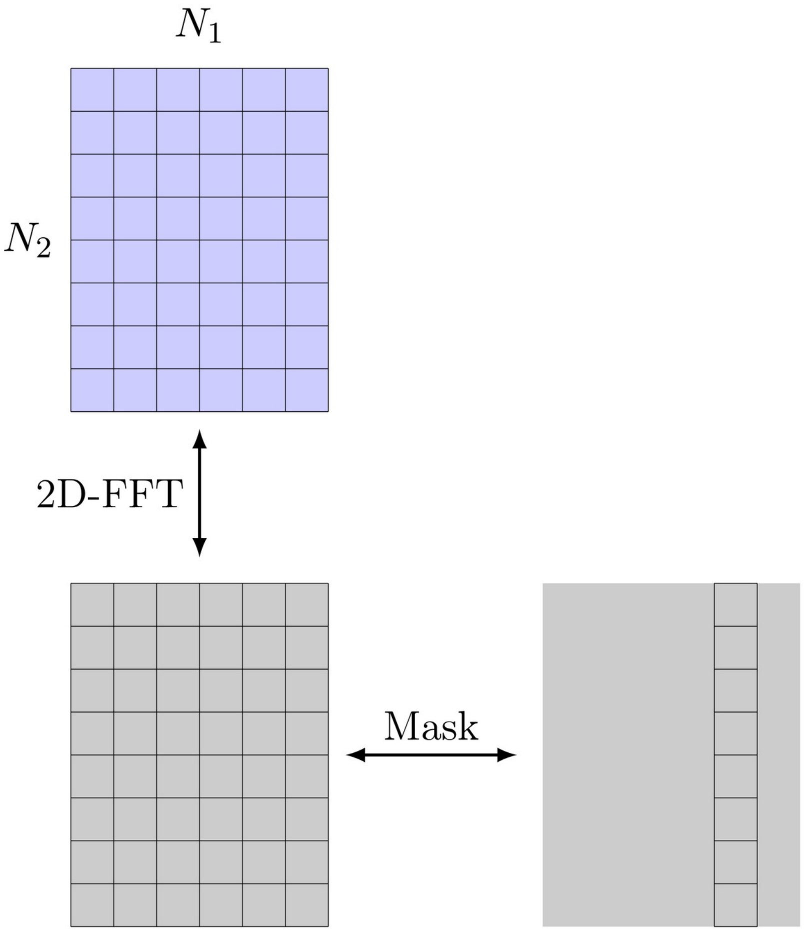
$$f_k(\boldsymbol{x}) = \tfrac{1}{2}\|\delta\boldsymbol{\Omega}_k\left(\boldsymbol{\mathcal{F}x} - \boldsymbol{y}\right)\|_2^2$$

  - Forward Backward Iteration:

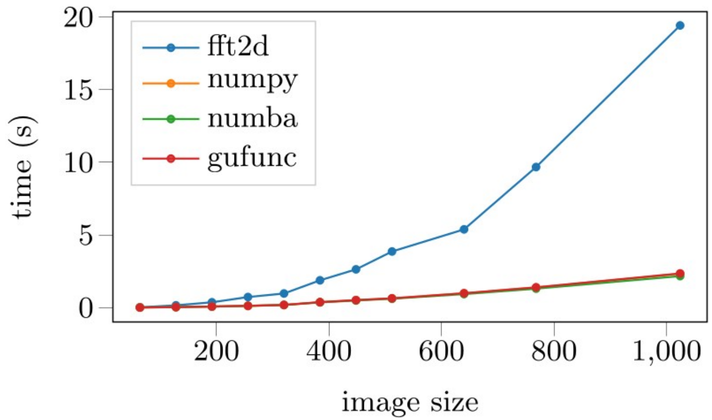
$$\hat{\boldsymbol{x}}_k = \mathrm{prox}_{\eta g/K}(Id - \boldsymbol{\nabla} f_k(\boldsymbol{x}_{k-1}))$$

  - Observed data is **iterated**
  - promising but hard to study
  - Memory and Time efficient
    - Faster FFT+mask !
  - Close to Machine Learning SGD framework

Online type II and Cartesian: Faster FFT + Mask



10^{-10} % rel. error



Only worth for a mask of a few columns
(High undersampling factor)

Online Reconstruction: Summary

$$\boldsymbol{x} = \arg\min_{x \in \mathbb{C}^N} f(\boldsymbol{x}) + g(\boldsymbol{x}) = \arg\min_{\boldsymbol{x} \in \mathbb{C}^N} \sum_{k=1}^{K} f_k(\boldsymbol{x}) + g(\boldsymbol{x})$$

Type	Offline	Online I	Online II
Available data	$\boldsymbol{y}$	$\boldsymbol{y}_k = \begin{bmatrix} \delta \boldsymbol{y}_1 & \dots & \delta \boldsymbol{y}_k \end{bmatrix}^T$	$\delta \boldsymbol{y}_k$
Gradient iteration	$\boldsymbol{\nabla} f(\boldsymbol{x}^{(n)})$	$\sum_{\kappa=1}^{k} \boldsymbol{\nabla} f_\kappa(\boldsymbol{x}^{(n,k)})$	$\boldsymbol{\nabla} f_k(\boldsymbol{x}^{(n,k)})$
Implementation	Expensive, but classic	Increasing Memory Cost	Low Memory, very fast
Theoretical Convergence	✔	✔	? (within bound)

