

## Some simple exercises.

1. Solve (in  $\mathbb{R}$ ) this equation  $x^2 + 3x + 3 = 0$ .
2. Find two complex numbers  $a$  and  $b$  such that for any complex  $z$ , different from 0 and  $-1$ , we have:  $\frac{1}{z(z+1)} = \frac{a}{z} + \frac{b}{z+1}$  (this is an example of decomposition into simple elements).
3. Solve this differential equation  $y' - 3y = 0$ .
4. Find the definition domain and **one** primitive of these functions:
  - (i)  $f : \mathbb{R} \longleftrightarrow \mathbb{R}, x \mapsto x^3 - 3$ ,
  - (ii)  $g : \mathbb{R}_+^* \longleftrightarrow \mathbb{R}, x \mapsto \ln(x) - 3$  (where  $\mathbb{R}_+^*$  is a shortcut for  $\mathbb{R} \cap (0, +\infty)$ ),
  - (iii)  $h : \mathbb{R} \longleftrightarrow \mathbb{R}, x \mapsto x^2 e^x$ .
5. What can you say regarding the behavior of this function, defined on  $\mathbb{R}$  by  $f(x) = e^{-x}$ , when  $x$  tends to  $+\infty$ ? Same question for the function  $g(x) = \frac{e^x}{x}$ , defined on  $\mathbb{R}^*$  ( $\mathbb{R}^* \stackrel{\text{def}}{=} \mathbb{R} \setminus \{0\}$ ).
6. Give the (non-negative) integer divisors of 24? Same question for 63, 125, et 2014.
7. Compute this integral  $\int_0^\pi \sin(2x) dx$ .
8. Let consider the real sequence  $(u_n)_{n \in \mathbb{N}}$ , defined by  $u_0 = 3$  and  $u_{n+1} = 5u_n + 3$  for any integer  $n$ . Explicit  $(u_n)_{n \in \mathbb{N}}$  (*i.e.* explicit the general term  $u_n$ ).
9. We call a *perfect number* a (integer) number equals to the *half* of the sum of its non-negative divisors. Could you find one? And a second one (if you have the time)?
10. Compute<sup>1</sup> this sum:  $\sum_{k=1}^{k=n} \frac{1}{k(k+1)}$ , for any positive integer number  $n > 0$ ?
11. How many *prime*<sup>2</sup> numbers is there? You should try to prove this **very rigorously**.
12. If  $\theta \in \mathbb{R}$  and  $n \in \mathbb{N}$ , compute this sum:  $\sum_{k=0}^{k=n} \cos(k\theta)$ .

<sup>1</sup>The well-known formula  $\frac{1}{k(k+1)} = \frac{1}{k} - \frac{1}{k+1}$  (for any  $k/ = 0$ ) might be useful...

<sup>2</sup>We recall that an integer number  $p \geq 2$  is *prime* if its only non-negative divisors are 1 and  $p$ .

13. If  $\theta \in \mathbb{R}$  et de  $n \in \mathbb{N}$ , compute this product:  $\prod_{k=0}^{k=n} 2 \cos(2^k \theta)$ .
14. We consider this differential equals  $E: y'(x) + y(x) = x$  (where the unknown is  $y$ , a differentiable function from  $\mathbb{R}$  to  $\mathbb{R}$ ). Can you give all the solutions of  $E$ ? Prove that there is an unique solution  $y_0$  which takes the value 0 in 0 (i.e.  $y_0$  satisfying  $E$  and  $y_0(0) = 0$ ). Could you give an explicit form of this peculiar solution  $y_0$ ?

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**Remark 1.** Remember that the symbols  $\sum$  and  $\prod$  are used to write sums and products in a short way.

That means:  $\sum_{k=1}^{k=n} f(k)$  is a shortcut for the sum  $f(1) + f(2) + \dots + f(n-1) + f(n)$  for any  $n \geq 1$ , and  $\prod_{k=1}^{k=n} g(k)$  is a shortcut for the product  $g(1) \times g(2) \times \dots \times g(n-1) \times g(n)$  for any  $n \geq 1$ .

**Remark 2.** Usually, the index  $k$  is only specified once, for the first value ( $k = 1$ ):

$\sum_{k=1}^{k=n} f(k) = \sum_{k=1}^n f(k)$ . And when it nos ambiguous, one can forget to precise the

name of the index in the first value:  $\sum_{k=1}^n f(k) = \sum_1^n f(k)$ .