

**NB :** This document sums up as much symbols, notations and conventions used for the Maths MA101 and MA102 courses (@MEC) as possible. If something is missing, please tell us.

**Warning:** For quizzes, tutorial sessions and assignments, homeworks, exams and lectures, you **have to** use these conventions.

---

### English shortcuts

*i.e.* : *id est*<sup>1</sup>, “that is”,

*e.g.* : *exempli gratia*<sup>1</sup>, “for example” (does **not** come from “example given”).

### Basic notations for functions or formulas

$f(x)$  : the *value* of the function  $f$  at the point  $x$ , defined when  $x$  belongs to the domain of  $f$  (written as  $D_f$ ),

$\longrightarrow$  : to specify the domain for a function, *e.g.*  $\sin : \mathbb{R} \longrightarrow \mathbb{R}$ ,

$f(A)$  : the *image set* of  $A$  by  $f$ , *i.e.* the set of values  $f(x)$  for all the  $x$  in the set  $A$ :  $\{f(x) : x \in A\}$ . Example:  $\sin(\mathbb{R}) = [-1, 1]$ ,

$f^{-1}(B)$  : the *pre-image set* of  $B$  by  $f$ , *i.e.* the set of points of  $A$  that are mapped to points of  $B$  by the function  $f$ :  $\{x \in A : f(x) \in B\}$ . Example:  $\sin^{-1}(\{0\}) = 2\pi\mathbb{Z}$ ,

$f|_A$  : the restriction of the function  $f : E \longrightarrow F$  to the subset  $A \subset E$ ,

$\forall$  : “**for all**”, *e.g.*  $\forall x \in \mathbb{R}, \exp(x) > 0$  says that all values  $\exp(x)$  are positive for any real number  $x$ ,

$\exists$  : “**there exists**”, *e.g.*  $\exists x > 0, \cos(x) = -1$  says that there is a positive real number  $x$  such that  $\cos(x) = -1$ .

### Logical binary predicates

$\implies$  : implies, *e.g.*  $(x > 0) \implies (x \geq 0)$ ,

$\Leftrightarrow$  : is equivalent to, *i.e.* “implies and is implied by”.  $A \Leftrightarrow B \equiv (A \implies B) \text{ and } (B \implies A)$ ,

$\neg$  : logical negation of a formula, *e.g.*  $\neg(\omega > 0) \equiv \omega \leq 0$ .

### Binary predicates for set

$\in$  : belongs to, *e.g.*  $n \in \mathbb{N}, i \in \mathbb{C}$  etc,

$\notin$  : does not belong to, *e.g.*  $-1 \notin \mathbb{R}_+$ ,

$\subset$  : is included in, *e.g.*  $\mathbb{N} \subset \mathbb{R}$ ,

$\supset$  : contains, *e.g.*  $\mathbb{Z} \supset \{-1, 0, 1\}$  (*we try to not use this one*),

$\subseteq$  : to precise that inclusion *can be an equality*, as the opposite of  $\subsetneq$  which excludes the equality,

$\setminus$  : difference of two sets, *e.g.*  $\mathbb{R} \setminus \mathbb{R}_- = \mathbb{R}_+^*$ . Especially used for  $\mathbb{K}^* = \mathbb{K} \setminus \{0\}$ ,

$\cap$  : intersection of two sets (also  $\bigcap$ ), *e.g.*  $\mathbb{C} \cap \mathbb{Z} = \mathbb{Z}$ ,

$\cup$  : union of two sets (also  $\bigcup$ ), *e.g.*  $\{0\} \cup \mathbb{N}^* = \mathbb{N}$ ,

$\uplus$  : disjoint union of two sets (also  $\bigsqcup$ ), *e.g.*  $\mathbb{R}_-^* \uplus \mathbb{R}_+^*$  (also  $\sqcup$  or  $\bigsqcup$ ),

$E_1 \times E_2$  : Cartesian product of two sets  $E_1$  and  $E_2$ , *i.e.*  $\{(x, y) : x \in E_1, y \in E_2\}$ ,

$E_1 \oplus E_n$  : direct sum of two sets  $E_1, E_2 \subset E$ , *i.e.*  $\{x + y : x \in E_1, y \in E_2\}$ , where  $E_1 \cap E_2 = \{0_E\}$ . (MA102)

---

<sup>1</sup>It is latin.

