NB: This document sums up as much symbols, notations and conventions used for the Maths 101 course as possible. If something is missing, please [tell us].

**Shortcuts**

*i.e.* : *id est* “that is”,

e.g.: *exempli gratia* “for example” (does not come from “example given”).

**Basic notations for functions**

\( f(x) \): the *value* of the function \( f \) at the point \( x \),

\( f(A) \): the *image set* of \( A \) by \( f \), i.e. \( \{ f(x) : x \in A \} \),

\( f(B) \): the *pre-image set* of \( B \) by \( f \), i.e. \( \{ y \in B : \exists x \in A / f(x) = y \} \),

\( \mapsto \): to specify the domain for a function, e.g. \( \sin : \mathbb{R} \mapsto \mathbb{R} \),

\( f |_A \): the restriction of the function \( f : E \mapsto F \) to the subset \( A \subset E \).

**Shortcuts for sums and products**

\( \sum_{k=1}^{n} f(k) \): sum \( f(1) + f(2) + \cdots + f(n-1) + f(n) \) for \( n \geq 1 \),

\( \prod_{k=1}^{n} g(k) \): product \( g(1) \times g(2) \times \cdots \times g(n-1) \times g(n) \) for \( n \geq 1 \),

\( \bigotimes_{k=1}^{n} E_k \): Cartesian product for spaces \( E_1 \times E_2 \times \cdots \times E_{n-1} \times E_n \) for \( n \geq 1 \),

\( \bigoplus_{k=1}^{n} E_k \): direct sum for spaces \( E_1 \oplus E_2 \oplus \cdots \oplus E_{n-1} \oplus E_n \) for \( n \geq 1 \).

**Logical binary predicates**

\( \mathsf{def} \Rightarrow \): implies, e.g. \( (x > 0) \Rightarrow (x \geq 0) \),

\( \Leftrightarrow \): is equivalent to, i.e. implies and is implied by,

\( \neg \): logical negation, e.g. \( \neg(x > 0) \equiv x \leq 0 \).

**Binary predicates for set**

\( \in \): belongs to, e.g. \( n \in \mathbb{N} \),

\( \notin \): do not belong to, e.g. \( -1 \notin \mathbb{R}^+ \),

\( \subseteq \): is included in, e.g. \( \mathbb{N} \subseteq \mathbb{R} \),

\( \supseteq \): contains, e.g. \( \mathbb{Z} \supseteq \{-1, 0, 1\} \),

\( \subseteq \): to precise that inclusion can be an equality, at the opposite of \( \supseteq \) which exclude the equality,

\( \setminus \): difference of two sets, e.g. \( \mathbb{R} \setminus \mathbb{R}^- = \mathbb{R}^+ \),

\( \cap \): intersection of two sets (also \( \bigcap \)), e.g. \( \mathbb{C} \cap \mathbb{Z} = \mathbb{Z} \),

\( \cup \): union of two sets (also \( \bigcup \)), e.g. \( \{0\} \cup \mathbb{N}^* = \mathbb{N} \),

\( \sqcup \): disjunct union of two sets (also \( \biguplus \)), e.g. \( \mathbb{R}^- \sqcup \mathbb{R}^+ \) (also \( \sqcup \) or \( \biguplus \)),

\( E_1 \times E_2 \): Cartesian product of two sets \( E_1 \) and \( E_2 \), i.e. \( \{(x, y) : x \in E_1, y \in E_2\} \),

\( E_1 \uplus E_n \): direct sum of two sets \( E_1, E_2 \subset E \), i.e. \( \{x + y : x \in E_1, y \in E_2\} \), where \( E_1 \cap E_2 = \{0_E\} \).

\( ^1 \)It is latin.

Please, report any issue to MA101@crans.org
Usual constants and functions

\pi : (pi), usually defined by the quotient of any circle perimeter by its diameter,

\e : base of the exponential function, i.e. \( \e^1 = \exp(1) \),

\(+\infty\) : positive infinite, \(-\infty\) negative infinite, \(\infty\) infinite,

\text{sign} : sign function, i.e. \( \text{sign} : \mathbb{R} \dashrightarrow \mathbb{R}, 0 \mapsto 0, x \mapsto 1 \) if \( x > 0 \), \(-1 \) if \( x < 0 \),

\exp : exponential function, i.e. \( \exp : \mathbb{R} \dashrightarrow \mathbb{R}, x \mapsto \exp(x) = \e^x \),

\ln : Neperian logarithm function, i.e. \( \ln : \mathbb{R}^+ \dashrightarrow \mathbb{R}, x \mapsto \ln(x) \),

cos : cosine, \sin : sinus, \tan : tangent (\( \mathbb{R} \dashrightarrow \mathbb{R} \)),