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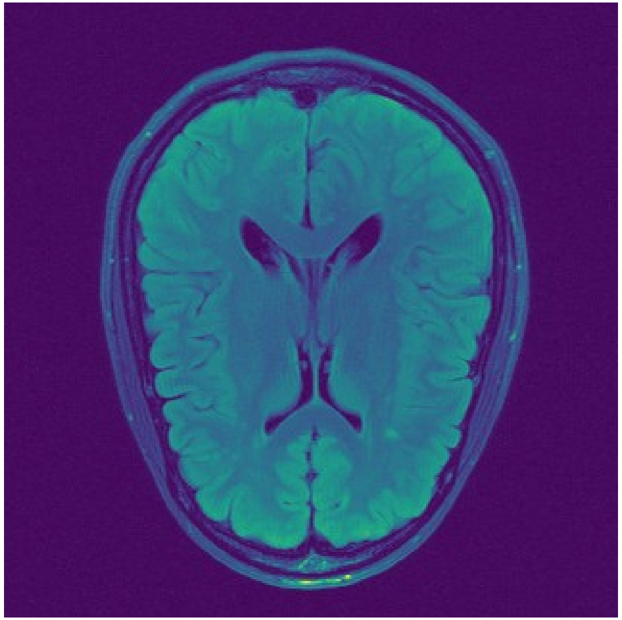
Results

- Materials & Methods
- Type I reconstruction
- Type II reconstruction
- Realtime example

Materials & Methods

- Fast MRI dataset
  - Cartesian reconstruction, *16 coils*
  - 640x320 images *noise-free*
  - R = 4 (80 column shots)
- Quantitative metrics:
  - PSNR and SSIM
  - Offline cost: $f(\boldsymbol{x}_k) + g(\boldsymbol{x}_k)$
  - Online cost:
    - Type I $\sum_{\kappa=1}^{k} f_\kappa(\boldsymbol{x}_k) + g(\boldsymbol{x}_k)$
    - Type II $f_k(\boldsymbol{x}_k) + g(\boldsymbol{x}_k)$



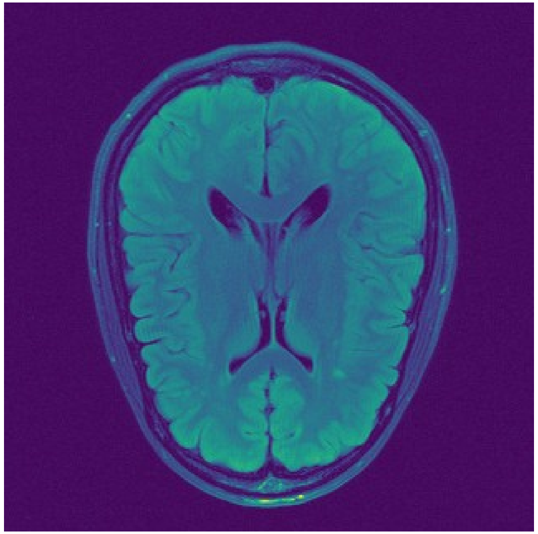
Reference Image (FLAIR)
Noise-free FastMRI dataset

Test Setup:

- **CPU :** AMD Ryzen 5 PRO 4650U
  6 cores/12 threads @2.1-4.0 GHz
- 16 Go RAM DDR4
- No GPU implementation *yet*

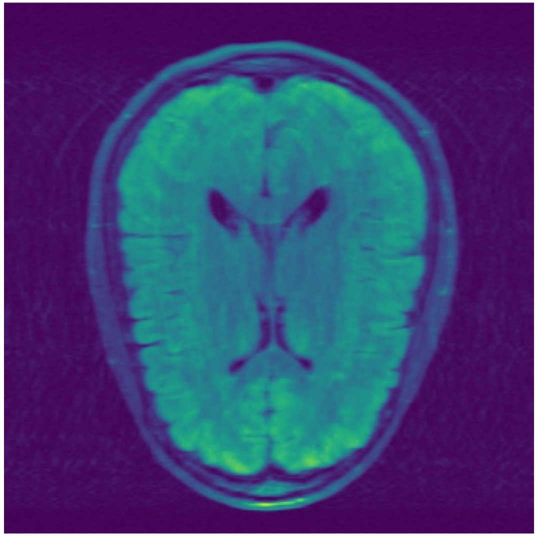
Offline multicoil Reference

Ground Truth



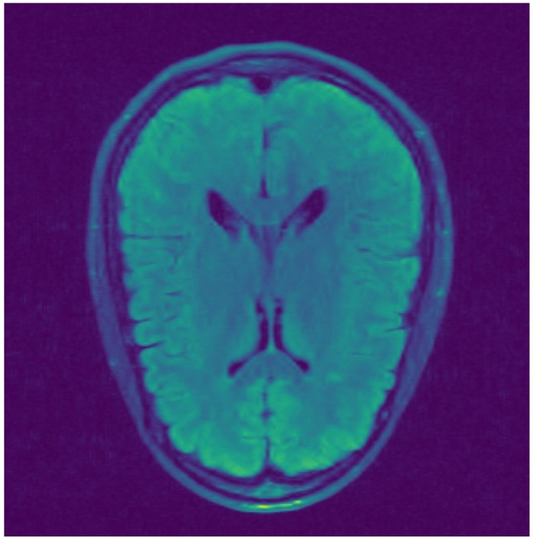
Zero-filled Reconstruction (IFFT)