**Shortcuts for sums and products**

$\sum_{k=1}^n f(k)$  : sum  $f(1) + f(2) + \dots + f(n-1) + f(n)$  for  $n \geq 0$ . The convention defines an empty sum as 0 (when  $n = 0$ ),

$\prod_{k=1}^n g(k)$  : product  $g(1) \times g(2) \times \dots \times g(n-1) \times g(n)$  for  $n \geq 0$ . An empty product as 1 (when  $n = 0$ ),

$\times_{k=1}^n E_k$  : Cartesian product (for spaces) :  $E_1 \times E_2 \times \dots \times E_{n-1} \times E_n$  for  $n \geq 1$ ,

$\bigoplus_{k=1}^n E_k$  : direct sum (for spaces) :  $E_1 \oplus E_2 \oplus \dots \oplus E_{n-1} \oplus E_n$  for  $n \geq 1$ . (MA102)

**Usual constants and functions**

$\pi$  : (pi), usually defined by the quotient of any circle perimeter by its diameter,  $\pi \simeq 3.14159265$ ,

$e$  : base of the exponential function, *i.e.*  $e \stackrel{\text{def}}{=} e^1 = \exp(1)$ ,  $e \simeq 2.71828183$ ,

$+\infty$  : positive infinite,  $-\infty$  negative infinite,  $\infty$  infinite,

sign : sign function, defined by  $\text{sign} : \mathbb{R} \rightarrow \mathbb{R}, 0 \mapsto 0, x \mapsto 1$  if  $x > 0, -1$  if  $x < 0$ ,

exp : exponential function, *i.e.*  $\exp : \mathbb{R} \rightarrow \mathbb{R}, x \mapsto \exp(x) = e^x$ ,

ln : Neperian logarithm function, *i.e.* logarithm in basis 10:  $\ln : \mathbb{R}_+^* \rightarrow \mathbb{R}, x \mapsto \ln(x)$  (also written  $\log(x)$ ),

$\log_y$  : logarithm function in basis  $y$  ( $y > 0, y \neq 1$ ), defined as:  $\log_y : \mathbb{R}_+^* \rightarrow \mathbb{R}, x \mapsto \ln(x)/\log(y)$ ,

cos : cosine, sin : sinus ( $\mathbb{R} \rightarrow \mathbb{R}$ ), tan : tangent ( $\mathbb{R} \setminus \frac{\pi}{2}\mathbb{Z} \rightarrow \mathbb{R}$ ),

arccos : arc cosine, arcsin : arc sinus, arctan : arc tangent (reciprocal),

cosh : hyperbolic cosine, sinh : hyperbolic sinus, tanh : hyperbolic tangent ( $\mathbb{R} \rightarrow \mathbb{R}$ ),

arccosh : arc hyperbolic cosine, arcsinh : arc hyperbolic sinus, arctanh : arc hyperbolic tangent (reciprocal),

floor : lower integer round, *i.e.*  $\text{floor}(x) \stackrel{\text{def}}{=} \lfloor x \rfloor$  the biggest integer  $n \in \mathbb{Z}$  to be smaller than  $x$ ,

ceil : upper integer round, *i.e.*  $\text{ceil}(x) \stackrel{\text{def}}{=} \lceil x \rceil$  the smallest integer  $n \in \mathbb{Z}$  to be bigger than  $x$ ,

det : determinant, com : co-matrix, (MA102)

$I_n$  : identity matrix of size  $n \geq 0$ . (MA102)

$\delta_{i,j}$  : KRONECKER'S symbol,  $\delta_{i,j} : (k, l) \mapsto 1$  if  $k = i$  and  $l = j, 0$  otherwise, (MA102)

**Usual sets**

$\emptyset$  : **the empty set** (*it is unique*, even if included in any set),

$\{x : P(x)\}$  : set of all elements  $x$  satisfying verifying a certain property<sup>2</sup>  $P(x)$ , *e.g.*  $\{x \in \mathbb{R} : x \geq 0\} = \mathbb{R}_+$ ,

$\mathbb{N}$  : non-negative natural numbers,  $\mathbb{N} = \{0, 1, \dots, n-1, n, n+1, \dots\}$ . **Warning:** this set  $\mathbb{N}$  starts from 0,

$\mathbb{Z}$  : (ring of the) integer numbers,  $\mathbb{Z} = \{\dots, -n-1, -n, -n+1, \dots, -1, 0, 1, \dots, n-1, n, n+1, \dots\}$ ,

$\mathbb{Q}$  : (field of the) rational numbers,  $\mathbb{Q} \stackrel{\text{def}}{=} \left\{ \frac{p}{q}, (p, q) \in \mathbb{Z} \times \mathbb{N}^* \right\}$ ,

$\mathbb{R}$  : (field of the) real numbers,

$- [a, b]$  : segment (close interval), *i.e.*  $[a, b] \stackrel{\text{def}}{=} \{x \in \mathbb{R} : a \leq x \leq b\}$ ,

<sup>2</sup>If you are curious, this page on Wikipédia explains *how* and *why* we have to limit the expressiveness of this way to define set, otherwise we can conclude paradoxes, as the famous Russel's paradox.