\text{arccos} : arc cosine, \text{arcsin} : arc sinus, \text{arctan} : arc tangent (reciprocal),

cosh : hyperbolic cosine, \sinh : hyperbolic sinus, \tanh : hyperbolic tangent (\( \mathbb{R} \dashrightarrow \mathbb{R} \)),

\text{arcosh} : arc hyperbolic cosine, \text{arcsinh} : arc hyperbolic sinus, \text{arctanh} : arc hyperbolic tangent (reciprocal),

det : determinant, com : comatrix,

\( \det \), \( \text{com} \),

\E : integer part (lower), i.e. \( E(x) \coloneqq \max\{n \in \mathbb{Z}, n \leq x\} \),

floor : lower integer round, i.e. \( \text{floor}(x) \coloneqq \lfloor x \rfloor = E(x) \),

ceil : upper integer round, i.e. \( \text{ceil}(x) \coloneqq \lceil x \rceil = E(x) + 1 \),

\( \delta_{i,j} \) : KRONECKER's symbol, \( \delta_{i,j} : (k,l) \mapsto 1 \) if \( k = i \) and \( l = j \), \( 0 \) otherwise,

\gamma : usually, EULER's constant,

\( \Gamma(x) \) : usually, EULER's Gamma function,

\( I_n \) : identity matrix of size \( n \geq 0 \).

Usual sets

\emptyset : the empty set (it is unique, even if included in any set),

\{ x : P(x) \} : set of all elements \( x \) satisfying verifying a certain property \({}^2 P(x) \), e.g. \( \{ x \in \mathbb{R} : x \geq 0 \} = \mathbb{R}_+ \),

\mathbb{N} : non-negative natural numbers, \( \mathbb{N} = \{0,1,\ldots,n-1,n,n+1,\ldots\} \),

\mathbb{Z} : (ring of the) integer numbers, \( \mathbb{Z} = \{\ldots,n-1,-n,-n+1,\ldots,-1,0,1,\ldots,n-1,n,n+1,\ldots\} \),

\mathbb{Q} : (field of the) rational numbers, \( \mathbb{Q} \coloneqq \left\{ \frac{p}{q} \mid (p,q) \in \mathbb{Z} \times \mathbb{N}^* \text{ being convergeant} \right\} \),

\mathbb{R} : (field of the) real numbers, \( \mathbb{R} \coloneqq \left\{ \lim_{n \to \infty} r_n, (r_n)\in\mathbb{N} \text{ being convergeant} \right\} \),

- \([a,b] \) : segment (close interval), i.e. \( [a,b] \coloneqq \{ x \in \mathbb{R} : a \leq x \leq b \} \),

- \((a,b)\) : open interval, i.e. \( (a,b) \coloneqq \{ x \in \mathbb{R} : a < x < b \} \),

- \([a,b)\) : half-open on the right interval, i.e. \( [a,b) \coloneqq \{ x \in \mathbb{R} : a \leq x < b \} \),

- \((a,b]\) : half-open on the left interval, i.e. \( (a,b] \coloneqq \{ x \in \mathbb{R} : a < x \leq b \} \),

\mathbb{C} : (field of the) complex numbers, \( \mathbb{C} \coloneqq \{ a + ib : a,b \in \mathbb{R} \} \),

\mathbb{K} : any (commutative) field, usually understood as “\( \mathbb{R} \) or \( \mathbb{C} \)”,

\( E^n \) : product set, set of the \( n \)-tuples of \( E \), i.e. \( \{ (x_1,\ldots,x_n) : \forall 1 \leq i \leq n, x_i \in E \} \), e.g. \( \mathbb{R}^2 \cong \mathbb{C} \),

\( M_{n,m}(A) \) : set of \( n \times m \)-sized matrices \( (n,m \in \mathbb{N}) \), also written as \( M_{n,m}(A) \),

\(^2\)If you are curious, this page on Wikipédia explains how and why we have to limit the expressiveness of this way to define set.
Operators for sets

Card : cardinal of a set, i.e. number of its elements, also written \( \text{Card}(E) \overset{\text{def}}{=} |E| \overset{\text{def}}{=} \#E \),

\( \mathcal{P}(E) : \) set of subsets of \( E \) (also written \( 2^E \)),

\( E^* : \) the set \( E \) without its zero \( \overset{\text{def}}{=} \{0\} \), i.e. \( E^* \overset{\text{def}}{=} E \setminus \{0\} \),

\( E_+ : \) the subset of non-negative elements of \( E \), i.e. \( E_+ \overset{\text{def}}{=} E \cap [0, +\infty) \),

\( E_- : \) the subset of non-positive elements of \( E \), i.e. \( E_- \overset{\text{def}}{=} E \cap (-\infty, 0] \),