PSNR=32.38, SSIM=0.899



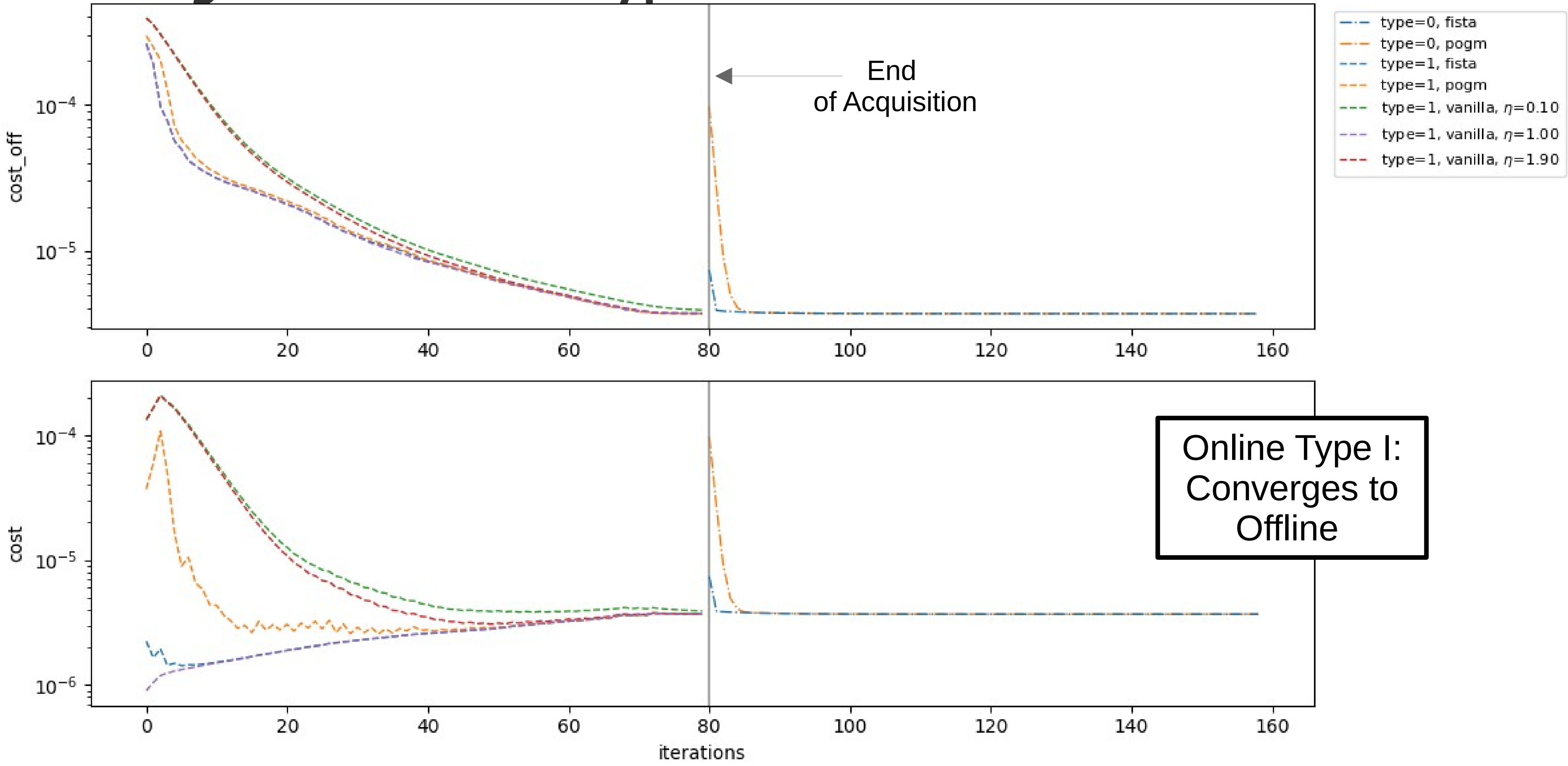
POGM+GroupLASSO

PSNR = 33.63 dB, ssim=0.909