- $(a, b)$  : open interval, *i.e.*  $(a, b) \stackrel{\text{def}}{=} \{x \in \mathbb{R} : a < x < b\}$ ,
- $[a, b)$  : half-open on the right interval, *i.e.*  $[a, b) \stackrel{\text{def}}{=} \{x \in \mathbb{R} : a \leq x < b\}$ ,
- $(a, b]$  : half-open on the left interval, *i.e.*  $(a, b] \stackrel{\text{def}}{=} \{x \in \mathbb{R} : a < x \leq b\}$ ,
- $\mathbb{C}$  : (field of the) complex numbers,  $\mathbb{C} \stackrel{\text{def}}{=} \{a + ib : a, b \in \mathbb{R}\}$ ,
- $\mathbb{K}$  : any (commutative) field, usually understood as “ $\mathbb{R}$  or  $\mathbb{C}$ ”,
- $E^n$  : product set, set of the  $n$ -tuples  $(x_1, \dots, x_n)$  of values of  $E$ , *i.e.*  $\{(x_1, \dots, x_n) : \forall 1 \leq i \leq n, x_i \in E\}$ , *e.g.*  $\mathbb{R}^2 \simeq \mathbb{C}$ ,
- $\mathbb{M}_{n,m}(A)$  : set of  $n \times m$ -sized matrices ( $n, m \in \mathbb{N}$ ), also written as  $M_{n,m}(A)$  or  $\mathcal{M}_{n,m}(A)$ , (MA102)

**Operators for sets**

- Card : cardinal of a set, *i.e.* number of its elements, also written  $\text{Card}(E) \stackrel{\text{def}}{=} |E| \stackrel{\text{def}}{=} (\text{also } \#E \text{ in some American books})$ ,
- $\mathcal{P}(E)$  : set of subsets of  $E$  (also written  $2^E$  in French books),
- $E^*$  : the set  $E$  without its zero<sup>3</sup> 0, *i.e.*  $E^* \stackrel{\text{def}}{=} E \setminus \{0\}$ ,
- $E_+$  : the subset of non-negative<sup>3</sup> elements of  $E$ , *i.e.*  $E_+ \stackrel{\text{def}}{=} E \cap [0, +\infty)$ ,
- $E_-$  : the subset of non-positive<sup>3</sup> elements of  $E$ , *i.e.*  $E_- \stackrel{\text{def}}{=} E \cap (-\infty, 0]$ ,
- $E^c$  : complementary set of  $E$  (**when this is not ambiguous**), *e.g.* in  $\{1, 2, 3, 4, 5\}$ ,  $\{1, 2, 5\}^c = \{3, 4\}$ ,
- $\delta E$  : frontier of the set  $E$ , *e.g.*  $\delta \mathcal{B}_2(0, 1) = \mathcal{C}_2(0, 1)$  (ball and sphere being centered at 0 and of radii 1, in 2D),
- $E^\circ$  : interior of the set  $E$ , *i.e.*  $E^\circ \stackrel{\text{def}}{=} E \setminus \delta E$  (also written  $\text{int}(E)$ ),
- $\bar{E}$  : cloture of  $E$ , *i.e.*  $\bar{E} \stackrel{\text{def}}{=} \left\{ \lim_{n \rightarrow +\infty} u_n, (u_n)_{n \in \mathbb{N}} \in E^{\mathbb{N}} \text{ being convergent} \right\}$ ,
- $d(x, y)$  : distance between two points  $x$  and  $y$ , usually  $d(x, y) = |x - y|$ ,
- $d(x, A)$  : distance from the point  $x$  to the subset  $A$  ( $\stackrel{\text{def}}{=} \inf_{a \in A} |x - a|$ ),
- $d(A, B)$  : distance from two subsets  $A$  and  $B$  ( $\stackrel{\text{def}}{=} \inf_{(a,b) \in A \times B} |a - b|$ ),
- $A + B$  : sum of two subsets  $A$  and  $B$  ( $= \{a + b : (a, b) \in A \times B\}$ ),
- $A - B$  : subtraction of two subsets  $A$  and  $B$  ( $= \{a - b : (a, b) \in A \times B\}$ ),
- $\text{diam}(A)$  : diameter of a subset  $A$ , *i.e.*  $\text{diam}(A) \stackrel{\text{def}}{=} \sup \{|x - y|, (x, y) \in A^2\}$ , (MA102)
- $\chi_A$  : characteristic function for  $A \subset E$ , *i.e.*  $E \longrightarrow \{0, 1\}, x \mapsto 1$  if  $x \in A$ , 0 otherwise,
- $\text{dim}(E)$  : dimension of a space  $E$  ( $\text{dim}_{\mathbb{K}}(A)$ ) : dimension of a subset  $A \subset E$ , as a  $\mathbb{K}$ -vectorial-space). (MA102)

