

Name: \_\_\_\_\_

Roll Number: \_\_\_\_\_

Section: \_\_\_\_\_

**Note:** Every question carries 1 mark. (You can use the back of the paper for rough work.)

As last time, **no need to justify your choice**, and for a multiple choice question **only one** (A, B, C or D) is correct.

• **Q.1)** Is the real sequence  $u = (u_n)_{n \in \mathbb{N}}$  defined by  $u_n = \frac{1}{\sqrt{5}} \left( \frac{1 + \sqrt{5}}{2} \right)^n$  convergent?

A) No,  $u_n \xrightarrow{n \rightarrow +\infty} +\infty$ C) No,  $u_n$  is not even correctly defined!B) Yes, and it converges to  $\frac{1}{\sqrt{5}}$ D) No,  $u_n \xrightarrow{n \rightarrow +\infty} -\infty$ 

• **Q.2)** Let  $S$  be the series sum of the terms  $a_n = \frac{1}{n(n+3)}$  for  $n > 0$ , ie  $S = \sum_{n=1}^{+\infty} a_n = \sum_{n=1}^{+\infty} \frac{1}{n(n+3)}$ . Is  $S$  convergent?

A) Yes (a series of positive terms always converges!)

D) Yes (thanks to the comparison test, the inequality  $a_n < \frac{1}{n^2}$  and because  $\sum_{n=1}^{+\infty} \frac{1}{n^2}$  converges)B) Yes (obvious, just because  $a_n \xrightarrow{n \rightarrow +\infty} 0$ )C) No ( $S$  will be infinite =  $+\infty$  because  $a_n > 0$ )

**Bonus: (for +2 marks)** Can you quickly (but carefully) compute the value of the sum of the series?

$$\sum_{n=1}^{+\infty} \frac{1}{n(n+3)} = ?$$

• **Q.3)** What are the solutions of this 1<sup>st</sup> order differential equation  $y' + 2y = 0$ ?

• **Q.4)** For this non-homogeneous equation  $y' + 2y = x^2$ , we add the initial value  $y(1) = 0$ . Which one of these functions is a solution?

A)  $y(x) = \frac{1}{2}x^2 - \frac{1}{2}x + 2$

C)  $y(x) = \frac{1}{2}x^2 + 4e^{-2x}$

B)  $y(x) = \frac{1}{2}x^2 - \frac{1}{2}x + \frac{1}{4} - \frac{1}{4}e^{2-2x}$

D)  $y(x) = \frac{1}{4} + \frac{7}{4}e^2e^{-2x}$

• **Q.5)** What are the solution(s) of this homogeneous 2<sup>nd</sup> order differential equation  $y'' + 2y' - 8y = 0$ ?

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As last time, **no need to justify your choice**, and for a multiple choice question **only one** (A, B, C or D) is correct.

- **Q.1)** What are the solution(s) of this homogeneous 2<sup>nd</sup> order differential equation  $y'' - 2y' - 8y = 0$ ?

- **Q.2)** For this non-homogeneous equation  $y' + 4y = -x^2$ , we ask the initial value  $y(1) = 0$ . Which one of these functions is a solution?

- A)  $y(x) = -\frac{1}{4}x^2 + \frac{1}{8}x - \frac{1}{32} + \frac{5}{32}e^{4-4x}$       C)  $y(x) = -\frac{1}{2}x^2 + \frac{1}{4}x - \frac{1}{16} + \frac{5}{16}e^{4(x-1)}$   
 B) there is no solution for this initial value problem!      D)  $y(x) = 2e^{-4x}$

- **Q.3)** Let  $S$  be the series sum of the terms  $a_n = \frac{1}{n(n+3)}$  for  $n > 0$ , ie  $S = \sum_{n=1}^{+\infty} a_n = \sum_{n=1}^{+\infty} \frac{1}{n(n+3)}$ . Is  $S$  convergent?

- A) No ( $S$  will be infinite =  $+\infty$  because  $a_n > 0$ )      C) Yes (obvious, just because  $a_n \xrightarrow{n \rightarrow +\infty} 0$ )  
 B) Yes (thanks to the comparison test, the inequality  $a_n < \frac{1}{n^2}$  and because  $\sum_{n=1}^{+\infty} \frac{1}{n^2}$  converges)      D) Yes (a series of positive terms always converges!)

**Bonus: (for +2 marks)** Can you quickly (but carefully) compute the value of the sum of the series?

$$\sum_{n=1}^{+\infty} \frac{1}{n(n+3)} = ?$$

- **Q.4)** Is the real sequence  $u = (u_n)_{n \in \mathbb{N}}$  defined by  $u_n = \frac{1}{\sqrt{5}} \left( \frac{1 + \sqrt{5}}{2} \right)^n$  convergent?

- A) No,  $u_n$  is not even correctly defined!      C) Yes, and it converges to  $\frac{1}{\sqrt{5}}$   
 B) No,  $u_n \xrightarrow{n \rightarrow +\infty} -\infty$       D) No,  $u_n \xrightarrow{n \rightarrow +\infty} +\infty$

- **Q.5)** What are the solution(s) of this homogeneous 1<sup>st</sup> order differential equation  $y' + 4y = 0$ ?