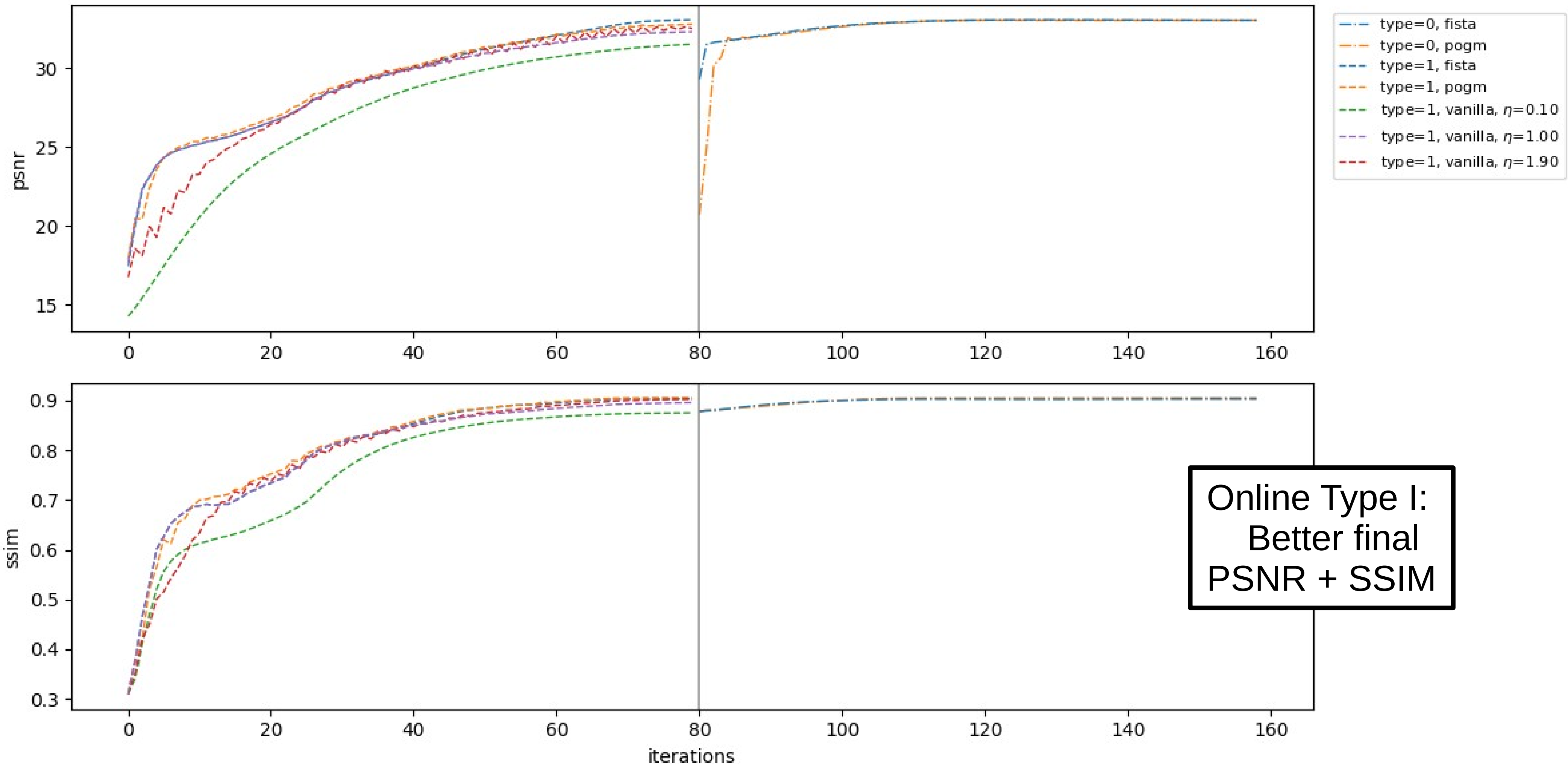


Online reconstruction should be as close as possible to this quality.