**Orders and relationships**

- $<$  : (strictly) lower than, *e.g.*  $1 < \sqrt{2}$ ,
- $\leq$  : lower or equal than, *e.g.*  $1 \leq 1$ ,
- $>$  : (strictly) greater than, *e.g.*  $1 > e^{-1}$ ,
- $\geq$  : greater or equal than, *e.g.*  $2 \geq 2$ ,
- $\equiv$  : “same meaning”, or semantical identity for formulas :  $\neg(\neg P) \equiv P$ ,
- $\simeq$  : similar, *e.g.* for numerical approximations :  $\pi \simeq 3.14$  or  $\sqrt{2} \simeq 1.4142$ ,
- $\sim$  : numerically equivalent, for a limit, *e.g.*  $\frac{\sin(7x)}{x} \underset{x \rightarrow 0, x \neq 0}{\sim} 7$ .

---

<sup>3</sup>When this as a meaning.

**Other binary operators**

$\circ$  : composition of functions, if  $f : E \rightarrow F, g : F \rightarrow G$ , then  $f \circ g : E \rightarrow G, x \mapsto f(g(x))$ ,

$\mathbf{x} \cdot \mathbf{y}$  : inner product, also written  $\langle \mathbf{x}, \mathbf{y} \rangle$ , e.g.  $[1; 2; 3] \cdot [4; 5; 6] = 1 * 4 + 2 * 5 + 3 * 6 = 32$ ,

$\mathbf{x} \times \mathbf{y}$  : vectorial product, e.g.  $[1; 2; 3] \times [4; 5; 6] = [-3; 6; -6]$ ,

$|$  : divisibility (for integers, or polynomials), e.g.  $5 | 2015$ ,

$\nmid$  : non-divisibility (for integers, or polynomials), e.g.  $2014 \nmid 2015$ ,

$\text{. mod .}$  : used to write equality *modulo* another number, e.g.  $17 = 2 \text{ mod } 5$ .

**Usual unary operators**

$\mathcal{Re}$  : real part for a complex number, i.e.  $\mathcal{Re}(a + ib) = a$ , also written as  $\mathcal{Real}(z)$ ,

$\mathcal{Im}$  : imaginary part for a complex number, i.e.  $\mathcal{Im}(a + ib) = b$ , also written as  $\mathcal{Imag}(z)$ ,

sup : best upper-bound of a set (smaller upper-bound), **warning:** when it exists,

inf : best lower-bound of a set (bigger lower-bound), **warning:** when it exists,

lim : limit, usually we specify  $\lim_{x \rightarrow 0}$  to say “when  $x$  tends to 0”,

$\lim_{x \rightarrow 0, x \neq 0}$  : a special case<sup>4</sup>, which specifies that “ $x$  tends to 0 while being always non-zero”,

arg : **one** argument (for a complex number), e.g.  $\arg(e^{i\pi/4}) = \pi/4$  but  $\pi/4 + 2\pi$  is also a valid argument.

**Warning:** the formula  $\arg(x + iy) = \arctan \frac{y}{x}$  **is not always true!** (only true if  $x > 0$  in fact).

**Notations for the limits**

$\rightarrow$  : for a limit :  $n \rightarrow +\infty$  means that “ $n$  tends to  $+\infty$ ”,

$x \rightarrow a^+$  : means that  $x$  tends to  $a$  **by being greater than  $a$**  (i.e.  $x > a, x \rightarrow a$ ),

$x \rightarrow a^-$  : means that  $x$  tends to  $a$  **by being lower than  $a$**  (i.e.  $x < a, x \rightarrow a$ ),

$f(x^+)$  : limit of  $f$  at  $x$  **for smaller values**, i.e.  $f(x^+) \stackrel{\text{def}}{=} \lim_{y \rightarrow x^+} f(y)$ ,

$f(x^-)$  : limit of  $f$  at  $x$  **for bigger values**, i.e.  $f(x^-) \stackrel{\text{def}}{=} \lim_{y \rightarrow x^-} f(y)$ .