\( E^c : \) complementary set of \( E \) (when this is not ambiguous),

\( \delta E : \) frontier of the set \( E \), e.g. \( \delta B_2(0, 1) = C_2(0, 1) \) (ball and sphere being centered at 0 and of radii 1, in 2D),

\( E^{\circ} : \) interior of the set \( E \), i.e. \( E^{\circ} \overset{\text{def}}{=} E \setminus \delta E \) (also written \( \text{int}(E) \)),

\( \overline{E} : \) clouture of \( E \), i.e. \( \overline{E} \overset{\text{def}}{=} \left\{ \lim_{n \to \infty} u_n \mid (u_n)_{n \in \mathbb{N}} \text{ being convergent} \right\} \),

\( d(x, y) : \) distance between two points \( x \) and \( y \), usually \( d(x, y) = |x - y| \),

\( d(x, A) : \) distance from the point \( x \) to the subset \( A \) (\( \overset{\text{def}}{=} \inf_{a \in A} |x - a| \)),

\( d(A, B) : \) distance from two subsets \( A \) and \( B \) (\( \overset{\text{def}}{=} \inf_{(a, b) \in A \times B} |a - b| \)),

\( A + B : \) sum of two subsets \( A \) and \( B \) (\( \overset{\text{def}}{=} \{a + b : (a, b) \in A \times B\} \)),

\( A - B : \) subtraction of two subsets \( A \) and \( B \) (\( \overset{\text{def}}{=} \{a - b : (a, b) \in A \times B\} \)),

\( \text{diam}(A) : \) diameter of a subset \( A \), i.e. \( \text{diam}(A) \overset{\text{def}}{=} \sup \{ |x - y|, (x, y) \in A^2 \} \),

\( \chi_A : \) characteristic function for \( A \subset E \), i.e. \( E \mapsto [0, 1], x \mapsto 1 \) if \( x \in A \), 0 otherwise,

\( \text{dim}(E) : \) dimension of a space \( E \), \( \text{dim}_K(A) : \) dimension of a subset \( A \subset E \), has a \( K \)-vectorial-space.

Orders and relationships

\(< : \) (strictly) lower than, e.g. \( 1 < \sqrt{2} \),

\( \leq : \) lower or equal than, e.g. \( 1 \leq 1 \),

\( > : \) (strictly) greater than, e.g. \( 1 > e^{-1} \),

\( \geq : \) greater or equal than, e.g. \( 2 \geq 2 \),

\( = : \) “same meaning”, or semantical identity for formulas : \( \neg(\neg P) = P \),

\( \simeq : \) similar, e.g. for numerical approximations : \( \pi \simeq 3.14 \),

\( \sim : \) numerically equivalent, for a limit, e.g. \( \frac{\sin(7x)}{x} \xrightarrow{x \to 0, x/\pi \to 0} 7 \).

Other binary operators

\( \circ : \) composition of functions, if \( f : E \leftrightarrow F, g : F \leftrightarrow G \), then \( f \circ g : E \leftrightarrow G, x \mapsto f(g(x)) \),

\( \cdot, \cdot : \) inner product, also written \( \langle \cdot, \cdot \rangle \), e.g. \( [1; 2; 3] \cdot [4; 5; 6] = 1 \cdot 4 + 2 \cdot 5 + 3 \cdot 6 = 32 \),

\( \times \times : \) vectorial product, e.g. \( [1; 2; 3] \times [4; 5; 6] = [-3; 6; -6] \),

\( | : \) divisibility (for integers, or polynomials), e.g. \( 1007 \div 2014 \),

\( \not| : \) non-divisibility (for integers, or polynomials), e.g. \( 2013 \not| 2014 \),

\( \mod : \) used to write equality modulo another number, e.g. \( 17 = 2 \mod 5 \).
Usual unary operators

\( \mathcal{R} \) : real part for a complex number, i.e. \( \mathcal{R}(a + ib) = a \),
\( \mathcal{I} \) : imaginary part for a complex number, i.e. \( \mathcal{I}(a + ib) = b \),
sup : best upper-bound of a set (smaller upper-bound),
inf : best lower-bound of a set (bigger lower-bound),
lim : limit, usually by precising \( \lim_{x \to 0} \) to say “when \( x \) tends to 0”,
lim sup : upper limit (also written \( \lim \), or sometimes \( \lim \) ),
lim inf : lower limit (also written \( \lim \), or sometimes \( \lim \ ) ),
arg : one argument (for a complex number), e.g. \( \arg \) \( p \) is also a valid argument.