Convergence of Online type I: ✔



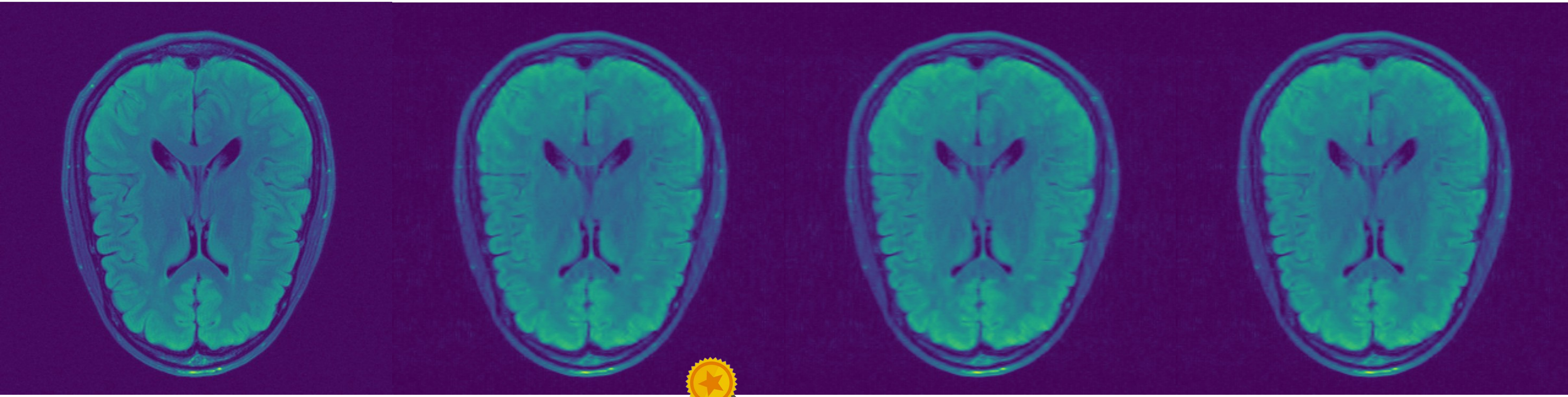
Quality of Type I



Online Type I:
Better final
PSNR + SSIM

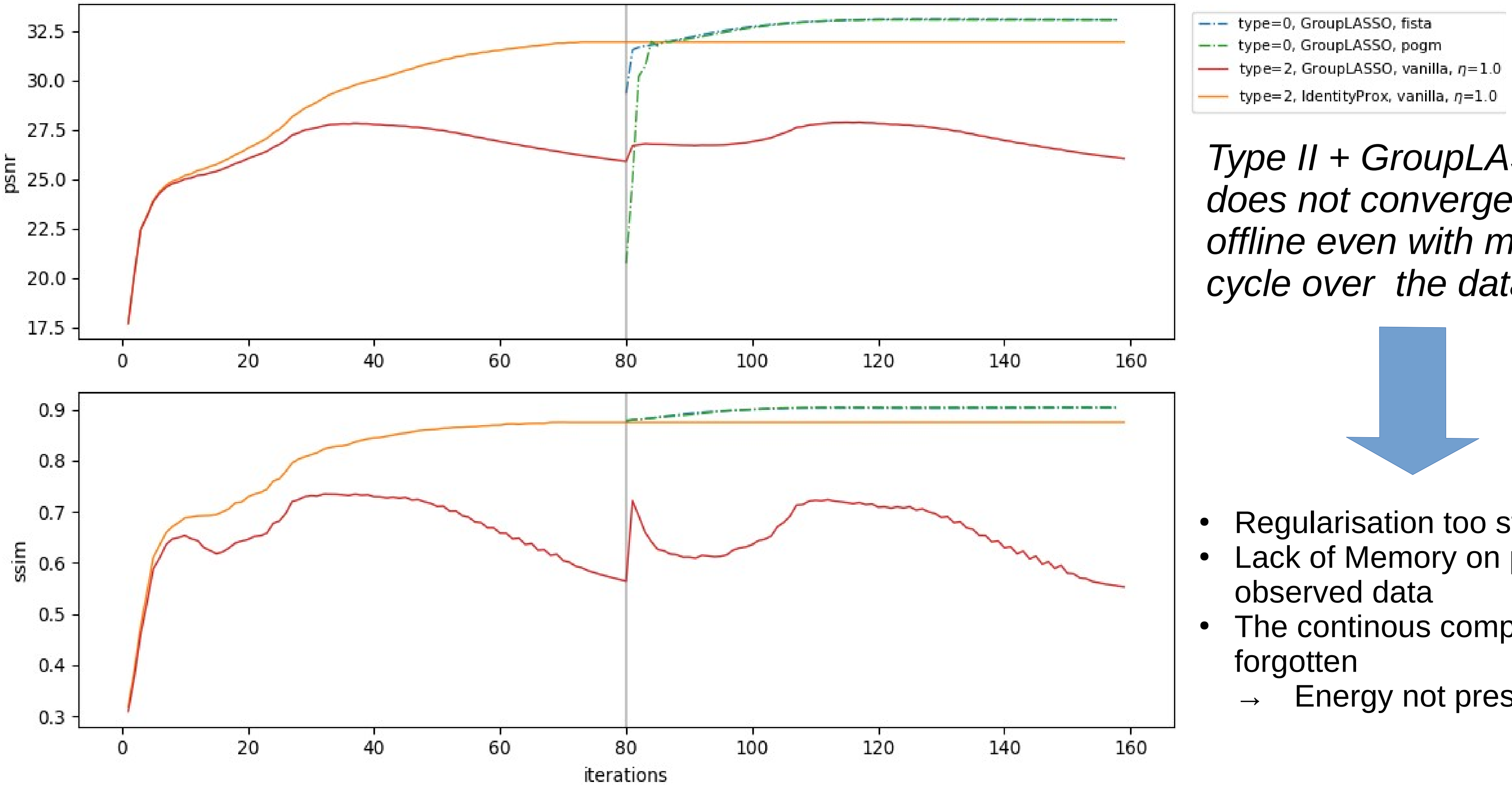
Online Type I: Multicoil CS reconstruction

reference image	Online FISTA	Online FB	Offline POGM
	PSNR = 33.67 dB, ssim=0.923	PSNR = 33.21 dB, ssim=0.922	PSNR = 33.63 dB, ssim=0.909



	Online FISTA	Online FB	Offline POGM
	80 iters 68s	80 iters 62s	200 iters ≈ 4 minutes