**Other notations**

$\bar{z}$  : conjugate of a complex number, i.e.  $\overline{a + ib} \stackrel{\text{def}}{=} a - ib$ ,

$|z|$  : absolute value of a number (real or complex), e.g.  $|a + ib| \stackrel{\text{def}}{=} \sqrt{a^2 + b^2}$  or  $|-2.73| = 2.73$ ,

$\mathbf{x}, \mathbf{y}$  : are for the vectors  $x, y$ . Sometimes also written as  $\vec{u}, \vec{v}$  etc,

$\|\mathbf{x}\|$  : norm of the vector  $\mathbf{x}$ , e.g.  $\|[1; 2; 3]\| = \sqrt{1^2 + 2^2 + 3^2} \simeq 3.741$ ,

$\|\cdot\|$  : unified notation for a norm. There is also the  $p$  norm:  $\|\cdot\|_p$ , and the infinite norm  $\|\cdot\|_{+\infty}$ ,

$(u_n)_{n \in \mathbb{N}}$  : the sequence of  $u_n \in E$ , i.e. the function  $u : \mathbb{N} \rightarrow E, n \mapsto u_n$ . A (bad) shortcut is  $\{u_n\}$ .

<sup>4</sup>It can be adapted to other situations, e.g.  $\lim_{(h,k) \rightarrow (0,0), (h,k) \neq (0,0)}$  for a limit when the vector  $(h, k)$  tends to  $(0, 0)$  but is never zero.

**Derivatives and differential calculus**

$f'$  : derivative<sup>5</sup> of  $f$ , e.g.  $\ln' : x \mapsto \frac{1}{x}, \forall x > 0$ . May be written  $\dot{x}$ , but *only in physics*,

$f''$  : second derivative of  $f$  (or  $\ddot{x}$  in physics),  $f'''$  third derivative,  $\dots$ ,  $f^{(n)}$   $n^{\text{th}}$  derivative for  $n \geq 0$ ,

$f^n$  :  $n^{\text{th}}$  power of  $f$  for  $n \geq 0$  (**be careful, it is not** a derivative  $f^{(n)}$ !),

$\frac{\partial f}{\partial x_j}$  : partial derivative of  $f$  for the  $j^{\text{th}}$  coordinate, also written as  $f_{x_j}$ . e.g. for  $f(x, y)$ , we have  $f_x$  and  $f_y$ ,

$\nabla f$  : gradient of  $f$  (also written **grad** $f$  or  $\overrightarrow{\text{grad}f}$ ),

$\text{div} f$  : divergence of  $f$ .

**Integrals**

$\int_a^b f(x)dx$  : integral of the function  $f : x \mapsto f(x)$  on  $(a, b)$ ,

$\int f$  : integral of the function  $f$  on its domain (**when it is not ambiguous**), or a primitive of  $f$  (i.e. an *anti-derivative*),

$\oint f(x)dx$  : curvilinear integral of  $f$ , (MA102)

$\iint_{(a,b) \times (c,d)} f(x, y)dx dy$  : double integral of the 2-variables function  $f : (a, b) \times (c, d), (x, y) \mapsto f(x, y)$ . (MA102)

**Some rare notation for functions spaces**

$\mathcal{F}(E, F)$  : functions from  $E$  to  $F$ , i.e.  $F^E$ ,

$\mathcal{C}(E, F)$  : continuous functions from  $E$  to  $F$ ,

$\mathcal{C}(E)$  : continuous functions from  $E$  to  $E$ ,

$\mathcal{C}_b(E, F)$  : bounded and continuous functions from  $E$  to  $F$ ,

$\mathcal{C}^k(E, F)$  :  $k$ -time differentiable functions, of  $k^{\text{th}}$  derivatives being continuous, from  $E$  to  $F$  ( $k \in \mathbb{N}$ ),

$\mathcal{C}^\infty(E, F)$  : infinitely differentiable functions, from  $E$  to  $F$ ,

$\mathcal{L}(E, F)$  : linear applications from  $E$  to  $F$ , (MA102)

$\mathcal{L}(E)$  : linear applications from  $E$  to  $E$ , (MA102)

$\mathcal{L}_c(E, F)$  : continuous and linear applications from  $E$  to  $F$ . (MA102)

**Want more?** A list of mathematical symbols : [http://en.wikipedia.org/wiki/Table\\_of\\_mathematical\\_symbols](http://en.wikipedia.org/wiki/Table_of_mathematical_symbols), on Wikipédia, and some others lists on mathematical notations, on Wikipédia.

---

<sup>5</sup>If it exists!