Notations for the limits
\( \xrightarrow[]{} \) : for a limit : \( \xrightarrow[]{\infty} \) means that “\( x \) tends to \( \infty \)”, \( \xrightarrow[]{\infty} \) : means that \( x \) tends to \( \infty \) by being greater than \( \infty \) (i.e. \( \infty \)), \( \xrightarrow[]{\infty} \) : means that \( x \) tends to \( \infty \) by being lower than \( \infty \) (i.e. \( \infty \)),
\( p \xrightarrow[]{} q \) : limit of \( x \) at \( \infty \) for smaller values, i.e. \( p \xrightarrow[]{} q \) \( \overset{\text{def}}{=} \lim_{x \to \infty} f(y) \),
\( p \xrightarrow[]{} q \) : limit of \( x \) at \( \infty \) for bigger values, i.e. \( p \xrightarrow[]{} q \) \( \overset{\text{def}}{=} \lim_{x \to \infty} f(y) \).

Other notations
\( g^+ \) : non-negative part of \( g \), i.e. \( g^+ \overset{\text{def}}{=} \max\{g(x), 0\} \),
\( g^- \) : non-positive part of \( g \), i.e. \( g^- \overset{\text{def}}{=} \min\{g(x), 0\} \),
\( \overline{z} \) : conjugate of a complex number, i.e. \( \overline{a + ib} \overset{\text{def}}{=} a - ib \),
|\( z \)| : absolute value of a number (real or complex), e.g. \( |a + ib| \overset{\text{def}}{=} \sqrt{a^2 + b^2} \) or \( |2.73| = 2.73 \),
\( x, y \) : vector \( \vec{x}, \vec{y} \) etc.,
|\( x \)| : norm of the vector \( x \), e.g. \( |[1; 2; 3]| = \sqrt{1^2 + 2^2 + 3^2} \approx 3.741 \),
\( ||.|| \) : unified notation for a norm, \( ||.||_p \) : norme \( p \), \( ||.||_{\infty} \) : infinite norm,
\( (u_n)_{n \in \mathbb{N}} \) : the sequence of \( u_n \in E \), i.e. the function \( u : \mathbb{N} \xrightarrow[]{} E, n \mapsto u_n \).

Derivatives and differential calculus
\( f' \) : derivative of \( f \), e.g. \( \ln'(x) = \frac{1}{x} \), \( \forall x > 0 \). May be written \( x \) (in physics),
\( f'' \) : second derivative of \( f \) (or \( \dot{x} \) in physics), \( f''' \) third derivative, \( \ldots \), \( f^{(n)} \) \( n^{th} \) derivative for \( n \geq 0 \),
\( f^n \) : \( n^{th} \) power of \( f \) for \( n \geq 0 \) (be careful, it is not a derivative \( f^{(n)} \)),
\( D_j f \) : partial derivative of \( f \) for the \( j^{th} \) coordinate,
\( \nabla f \) : gradient of \( f \) (also written \( \text{Grad} f \)),
\( \text{Div } f \) : divergence of \( f \).
Integrals
\[
\int_a^b f(x)\,dx \quad \text{integral of the function } f : x \mapsto f(x) \text{ on } (a,b),
\]
\[
\int f \quad \text{integral of the function } f \text{ on its domain (when it is not ambiguous)},
\]
\[
\oint f(x)\,dx \quad \text{curvilinear integral of } f,
\]
\[
\iint_{(a,b)\times(c,d)} f(x,y)\,dxdy \quad \text{double integral of the 2-variables function } f : (a,b) \times (c,d) \mapsto f(x,y).
\]

Usual functions spaces
\[
\mathcal{F}(E,F) \quad \text{functions from } E \text{ to } F, \text{ i.e. } F^E,
\]
\[
\mathcal{C}(E,F) \quad \text{continuous functions from } E \text{ to } F,
\]
\[
\mathcal{C}(E) \quad \text{continuous functions from } E \text{ to } E,
\]
\[
\mathcal{C}_b(E,F) \quad \text{bounded and continuous functions from } E \text{ to } F,
\]
\[
\mathcal{C}^k(E,F) \quad k\text{-time differentiable functions, of } k^{\text{th}} \text{ derivatives being continuous, from } E \text{ to } F \text{ (} k \in \mathbb{N} \text{)},
\]
\[
\mathcal{C}^\infty(E,F) \quad \text{infinitely differentiable functions, from } E \text{ to } F,
\]
\[
\mathcal{L}(E,F) \quad \text{linear applications from } E \text{ to } F,
\]
\[
\mathcal{L}(E) \quad \text{linear applications from } E \text{ to } E.
\]
\[
\mathcal{L}_c(E,F) \quad \text{continuous and linear applications from } E \text{ to } F,
\]

Want more? A list of mathematical symbols: [en.wikipedia.org/wiki/Table_of_mathematical_symbols](https://en.wikipedia.org/wiki/Table_of_mathematical_symbols) on Wikipédia, and some others lists on mathematical notations, on Wikipédia.