Convergence Online Type II , Forward-Backward: ✘



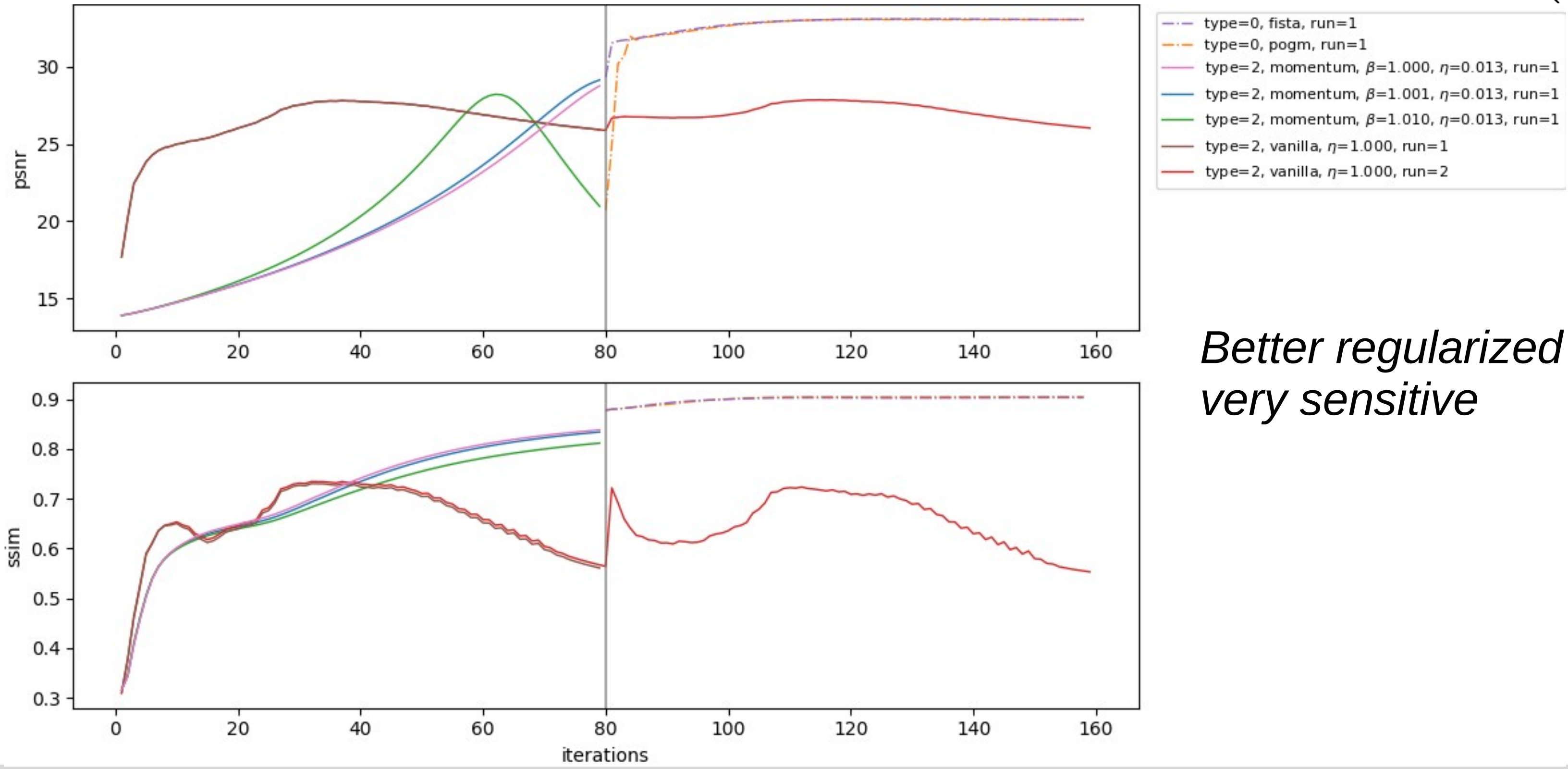
Type II + GroupLASSO does not converge to offline even with multiple cycle over the data.

- Regularisation too strong
- Lack of Memory on previous observed data
- The continous component is forgotten
  → Energy not preserved.

Forward Backward with momentum

"Heavy-Ball" method + Regularisation

$$\boldsymbol{v}^{(n,k)} = \beta \boldsymbol{v}^{(n,k-1)} + \boldsymbol{\nabla} f_k(\boldsymbol{x}^{(n,k)})$$
$$\boldsymbol{x}^{(n,k+1)} = \text{prox}_{\eta g}\left(\boldsymbol{x}^{(n,k)} - \eta \boldsymbol{v}^{(n,k)}\right)$$



