

Roll No.:

Section:

**Consigns:** You should be extra cautious about small mistakes, try to explain as much as possible, and do not forget to justify every statements and every steps in your answers.

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### Ex. 1: Corner Circle

A unit circle is placed against a right angle.

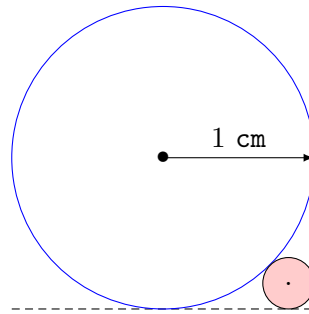


Figure 1: Corner Circle

What is radius of the smaller circle (in cm) ?

**Answer:**

### Ex. 2: Cubes and Multiples of 7

Prove that for any number that is not a multiple of seven, its cube is one more or one less than a multiple of seven.

Formally, this would be written as:  $\forall n \in \mathbb{N}^*, 7 \nmid n \stackrel{\text{def}}{\implies} 7 \mid n^3 \pm 1$ .

**Answer:**

### Ex. 3: About continuity

Let  $f$  be a function from an interval  $[a, b]$  to  $\mathbb{R}$  (with  $a, b \in \mathbb{R}, a < b$ ), and  $x_0 \in [a, b]$ .

1. Recall the *formal* definition of  $f$  being continuous at this point  $x_0$  (as a pure formula with variables, well introduced with quantifiers  $\forall$  and  $\exists$ ).
2. If  $f$  is continuous at  $x_0$ , does this mean  $f$  is differentiable at  $x_0$ ?  
If yes, (try to) prove it. If not, (try to) give a counter-example.

**Answer:**