Better regularized results, but very sensitive

Online Type II

reference image

Online FB(η=1) + GroupLASSO

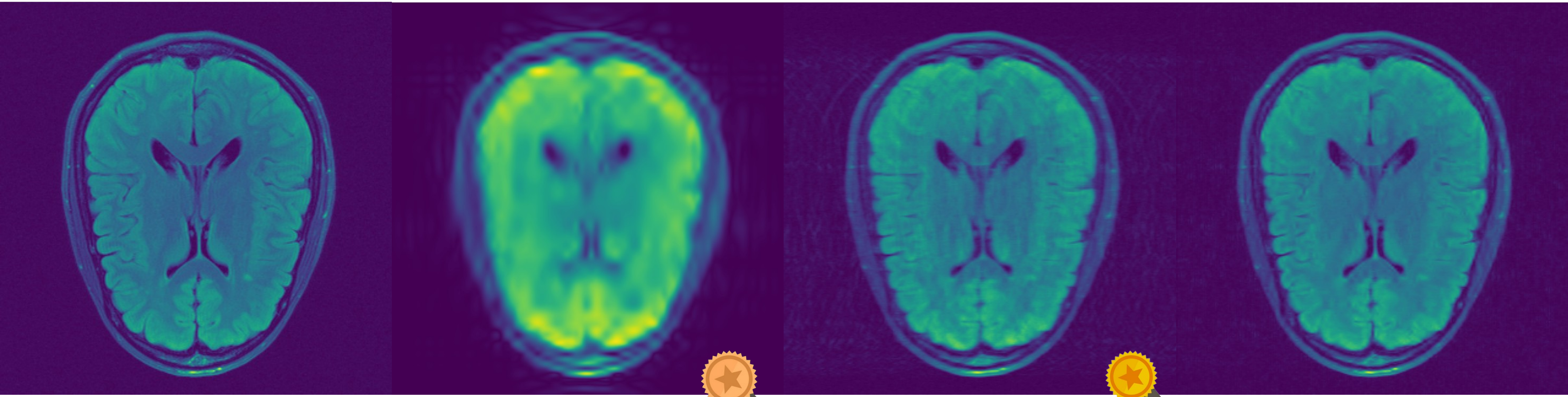
PSNR = 23.35 dB, ssim=0.608

Online Gradient Descent

PSNR = 29.37 dB, ssim=0.890

Offline POGM

PSNR = 33.63 dB, ssim=0.909



Online FB(η=1) + GroupLASSO:
80 it
≈ 22s
1 it = 240ms

Online Gradient Descent:
80 it
≈ 3s
1 it = 42ms

Offline POGM:
200 its
≈ 4 minutes

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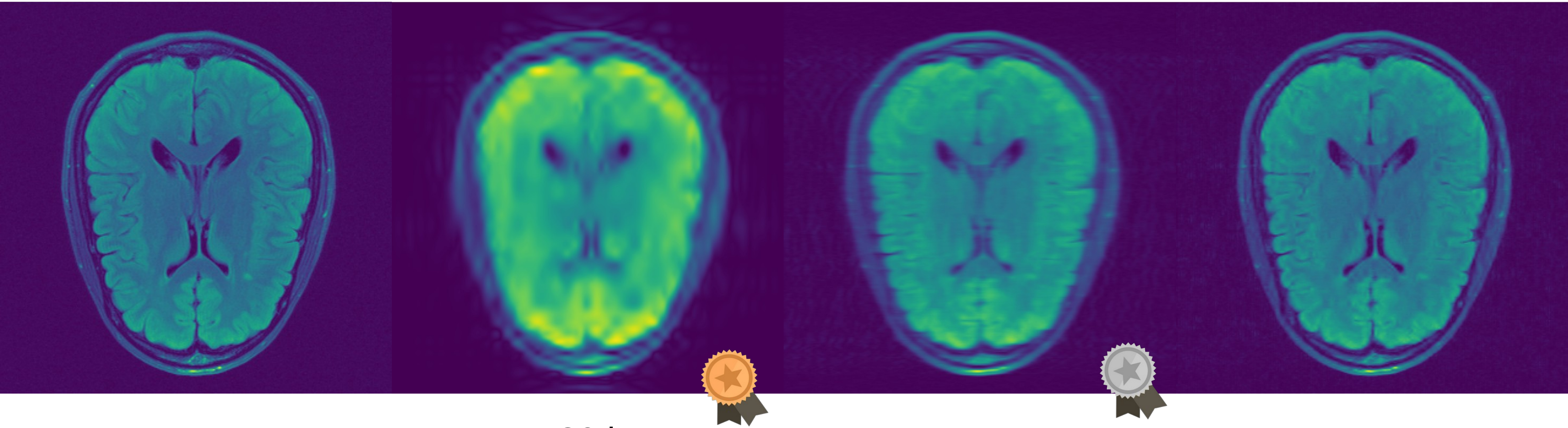
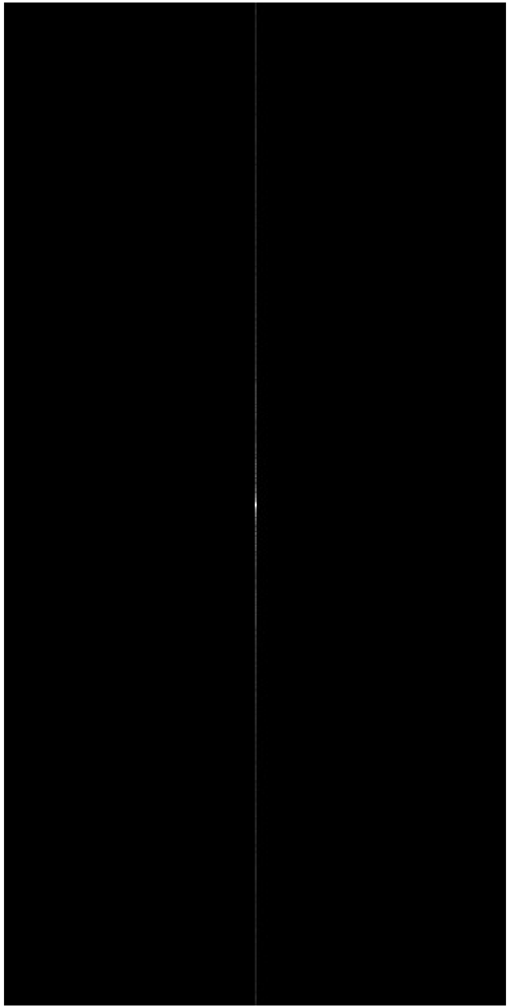
Online Type II, with momentum

reference image	Vanilla FB (η=1) + GroupLASSO	Momentum (η=1/80, β=1.001) + GroupLASSO	Offline POGM
	PSNR = 23.35 dB, ssim=0.608	PSNR = 26.46 dB, ssim=0.839	PSNR = 33.63 dB, ssim=0.909



	80 it ≈ 22s 1 it ≈ 240ms	80 it ≈ 23s 1 it ≈ 240ms	200 its ≈ 4 minutes

“Real-time” reconstruction:



Type I + FISTA

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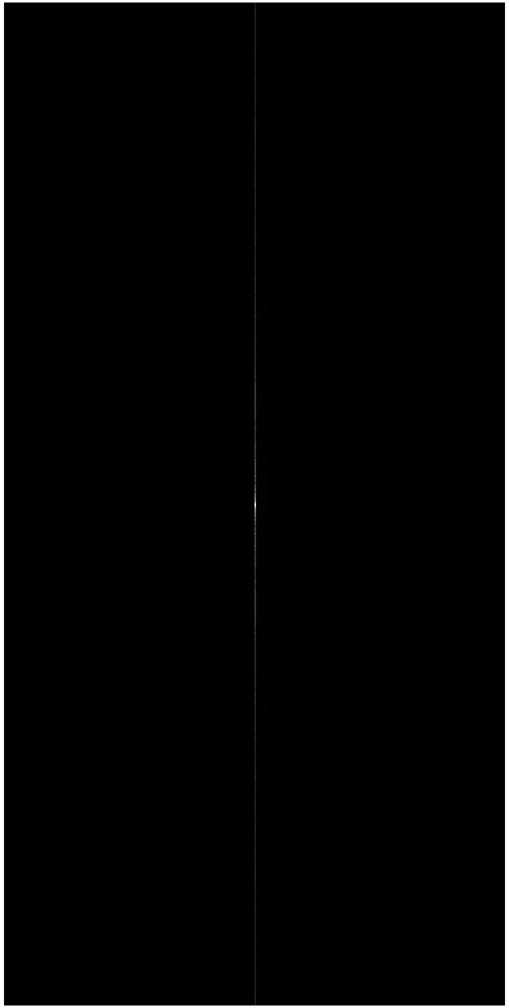
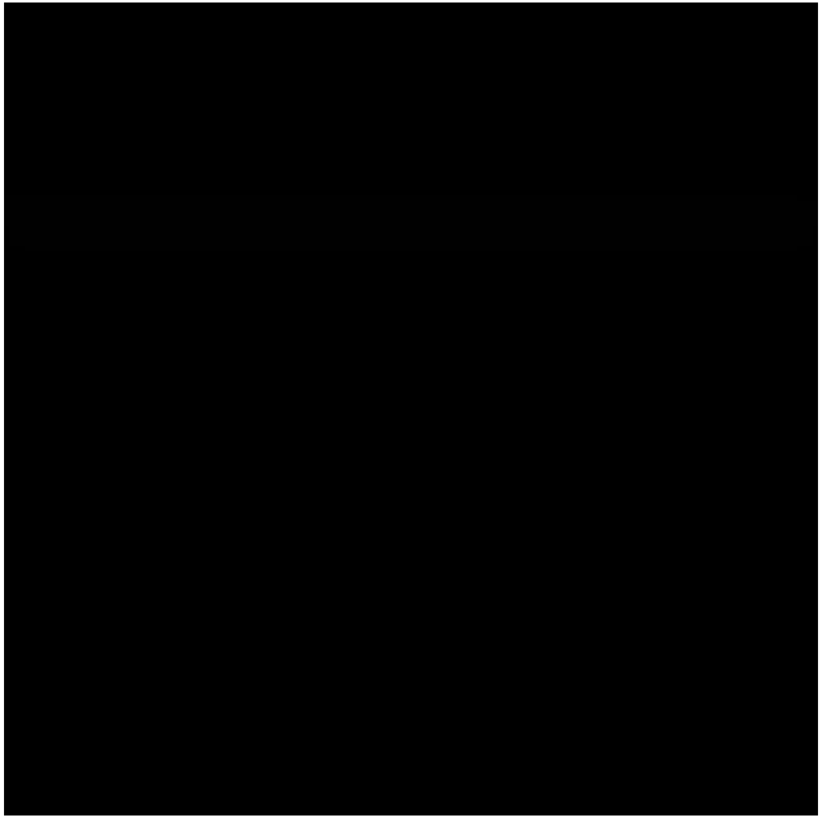
“Real-time” Reconstruction



Type II + Momentum



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Conclusions and Perspectives

- Type I: Online Results ≈ Offline Result
- Type II: Time↘ Memory↘
- Regularisation expensive, sometimes detrimental.

Perspectives:

- Extend Online Algorithm to non-Cartesian acquisition

  → Use the NUFFT (via gpuNUFFT/cufiNUFFT)
- Test on higher acceleration factor
- Proof of convergence of type II with less constraints.

Thank you for your attention

Any Questions?

Offline reconstruction

- Solve using **Forward-Backward algorithm**

$$(\forall n \in \mathbb{N}), \quad \boldsymbol{x}_{(n+1)} = \operatorname{prox}_{\eta g}\left(\boldsymbol{x}_{(n)} - \eta \boldsymbol{\nabla} f(\boldsymbol{x}_{(n)})\right)$$

- η is the gradient descent step: “learning rate” in ML
- FB algorithm well studied and improved in the literature
  - FISTA, POGM, BARISTA, *etc*
- Regularisation: GroupLASSO (GL) on wavelets coefficients/coils

$$\operatorname{prox}_{R_{GL}} : \begin{array}{ccc} \mathbb{C}^{L \times N_\Psi} & \to & \mathbb{C}^{L \times N_\Psi} \\ \begin{bmatrix} \boldsymbol{\alpha}_1 \\ \vdots \\ \boldsymbol{\alpha}_L \end{bmatrix} & \mapsto & \begin{bmatrix} \boldsymbol{\alpha}'_1 \\ \vdots \\ \boldsymbol{\alpha}'_L \end{bmatrix} \end{array} \quad \text{with} \quad \alpha'_{\ell,j} = \begin{cases} 0 & \text{if } \sqrt{\sum_{\ell=1}^{L} \|\alpha_{\ell,j}\|_2^2} < w_j \\ \alpha_{\ell,j} & \text{else} \end{cases}$$